ELG4125: Lecture 2

Power Transformers

The Ideal Transformer

The Real Transformer

Equivalent Circuits for Practical Transformers

The Per-Unit System

Three-Phase Transformers

Autotransformers

Transformers in Power Systems

- Typically in power systems, voltages get transformed approximately five times between generation and delivery to the users.
- Generation in power systems, primarily by synchronous generators, takes place at around 20-kV level.
- Transmission voltages of 230 kV, 345 kV, 500 kV, and 765 kV is common.
- At the load, these voltages are stepped down to 120/240 V single-phase in residential usage.
- Another advantage of transformers in many applications is to provide electrical insulation for safety purposes.

Real Transformers

Transformer Types

Power Transformers

Current Transformers

Voltage Transformers

Series Transformers

Transformer Purchasing Issues

Efficiency

Audible Noise

Installation Costs

Manufacturing Facilities

Performance Record

Transformer Connections

Each leg is a single phase transformer

Y-Y connections (no phase shift)

Δ–Δ connections (no phase shift)

Y-Δ connections (-30 degrees phase shift)

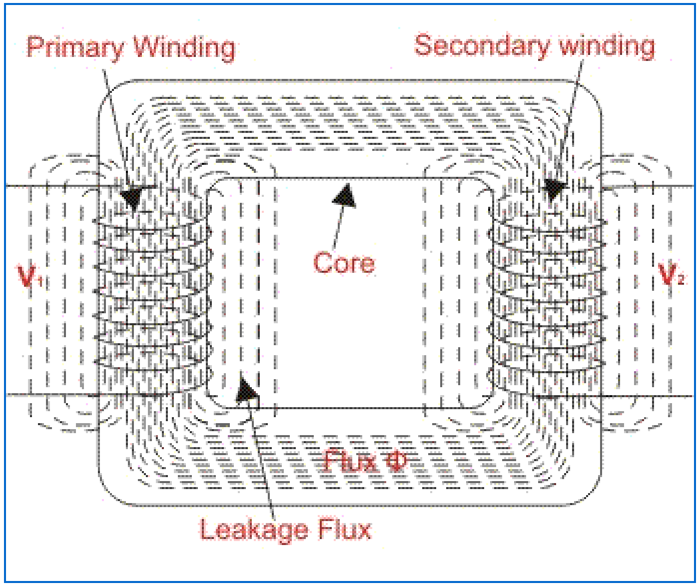
Δ–Y connections (+30 degrees phase shift)

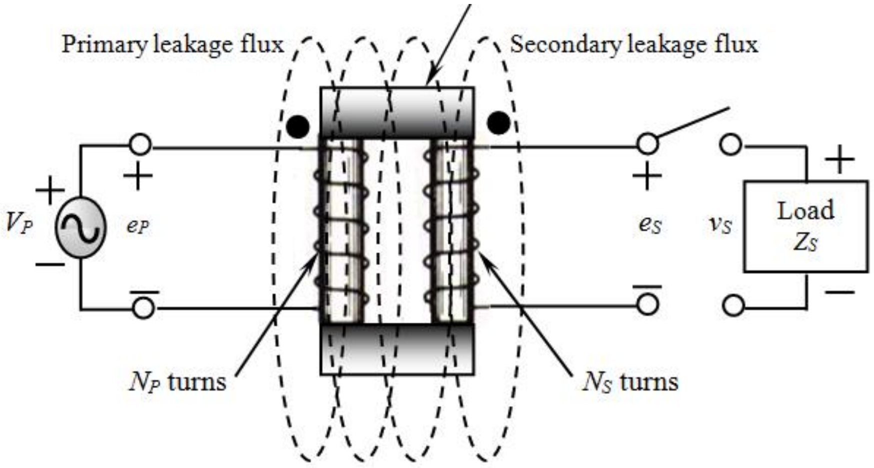
Nameplate Information

- kVA is the apparent power.
- Voltage ratings for high and low side are no-load values. The symbol between the values indicate how the voltages are related:
- Long dash (—) From different windings.
- Slant (/) …from same winding:
    - 240/120 is a 240 V winding with a center tap.
- Cross (X) Connect windings in series or parallel. Not used in wye-connected winding:
    - 240X120 two part winding connected in series for 240 or parallel for 120.
- Wye (Y) A wye-connect winding.

