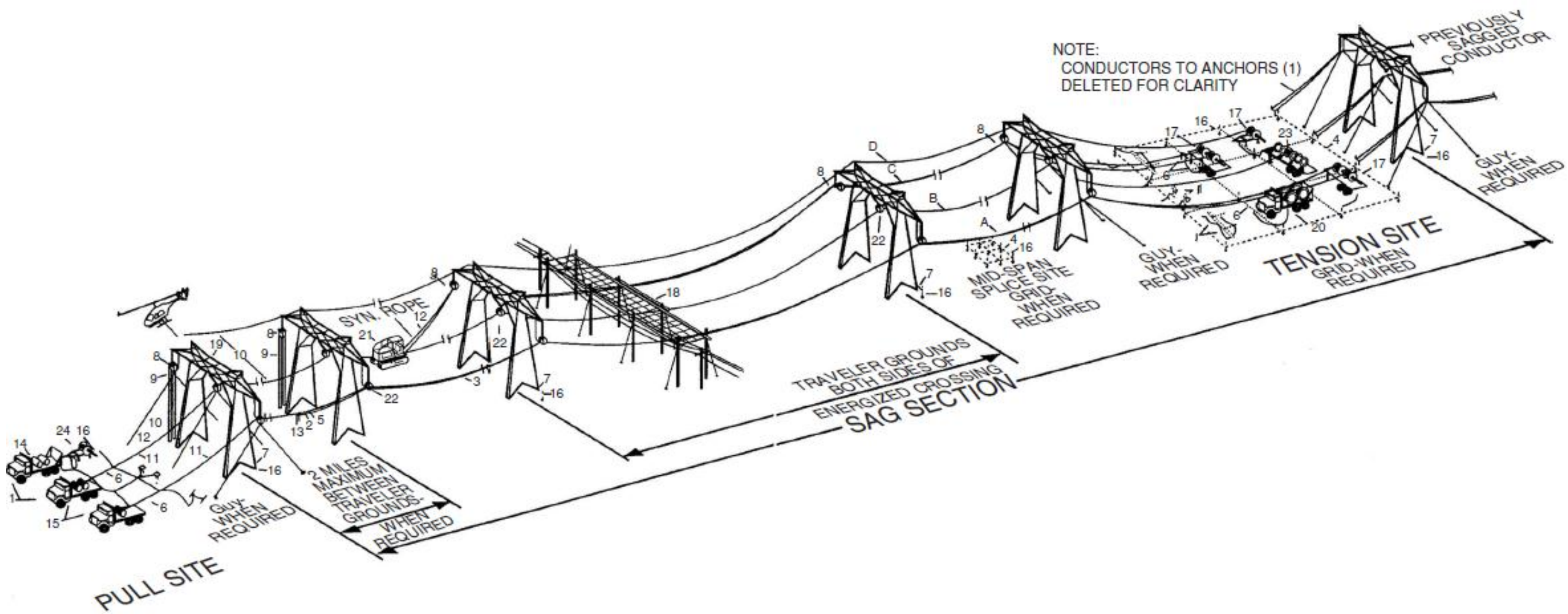
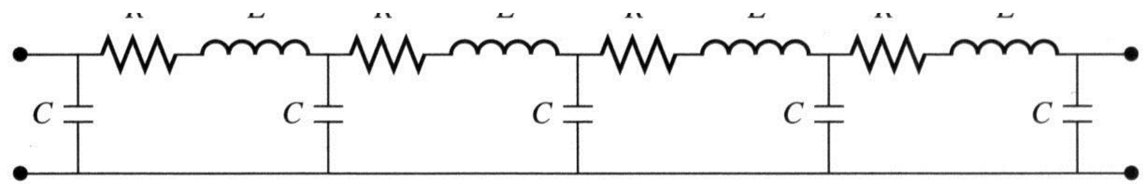
# ELG4125: Power Transmission Lines

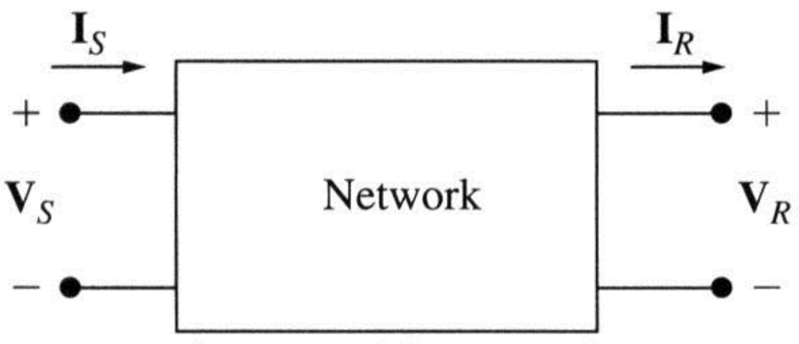
## Steady State Operation



# Two-Port Networks and ABCD Models

A transmission line can be represented by a two-port network, that is a network that can be isolated from the outside world by two connections (ports) as shown:.




