Introduction to the Power Flow Problem

Introduction

- It is possible to analyze the performance of power systems both in normal operating conditions and under fault (short-circuit) condition. The analysis in normal steady-state operation is called a **power-flow study** (load-flow study) with an objective to determine the voltages, currents, and real and reactive power flows in a system under a given load conditions.
- The **purpose** of power flow studies is to plan ahead and account for various hypothetical situations. For instance, what if a transmission line within the power system properly supplying loads must be taken off line for maintenance. Can the remaining lines in the system handle the required loads without exceeding their rated parameters?



Power Flow Approach

- A **power-flow study** (**load-flow study**) is an analysis of the voltages, currents, and power flows in a power system under steady-state conditions. In such a study, we make an assumption about either a voltage at a bus or the power being supplied to the bus for each bus in the power system and then determine the magnitude and phase angles of the bus voltages, line currents, etc. that would result from the assumed combination of voltages and power flows.
- The simplest way to perform power-flow calculations is by iteration:
 - 1. Create a bus admittance matrix Y_{bus} for the power system;
 - 2. Make an initial estimate for the voltages at each bus in the system;
 - 3. Update the voltage estimate for each bus (one at a time), based on the estimates for the voltages and power flows at every other bus and the values of the bus admittance matrix: since the voltage at a given bus depends on the voltages at all of the other busses in the system (which are just estimates), the updated voltage will not be correct. However, it will usually be closer to the answer than the original guess.
 - 4. Repeat this process to make the voltages at each bus approaching the correct answers closer and closer.

Load Flow Problem

- Load flow calculations are used to determine the voltage, current, and real and reactive power at various points in a power system under normal steady-state conditions.
- For power systems with a large number of buses, the load flow problem becomes computationally intensive. Therefore, for large power systems, the load flow is solved using specific programs based on iterative techniques, such as the Newton-Raphson method.
- Power systems of smaller size, however, require considerably less computational effort, and load flow algorithms can be developed which function easily on personal computers.
- The approach used here for solving the load flow is based on the Newton-Raphson iterative method. The required input to the problem is the generated and load power at each bus and the voltage magnitude on generating buses.
- This information is acquired from load data and the normal system operating conditions. The solution provides the voltage magnitude and phase angle at all buses and the power flows and losses of the transmission lines.

Load Flow Problem

For load flow calculations, the system buses are classified into three types:

- The slack bus: There is only one such bus in the system. Due to losses in the network, the real and reactive power cannot be known at all buses. Therefore, the slack bus will provide the necessary power to maintain the power balance in the system. The slack bus is usually a bus where generation is available. For this bus, the voltage magnitude and phase angle are specified (normally the voltage phase angle is set to zero degrees). The voltage phase angle of all other buses is expressed with the slack bus voltage phasor as reference.
- **The generating or PV-bus:** This bus type represents the generating stations of the system. The information known for PV-buses is the net real power generation and bus-voltage magnitude. The net real power generation is the generated real power minus the real power of any local load.
- **The load or PQ-bus:** For these buses, the net real and reactive power is known. PQ-buses normally do not have generators. However, if the reactive power of a generator reaches its limit, the corresponding bus is treated as a PQ-bus. This is equivalent to adjusting the bus voltage until the generator reactive power falls within the prescribed limits.
- Distribution substations and feeders may be treated as generating buses in distribution networks.

Power Flow Analysis

- We know:
 - The system topology (the circuit diagram)
 - The impedance of each line
 - The load at each load bus (S = P + jQ)
 - The capability of each generator (P, V)
- We want to know:
 - The output of each generator (S)
 - The voltage at each bus ($\mathbf{V} = \mathbf{V}\angle \mathbf{\theta}$)
 - The power flow on each line (P_{flow})

Running a Power Flow Program

- We may use popular simulation programs like:
 - MATPower
 - Tabular data input and output
 - Relatively easy to use
 - Powerworld
 - Visual
 - More difficult to use
- Terminology:
 - One-line diagrams
 - Per unit system (normalize all values).