Basic Principles of Transformer Operation

- Transformers consist of two or more coupled windings where almost all of the flux produced by one winding links the other winding.



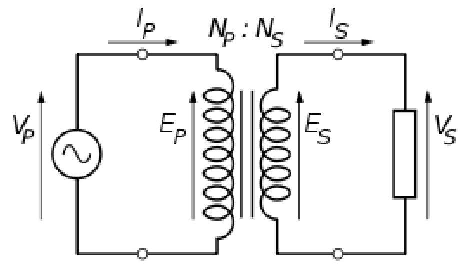


Transformer Exciting Current

- Transformers use ferromagnetic materials that guide magnetic flux due to their high permeability, require small ampere-turns to produce the desired flux density.
- The magnetising current is that current which flows in the primary winding when the primary voltage is applied with the secondary unloaded. It is the necessary current that satisfies the excitation condition as determined by the fundamental transformer equation.
- In modern power transformer of large kVA ratings, the magnetizing currents in their normal operating region are very small, for example, well below 0.2% of the rated current at the rated voltage.

Voltage Transformation

$$v_P = e_P = N_P \frac{d\phi}{dt}$$



$$v_S = e_S = N_S \frac{d\phi}{dt}$$

$$\frac{V_P}{V_S} = \frac{I_S}{I_P} = \frac{N_P}{N_S}$$

- The EMF of a transformer at a given flux density increases with frequency. By operating at higher frequencies, transformers can be physically more compact because a given core is able to transfer more power without reaching saturation and fewer turns are needed to achieve the same impedance. However, core loss and conductor skin effect also increase with frequency.
- Aircraft and military equipment employ 400 Hz power supplies which reduce core and winding weight. Conversely, frequencies used for some railway electrification systems were much lower (16.7 Hz and 25 Hz) than normal utility frequencies (50/60 Hz) for historical reasons concerned mainly with the limitations of early electric traction systems.
- As such, the transformers used to step down the high over-head line voltages (15 kV) were much heavier for the same power rating than those designed only for the higher frequencies.

$$E_P = \frac{2\pi}{\sqrt{2}} f N_P \varphi_{\max} = 4.44 f N_P \varphi_{\max}$$

$$\phi_{\max} = \frac{V_P}{4.44 f N_P}$$

- Where V_P is the RMS value of the applied voltage with the assumption that $e_P = V_P$. The second Equation shows that the flux is determined by the applied voltage, the frequency of operation, and number of turns in the winding.

Example

A 400-turn winding on a magnetic core is excited by a 60-Hz sinusoidal primary voltage of 200 V (RMS). Find the maximum flux density in the core if the uniform cross-sectional area of the core is 10 cm × 10 cm. Ignore the resistance of the winding and any leakage flux.

Solution: Since the resistance of the winding is zero, the voltage drop in the winding is ignored. Accordingly $e_P = V_P = 400$ V.