If the network is linear, an elementary circuits theorem (analogous to Thevenin's theorem) establishes the relationship between the sending and receiving end voltages and currents as

$$V_s = AV_R + BI_R$$

$$I_s = CV_R + DI_R$$

Constants *A* and *D* are dimensionless; constant *B* has units of Ω, and constant *C* is measured in siemens. These constants are sometimes referred to as generalized circuit constants, or *ABCD* constants.

# *ABCD* Model

- The ABCD constants can be physically interpreted. Constant $A$ represents the effect of a change in the receiving end voltage on the sending end voltage; and constant $D$ models the effect of a change in the receiving end current on the sending end current. Naturally, both constants $A$ and $D$ are dimensionless.

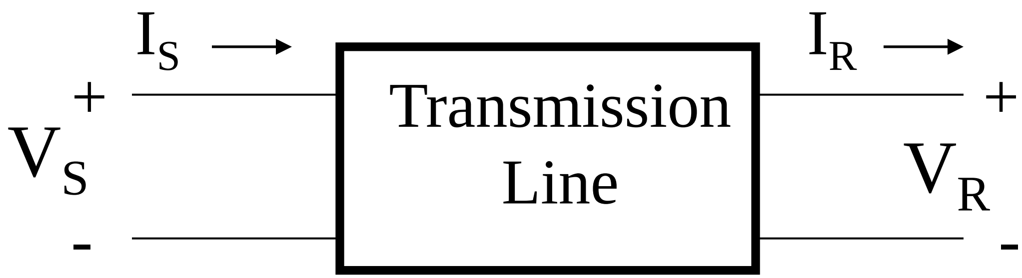
- The constant $B$ represents the effect of a change in the receiving end current on the sending end voltage. The constant $C$ denotes the effect of a change in the receiving end voltage on the sending end current.

- Transmission lines are two-port linear networks, and they are often represented by ABCD models. For the short transmission line model, $I_S = I_R = I$, and the *ABCD* constants are

# Transmission Matrix Model

Oftentimes we're only interested in the terminal characteristics of the transmission line. Therefore we can model it as a "black box".

With $\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$




With $\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$

Use voltage/current relationships to solve for A,B,C,D

$$V_S = V_R \cosh \gamma l + Z_c I_R \sinh \gamma l$$

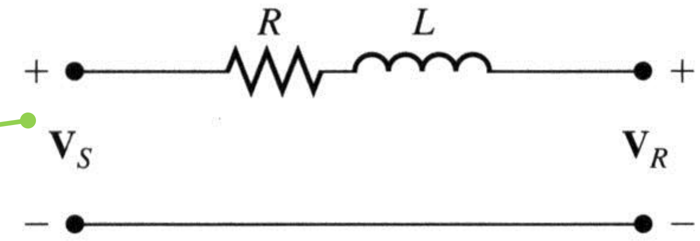
$$I_S = I_R \cosh \gamma l + \frac{V_R}{Z_c} \sinh \gamma l$$

$$\mathbf{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_c \sinh \gamma l \\ \frac{1}{Z_c} \sinh \gamma l & \cosh \gamma l \end{bmatrix}$$

Prof. Tom Overbye

# Short Transmission Line

The per-phase equivalent circuit of a short line




$V_S$ and $V_R$ are the sending and receiving end voltages; $I_S$ and $I_R$ are the sending and receiving end currents. Assumption of no line admittance leads to

$$I_s = I_R$$

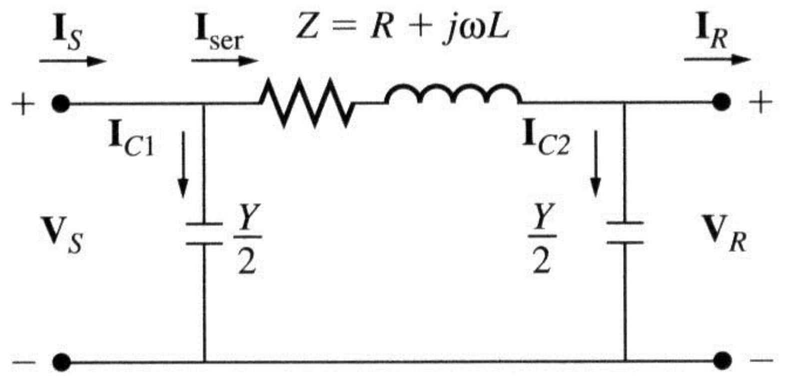
We can relate voltages through the Kirchhoff's voltage law

$$V_R = V_s - RI - jX_L I$$

which is very similar to the equation derived for a synchronous generator.