Using the Power Flow Simulation Tool



Power System Diagram

One-line diagram



Transmission lines are shown as a single line

Power System Diagram



Load Models

- The ultimate goal is to supply loads with electricity at constant **frequency** and **voltage**.
- Electrical characteristics of individual loads matter, but usually they can only be estimated
 - Actual loads are constantly changing, consisting of a large number of individual devices,
 - Only limited network observability of load characteristics
- Aggregate models are typically used for analysis
- Two common models:
 - Constant power: $S_i = P_i + jQ_i$
 - Constant impedance: $S_i = |V|^2 / Z_i$

Generator Models

- Engineering models depend on the application.
- Generators are usually synchronous machines: Exception is wind generators,
- For the generator, two models may be used:
 - A steady-state model, treating the generator as a constant power source operating at a fixed voltage; this model will be used for power flow and economic analysis.
 - A short term model treating the generator as a constant voltage source behind a possibly time-varying reactance.

Gauss-Seidel Iterative Technique

- Slow iterative problem-solving technique
- Use a full matrix
- Require a large number of processors
- Comparably slow convergence rate
- Matrix is sparse and can't be inverted

$$V_{k}(i+1) = \frac{1}{Y_{kk}} \left[\frac{P_{k} - jQ_{k}}{V_{k}^{*}(i)} - \sum_{n=1}^{k-1} Y_{kn}V_{n}(i+1) - \sum_{n=k+1}^{N} Y_{kn}V_{n}(i) \right]$$
$$Q_{k} = \operatorname{Im} \left\{ \mathbf{V}_{k} \sum_{j=1}^{N} \mathbf{Y}_{jk}^{*} \mathbf{V}_{j}^{*} \right\}$$

An iterative procedure in which a correction for each of the voltages of the network is computed in one step, and the corrections applied all at once is called **Gaussian** Iteration. If, on the other hand, the improved variables are used immediately, the procedure is called Gauss–Seidel Iteration.

Newton-Raphson Technique

- Solve non-square and nonlinear problems
- Relatively high iteration
- Require a good initial guess of the solution
- Active power cost optimization
- Active power loss minimization
- Minimum control-shift
- Minimum number of controls rescheduled
- Very complicated
- Lead to partial derivations of nonlinear complex
- It involves Jacobian -J complex form
- It has relatively slow convergence time.

Comparison of Power Flow Methods

	Newton		Jacobi		Gauss-Seidel	
Bus #	Iter. #	time [s]	Iter. #	time [s]	Iter. #	time [s]
14	4	0.0050	76	0.0217	56	0.0288
118	5	0.0287	580	0.505	388	2.738
1228	5	0.210	454	5.120	224	112.4
11856	4	3.15	340	399.0	173	9112

Types of Busses

The equations used to update the estimates differ for different types of busses. Each bus in a power system can be classified to one of three types:

- Load Bus (PQ Bus): A buss at which the real and reactive power are specified, and for which the bus voltage will be calculated. Real and reactive powers supplied to a power system are defined to be positive, while the powers consumed from the system are defined to be negative. All busses having no generators are load busses.
- Generator bus/Voltage Controlled (PV Bus): A bus at which the magnitude of the voltage is kept constant by adjusting the field current of a synchronous generator on the bus (as we learned, increasing the field current of the generator increases both the reactive power supplied by the generator and the terminal voltage of the system). We assume that the field current is adjusted to maintain a constant terminal voltage V_T. We also know that increasing the prime mover's governor set points increases the power that generator supplies to the power system. Therefore, we can control and specify the magnitude of the bus voltage and real power supplied.
- Slack Bus (or Swing Bus): A special generator reference bus serving as the reference bus for the power system. Its voltage is assumed to be fixed in both magnitude and phase (for instance, 1∠0° per unit). The real and reactive powers are uncontrolled: the bus supplies whatever real or reactive power is necessary to make the power flows in the system balance.

In Practice

- In practice, a voltage on a load bus may change with changing loads. Therefore, load busses have specified values of P and Q, while V varies with load conditions.
- Real generators work most efficiently when running at full load. Therefore, it is desirable to keep all but one (or a few) generators running at 100% capacity, while allowing the remaining (swing) generator to handle increases and decreases in load demand. Most busses with generators will supply a fixed amount of power and the magnitude of their voltages will be maintained constant by field circuits of generators. These busses have specific values of P and $|V_i|$.
- The controls on the swing generator will be set up to maintain a constant voltage and frequency, allowing *P* and *Q* to increase or decrease as loads change.