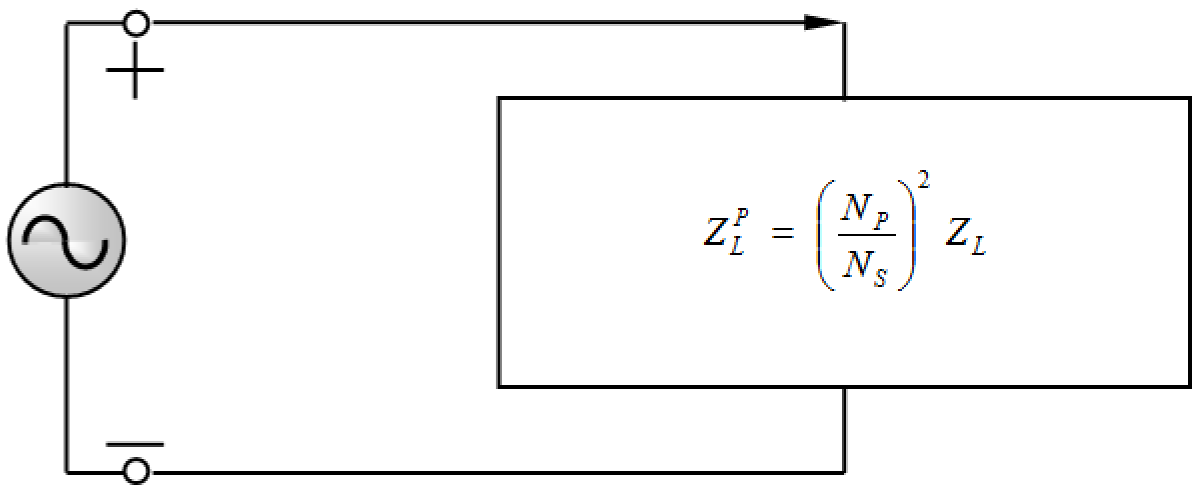
$$\phi_{max} = \frac{200}{4.44 \times 60 \times 400} = 0.0018 \text{ Wb}$$

The corresponding flux density is given by

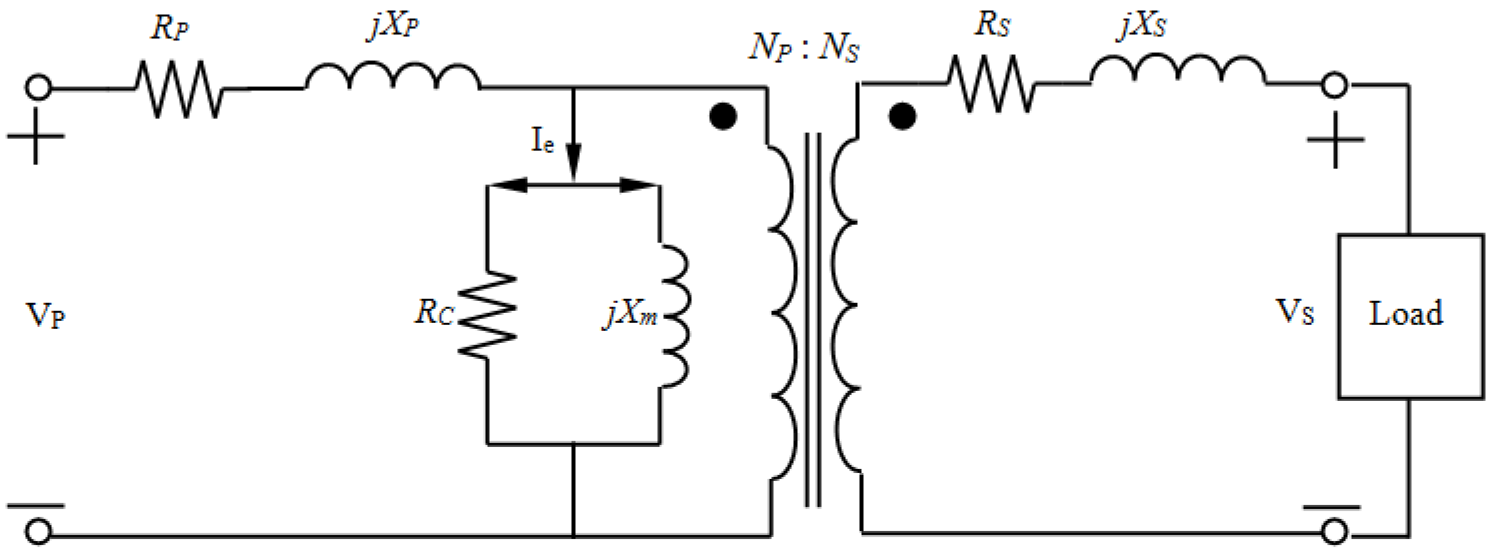
$$B_{max} = \frac{0.0018}{0.1 \times 0.1} = 0.18 \text{ Wb/m}^2$$