# Medium-Length Transmission Line

Considering medium-length lines (50 to 150 mile-long), the shunt admittance must be included in calculations. However, the total admittance is usually modeled ($\pi$ model) as two capacitors of equal values (each corresponding to a half of total admittance) placed at the sending and receiving ends.




The current through the receiving end capacitor can be found as

$$I_{C2} = V_R \frac{Y}{2}$$

And the current through the series impedance elements is

$$I_{ser} = V_R \frac{Y}{2} + I_R$$

# Medium-Length Transmission Line

(Between 80 km and 250 km)

From the Kirchhoff's voltage law, the sending end voltage is:

$$V_s = ZI_{ser} + V_R = \left(\frac{YZ}{2} + 1\right)V_R + ZI_R$$

The source current will be

$$V_s = I_{c1} + I_{ser} = I_{c1} + I_{c2} + I_R = Y\left(\frac{YZ}{2} + 1\right)V_R + (\frac{ZY}{2} + 1)I_R$$

Therefore, the *ABCD* constants of a medium-length transmission line are:

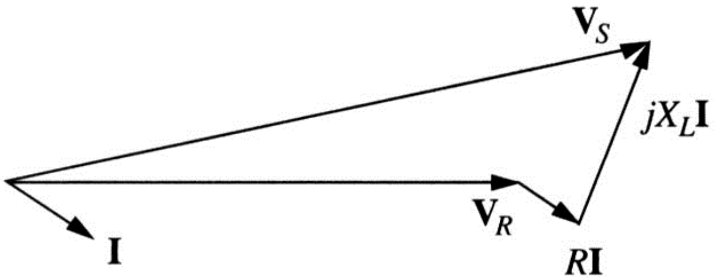
If the shunt capacitance of the line is ignored, the *ABCD* constants are the constants used for a short transmission line.

$$A = \frac{ZY}{2} + 1 \qquad B = Z \qquad C = Y(\frac{ZY}{4} + 1) \qquad D = \frac{ZY}{2} + 1$$

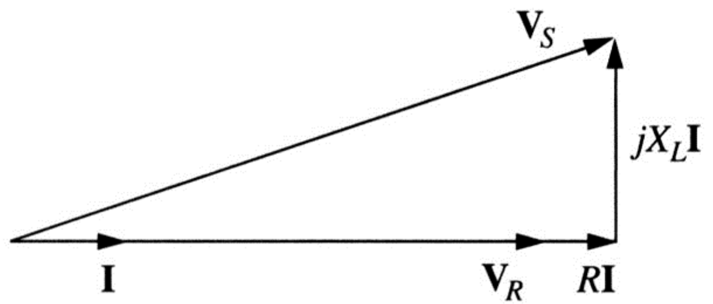
# Short Transmission Line: Phasor Diagram

AC voltages are usually expressed as phasors.

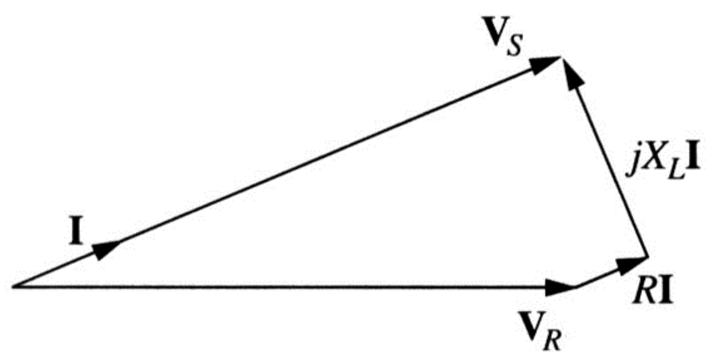
Load with lagging power factor.




Load with unity power factor.




Load with leading power factor.