Y_{bus} for the Power Flow

The most common approach to power-flow analysis is based on the bus admittance matrix Y_{bus} . However, this matrix is slightly different from the one studied previously since the internal impedances of generators and loads connected to the system are not included in Y_{bus} . **Example:** Consider a simple power system with 4 busses, 5 transmission lines, 1 generator, and 3 loads. Series per-unit impedances are given in the following table

lin e #	Bus to bus	Series Z (<u>pu</u>)	Series Y (<u>pu</u>)
1	1-2	0.1+j0.4	0.5882-j2.3529
2	2-3	0.1+j0.5	0.3846-j1.9231
3	2-4	0.1+j0.4	0.5882-j2.3529
4	3-4	0.5+j0.2	1.1765-j4.7059
5	4-1	0.5+j0.2	1.1765-j4.7059



- The shunt admittances of the transmission lines are ignored. In this case, the Y_{ii} terms of the bus admittance matrix can be constructed by summing the admittances of all transmission lines connected to each bus, and the Y_{ij} ($i \neq j$) terms are just the negative of the line admittances stretching between busses *i* and *j*. Therefore, for instance, the term Y_{11} will be the sum of the admittances of all transmission lines 1 and 5, so $Y_{11} = 1.7647 j7.0588$ per unit.
- If the shunt admittances of the transmission lines are not ignored, the self admittance Y_{ii} at each bus would also include half of the shunt admittance of each transmission line connected to the bus.
- The term Y_{12} will be the negative of all the admittances stretching between bus 1 and bus 2, which will be the negative of the admittance of transmission line 1, so $Y_{12} = -0.5882 + j2.3529$.

The Power-Flow Problem

- The starting point for a power-flow problem is a single-line diagram of the power system, from which the input data for computer solutions can be obtained. Input data consist of bus data, transmission line data, and transformer data.
- As shown in the following figure, four variables are associated with each bus k: voltage magnitude V_k , phase angle δ_k , net real power P_k , and reactive power Q_k supplied to the bus.
- At each bus, two of these variables are specified as input data, and the other two are unknowns to be computed by the power-flow program.

$$\mathsf{P}_{\mathsf{k}} = \mathsf{P}_{\mathsf{GK}} - \mathsf{P}_{\mathsf{LK}}$$

$$\mathbf{Q}_{\mathbf{k}} = \mathbf{Q}_{\mathbf{GK}} - \mathbf{Q}_{\mathbf{LK}}$$

Bus Variables



Bus Admittance Matrix or Y_{bus}

- First step in solving the power flow is to create what is known as the **bus admittance matrix**, also called the **Y**_{bus}.
- The Y_{bus} gives the relationships between all the bus current injections, I, and all the bus voltages, V; I = Y_{bus} V.
- The Y_{bus} is developed by applying KCL at each bus in the system to relate the bus current injections, the bus voltages, and the branch impedances and admittances.

Y_{bus} Example

Determine the bus admittance matrix for the network shown below, assuming the current injection at each bus *i* is $I_i = I_{Gi} - I_{Di}$ where I_{Gi} is the current injection into the bus from the generator and I_{Di} is the current flowing into the load.



Y_{bus} Example

We can get similar relationships for buses 3 and 4 The results can then be expressed in matrix form

 $\mathbf{I} = \mathbf{Y}_{\text{bus}} \mathbf{V}$ $\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_A + Y_B & -Y_A & -Y_B & 0 \\ -Y_A & Y_A + Y_C + Y_D & -Y_C & -Y_D \\ -Y_B & -Y_C & Y_B + Y_C & 0 \\ 0 & -Y_D & 0 & Y_D \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$

For a system with *n* buses, Y_{bus} is an *n* by *n* symmetric matrix (i.e., one where $Y_{buskl} = Y_{buslk}$). From now on, we will mostly write Y for Y_{bus} , but be careful to distinguish Y_{kl} from line admittances.

Y_{bus} General Form

- The diagonal terms, Y_{kk}, are the self admittance terms, equal to the sum of the admittances of all devices incident to bus k.
- The off-diagonal terms, Y_{kl} , are equal to the negative of the admittance joining the two buses.
- With large systems Y_{bus} is a sparse matrix (that is, most entries are zero): sparsity is key to efficient numerical calculation.
- Shunt terms, such as in the equivalent π line model, only affect the diagonal terms.