Transformer Equivalent Circuit

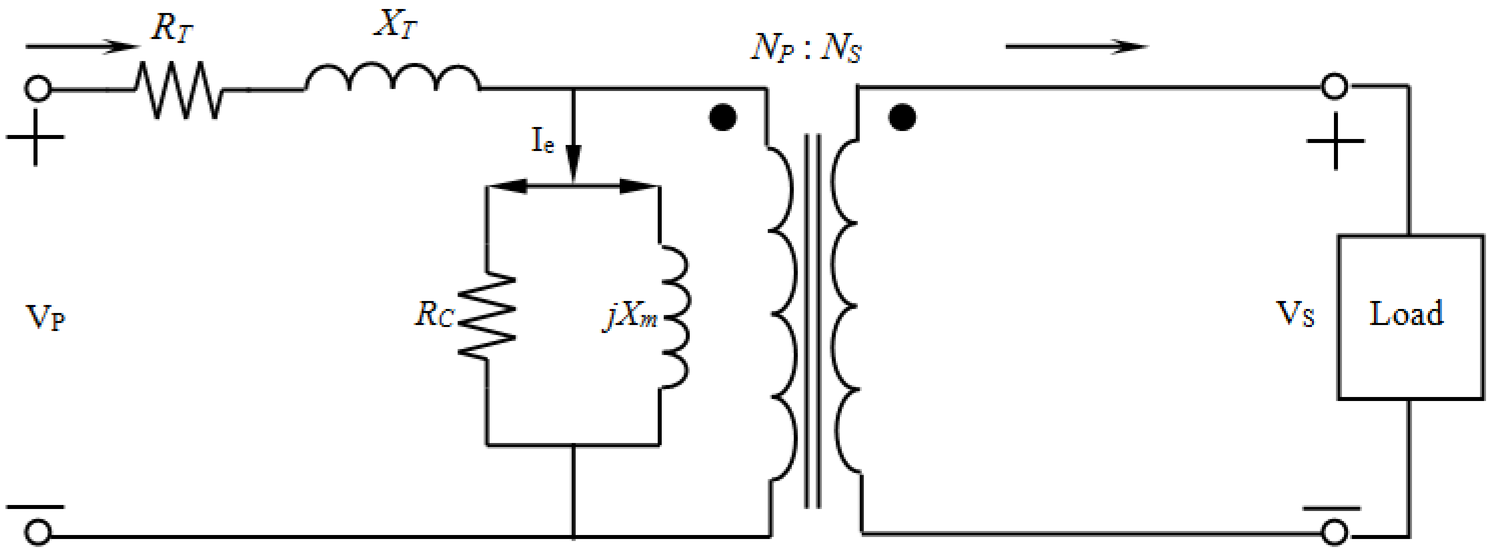
Viewed from the Primary Side



Modeling a Real Transformer

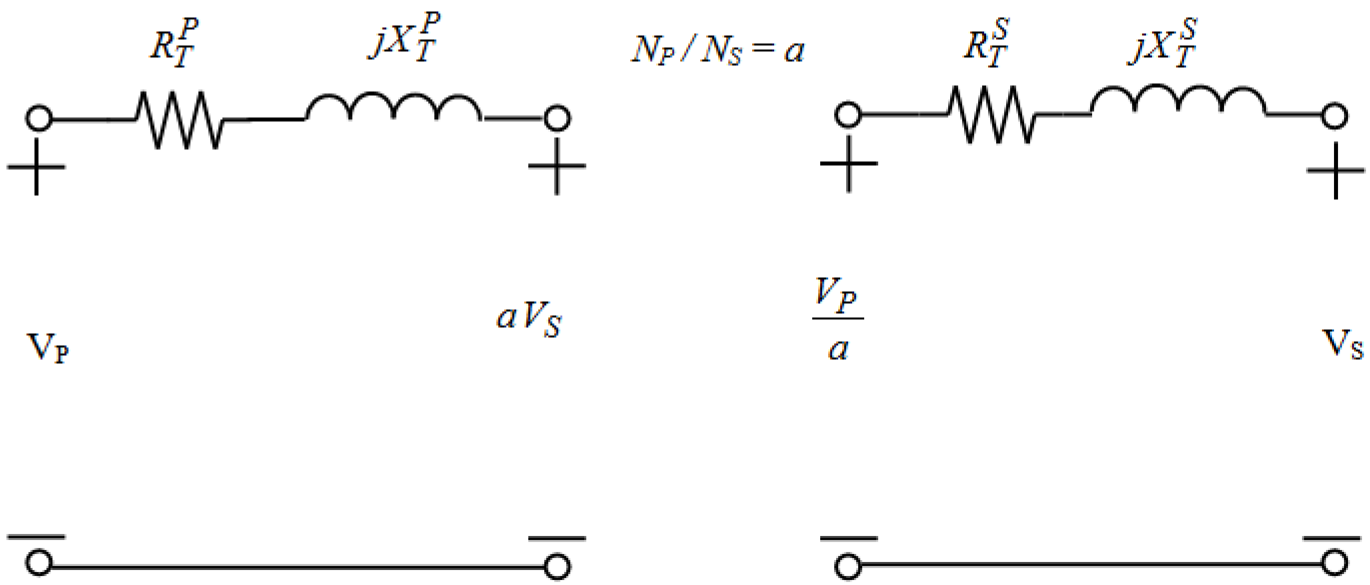


Equivalent Circuit with Parameters Moved to the Primary Side