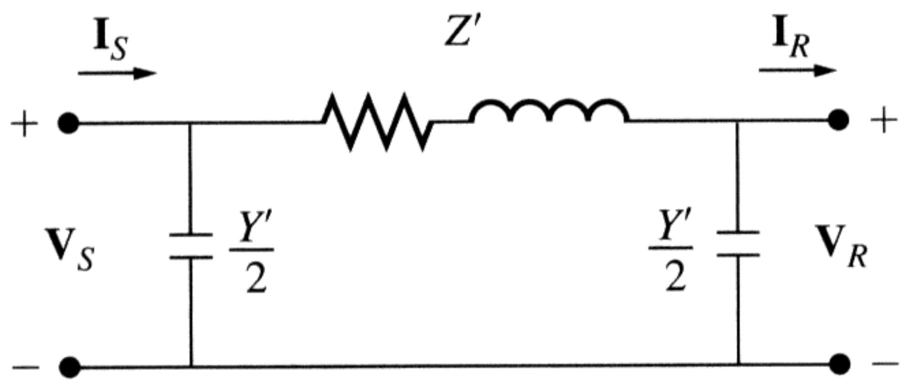

For a given source voltage $V_S$ and magnitude of the line current, the received voltage is lower for lagging loads and higher for leading loads.

# Long Transmission Line

## Lines of length above 250 km and voltage above 100 kV

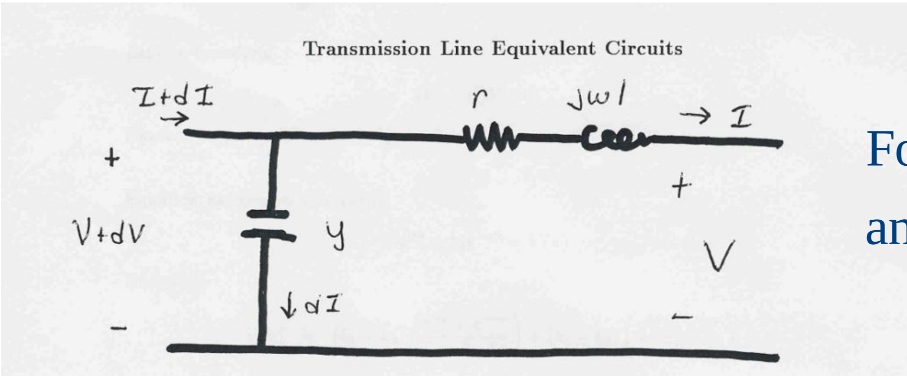
For long lines, it is not accurate enough to approximate the shunt admittance by two constant capacitors at either end of the line. Instead, both the shunt capacitance and the series impedance must be treated as **distributed quantities**; the voltages and currents on the line should be found by solving **differential equations** of the line.

It is possible to model a long transmission line as a $\pi$ model with a modified series impedance Z' and a modified shunt admittance Y' and to perform calculations on that model using ABCD constants. The modified values of series impedance and shunt admittance are:




$$Z' = Z\,\frac{\sinh \gamma d}{\gamma d} \qquad Y' = Y\,\frac{\tanh(\gamma d/2)}{\gamma d/2}$$

# Derivation of Voltage and Current Relationships




For operation at frequency $\omega$, let z = r + j$\omega$L and y = g +j$\omega$C (with g usually equal 0)

We can then derive the following relationships:

$$dV = I\ z\ dx$$

$$dI = (V + dV)y\ dx \approx V\ y\ dx$$

$$\frac{dV(x)}{dx} = zI \qquad \frac{dI(x)}{dx} = yV$$

$$\frac{dV(x)}{dx} = zI \qquad \frac{dI(x)}{dx} = yV$$

We can rewrite these two, first order differential equations as a single second order equation

$$\frac{d^2V(x)}{dx^2} = z\frac{dI(x)}{dx} = zyV$$

$$\frac{d^2V(x)}{dx^2} - zyV = 0$$

# Voltage and Current Relationships

Define the propagation constant $\gamma$ as

$$\gamma = \sqrt{yz} \;=\; \alpha + j\beta$$

where

$$\alpha \;=\; \text{the attenuation constant}$$

$$\beta \;=\; \text{the phase constant}$$

Use the Laplace Transform to solve. System has a characteristic equation

$$(s^2 - \gamma^2) = (s-\gamma)(s+\gamma) \;=\; 0$$

The general equation for V is

$$V(x) \;=\; k_1 e^{\gamma x} + k_2 e^{-\gamma x}$$

Which can be rewritten as

$$V(x) \;=\; (k_1 + k_2)\left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) + (k_1 - k_2)\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)$$

Let $\mathrm{K}_1 = k_1 + k_2$ and $\mathrm{K}_2 = k_1 - k_2$. Then

$$V(x) \;=\; K_1\left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) + K_2\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)$$

$$\;=\; K_1 \cosh(\gamma x) + K_2 \sinh(\gamma x)$$

# Long Transmission Line

*Z* is the series impedance of the line; *Y* is the shunt admittance of the line; *d* is the length of the line; $\gamma$ is the propagation constant of the line:

$$\gamma = \sqrt{yz}$$

where *y* is the shunt admittance per kilometer and *z* is the series impedance per km.

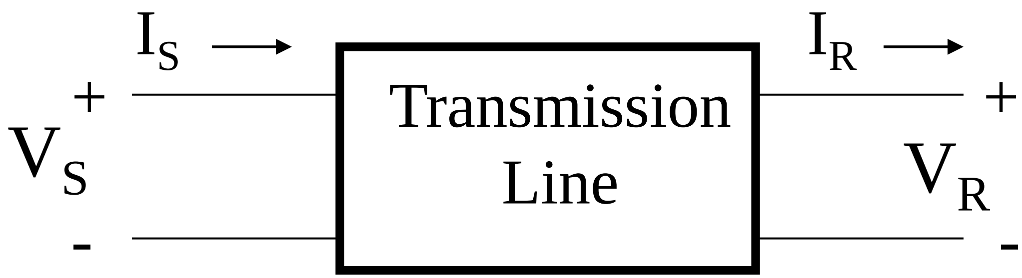
As $\gamma d$ gets small, the ratios approach 1.0 and the model becomes a medium-length line model. The ABCD constants for a long transmission line are

$$A = \frac{Z'Y'}{2} + 1 \qquad B = Z' \qquad C = Y'(\frac{Z'Y'}{4} + 1) \qquad D = \frac{Z'Y'}{2} + 1$$

# Transmission Matrix Model

Oftentimes we're only interested in the terminal characteristics of the transmission line. Therefore we can model it as a "black box".

With $$\begin{bmatrix} \mathrm{V_S} \\ \mathrm{I_S} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$




With $$\begin{bmatrix} \mathrm{V_S} \\ \mathrm{I_S} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Use voltage/current relationships to solve for A,B,C,D

$$V_S = V_R \cosh \gamma l + Z_c I_R \sinh \gamma l$$

$$I_S = I_R \cosh \gamma l + \frac{V_R}{Z_c} \sinh \gamma l$$

$$\mathbf{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_c \sinh \gamma l \\ \frac{1}{Z_c} \sinh \gamma l & \cosh \gamma l \end{bmatrix}$$

# Equivalent Circuit Parameters

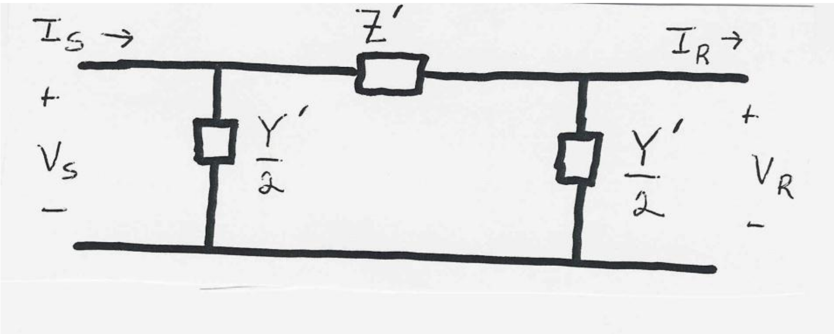
$$\frac{V_S - V_R}{Z'} - V_R \frac{Y'}{2} = I_R$$

$$V_S = \left(1 + \frac{Z'Y'}{2}\right) V_R + Z' I_R$$

$$I_S = V_S \frac{Y'}{2} + V_R \frac{Y'}{2} + I_R$$

$$I_S = Y'\left(1 + \frac{Z'Y'}{4}\right) V_R + \left(1 + \frac{Z'Y'}{2}\right) I_R$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z'Y'}{2} & Z' \\ Y'\left(1 + \frac{Z'Y'}{4}\right) & \left(1 + \frac{Z'Y'}{2}\right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$




We now need to solve for Z' and Y'. Using the B element solving for Z' is straightforward

$$B = Z_C \sinh \gamma l = Z'$$

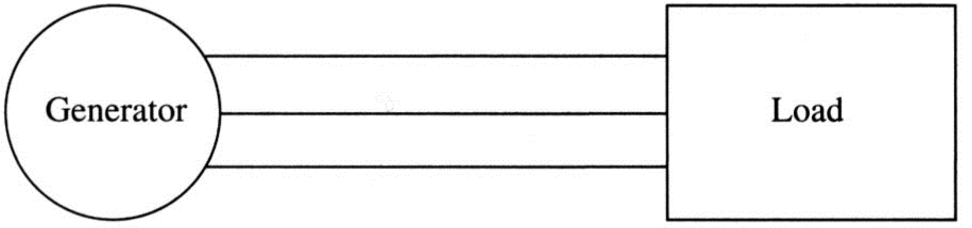
Then using A we can solve for Y'

$$A = \cosh \gamma l = 1 + \frac{Z'Y'}{2}$$

$$\frac{Y'}{2} = \frac{\cosh \gamma l - 1}{Z_c \sinh \gamma l} = \frac{1}{Z_c} \tanh \frac{\gamma l}{2}$$

# Transmission Line Characteristics

In real overhead transmission lines, the line reactance $X_L$ is normally much larger than the line resistance $R$; therefore, the line resistance is often neglected. We consider next some important transmission line characteristics.




Assuming that a single generator supplies a single load through a transmission line, we consider consequences of increasing load.

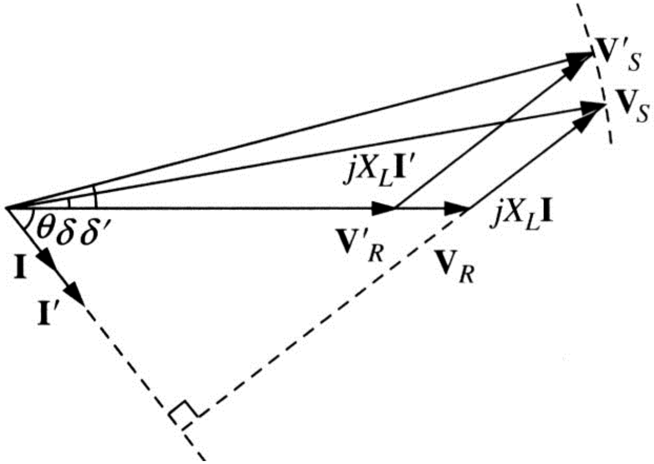
Assuming that the generator is ideal, an increase of load will increase a real and (or) reactive power drawn from the generator and, therefore, the line current, while the voltage and the current will be unchanged.

1. If more load is added with the same lagging power factor, the magnitude of the line current increases but the current remains at the same angle $\theta$ with respect to $V_R$ as before.

# Transmission Line Characteristics

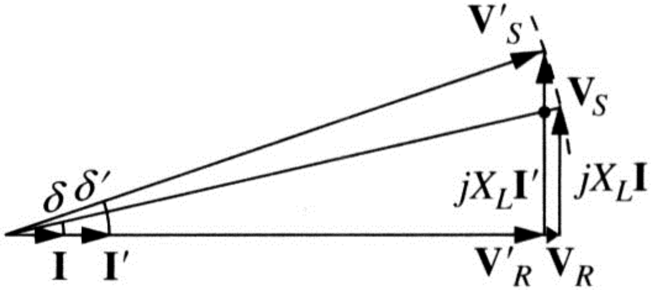
The voltage drop across the reactance increases but stays at the same angle.

Assuming zero line resistance and remembering that the source voltage has a constant magnitude:



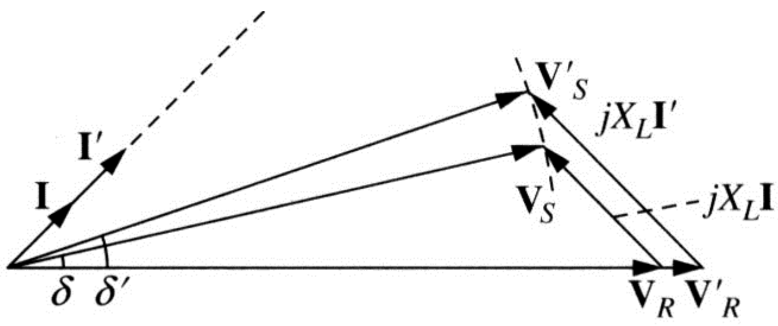

voltage drop across reactance $jX_LI$ will stretch between $V_R$ and $V_S$.

2. An increase in a unity PF load, on the other hand, will slightly decrease the received voltage at the end of the transmission line.



# Transmission Line Characteristics

3. Finally, an increase in a load with leading PF increases the received (terminal) voltage of the transmission line.




- If lagging (inductive) loads are added at the end of a line, the voltage at the end of the transmission line **decreases significantly** – large positive VR.
- If unity-PF (resistive) loads are added at the end of a line, the voltage at the end of the transmission line **decreases slightly** – small positive VR.
- If leading (capacitive) loads are added at the end of a line, the voltage at the end of the transmission line **increases** – negative VR.

The **voltage regulation** of a transmission line is

$$V_R = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\%$$

where $V_{nl}$ and $V_{fl}$ are the no-load and full-load voltages at the line output.

# Transmission Line Characteristics

The real power input to a 3-phase transmission line can be computed as

$$P_{in} = 3V_s I_s cos\theta_s$$

where $V_S$ is the magnitude of the source (input) line-to-neutral voltage and $V_{LL,S}$ is the magnitude of the source (input) line-to-line voltage. Note that Y-connection is assumed! Similarly, the real output power from the transmission line is

$$P_{out} = 3V_R I_R cos\theta_R$$

The reactive power input to a 3-phase transmission line can be computed as

$$Q_{in} = 3V_s I_s sin\theta_s$$

# Transmission Line Characteristics

And the reactive output power is

$$Q_{out} = 3V_R I_R sin\theta_R$$

The apparent power input to a 3-phase transmission line can be computed as

$$S_{in} = 3V_s I_s$$

And the apparent output power is

$$S_{out} = 3V_R I_R$$

# Transmission Line Characteristics

The efficiency of the transmission line is

$$\text{Efficiency} = \frac{P_{out}}{P_{in}} \times 100\%$$

Several practical constrains limit the maximum real and reactive power that a transmission line can supply. The most important constrains are:

The maximum steady-state current must be limited to prevent the overheating in the transmission line. The power lost in a line is approximated as

$$P_{loss} = 3I^2R$$

The voltage drop in a practical line should be limited to approximately 5%. In other words, the ratio of the magnitude of the receiving end voltage to the magnitude of the sending end voltage should be

$$\frac{V_R}{V_s} \leq 0.95$$

The angle $\delta$ in a transmission line should typically be $\leq 30^0$ ensuring that the power flow in the transmission line is well below the static stability limit and, therefore, the power system can handle transients.

# Line Voltage and Line Current

$$V(x) = K_1 \cosh(\gamma x) + K_2 \sinh(\gamma x)$$

$$V(0) = V_R = K_1 \cosh(0) + K_2 \sinh(0)$$

$$\text{Since } \cosh(0) = 1 \,\&\, \sinh(0) = 0 \Rightarrow K_1 = V_R$$

$$\frac{dV(x)}{dx} = zI = K_1 \gamma \sinh(\gamma x) + K_2 \gamma \cosh(\gamma x)$$

$$\Rightarrow K_2 = \frac{zI_R}{\gamma} = \frac{I_R\, z}{\sqrt{yz}} = I_R \sqrt{\frac{z}{y}}$$

$$V(x) = V_R \cosh(\gamma x) + I_R Z_c \sinh(\gamma x)$$

$$\text{where } \mathrm{Z_c} = \sqrt{\frac{z}{y}} = \text{characteristic impedance}$$

By similar reasoning we can determine I(x)

$$I(x) = I_R \cosh(\gamma x) + \frac{V_R}{Z_c} \sinh(\gamma x)$$

where x is the distance along the line from the receiving end.

Define transmission efficiency as $\eta = \dfrac{P_{out}}{P_{in}}$

# Transmission Line Example

Assume we have a 765 kV transmission line with a receiving end voltage of 765 kV(line to line), a receiving end power $S_R = 2000 + j1000$ MVA and

$$z = 0.0201 + j0.535 = 0.535\angle 87.8^\circ \ \Omega/\text{mile}$$

$$y = j7.75\times 10^{-6} = 7.75\times 10^{-6}\angle 90.0^\circ \ \square/mile$$

Then

$$\gamma = \sqrt{zy} = 2.036\ \angle 88.9^\circ\ /\ \text{mile}$$

$$Z_c = \sqrt{\frac{z}{y}} = 262.7\ \angle\text{-}1.1^\circ\ \Omega$$

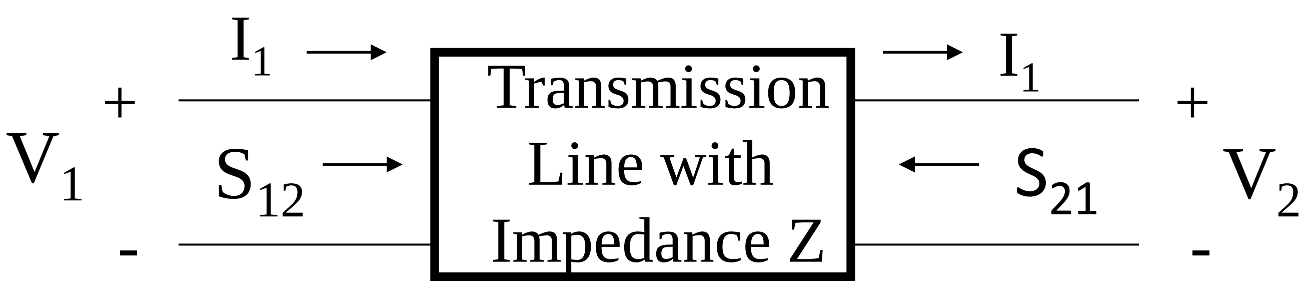
Do per phase analysis, using single phase power and line to neutral voltages. Then

$$V_R = 765/\sqrt{3} = 441.7\angle 0^\circ \text{ kV}$$

$$I_R = \left[\frac{(2000+j1000)\times 10^6}{3\times 441.7\angle 0^\circ\times 10^3}\right]^* = 1688\angle -26.6^\circ \text{ A}$$

$$\begin{aligned} V(x) &= V_R\cosh(\gamma x) + I_R Z_c \sinh(\gamma x) \\ &= 441{,}700\angle 0^\circ \cosh(x\times 2.036\angle 88.9^\circ) + \\ &\quad 443{,}440\angle -27.7^\circ \times \sinh(x\times 2.036\angle 88.9^\circ) \end{aligned}$$

# Power Transfer




$$S_{12} = V_1 I_1^* = V_1 \left( \frac{V_1 - V_2}{Z} \right)^*$$

with $V_1 = |V_1| \angle \theta_1, \quad V_2 = |V_2| \angle \theta_2 \quad Z = |Z| \angle \theta_Z$

$$S_{12} = \frac{|V_1|^2}{|Z|} \angle \theta_Z - \frac{|V_1||V_2|}{|Z|} \angle \theta_Z + \theta_{12}$$

If we assume a line is lossless with impedance jX and are just interested in real power transfer then:

$$P_{12} + jQ_{12} = \frac{|V_1|^2}{|Z|} \angle 90^\circ - \frac{|V_1||V_2|}{|Z|} \angle 90^\circ + \theta_{12}$$

Since $-\cos(90^\circ + \theta_{12}) = \sin \theta_{12}$, we get

$$P_{12} = \frac{|V_1||V_2|}{X} \sin \theta_{12}$$

Hence the maximum power transfer is

$$P_{12}^{Max} = \frac{|V_1||V_2|}{X}$$

# Surge Impedance Loading

- The surge impedance loading or SIL of a transmission line is the MW loading of a transmission line at which a natural reactive power balance occurs. Transmission lines produce reactive power (Mvar) due to their natural capacitance. The amount of Mvar produced is dependent on the transmission line's capacitive reactance ($X_c$) and the voltage (kV) at which the line is energized. In equation form the Mvar **produced** is:

$$\mathrm{MVAr} = \frac{(\mathrm{kV})^2}{X_c}$$

Transmission lines also utilize reactive power to support their magnetic fields. The magnetic field strength is dependent on the magnitude of the current flow in the line and the line's natural inductive reactance (XL). It follows then that the amount of Mvar used by a transmission line is a function of the current flow and inductive reactance. In equation form the Mvar **used** by a transmission line is:

$$\mathrm{MVAr} = I^2 X_L$$

- A transmission line's surge impedance loading or SIL is simply the MW loading (at a unity power factor) at which the line's Mvar usage is equal to the line's Mvar production. In equation form we can state that the SIL occurs when:

$$I^2 X_L = \frac{(\mathrm{kV})^2}{X_c}$$

$$X_L X_c = \frac{(\mathrm{kV})^2}{I^2}$$

$$\frac{2\pi f L}{2\pi f C} = \sqrt{\frac{V^2}{I^2}}$$

The Surge Impedance $$\frac{V}{I} = \sqrt{\frac{L}{C}}$$

The concept of a surge impedance is more readily applied to telecommunication systems than to power systems. However, we can extend the concept to the power transferred across a transmission line. The surge impedance loading or SIL (in MW) is equal to the voltage squared (in kV) divided by the surge impedance (in ohms). In equation form:

$$\text{SIL (MW)} = \frac{(V_{LL})^2}{\text{Surge Impedance}}$$

This formula that the SIL is dependent only on the kV the line is energized at and the line's surge impedance. The line length is not a factor in the SIL or surge impedance calculations. Therefore the SIL is not a measure of a transmission line's power transfer capability as it does not take into account the line's length nor does it consider the strength of the local power system.

The value of the SIL to a system operator is realizing that when a line is loaded above its SIL it acts like a shunt reactor - absorbing Mvar from the system and when a line is loaded below its SIL it acts like a shunt capacitor - supplying Mvar to the system.