$$R_T = R_P + \left(\frac{N_p}{N_s}\right)^2 R_S \qquad X_T = X_P + \left(\frac{N_p}{N_s}\right)^2 X_S$$

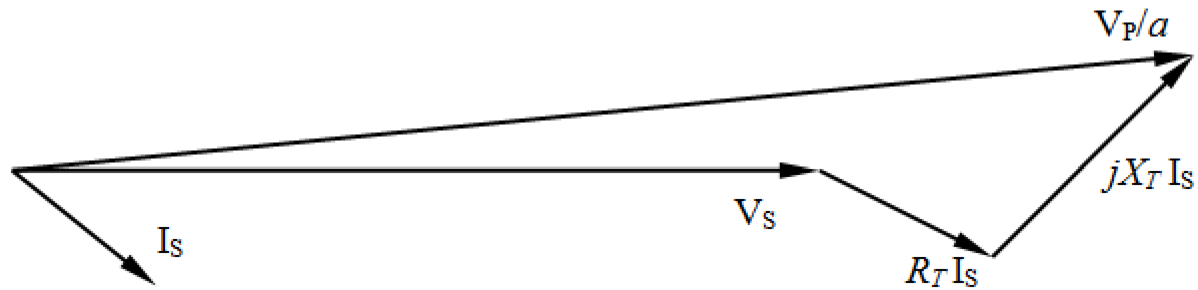
Transformer Phasor Diagram



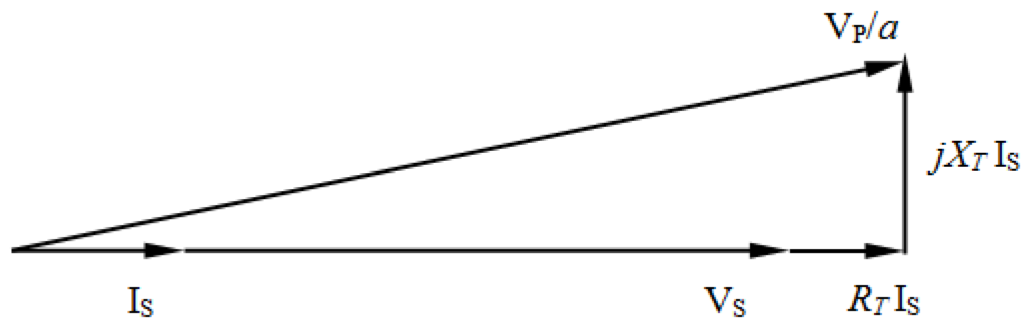
(a) (b)

$$R_T^P = R_P + a^2 R_S$$
$$X_T^P = X_P + a^2 X_S$$

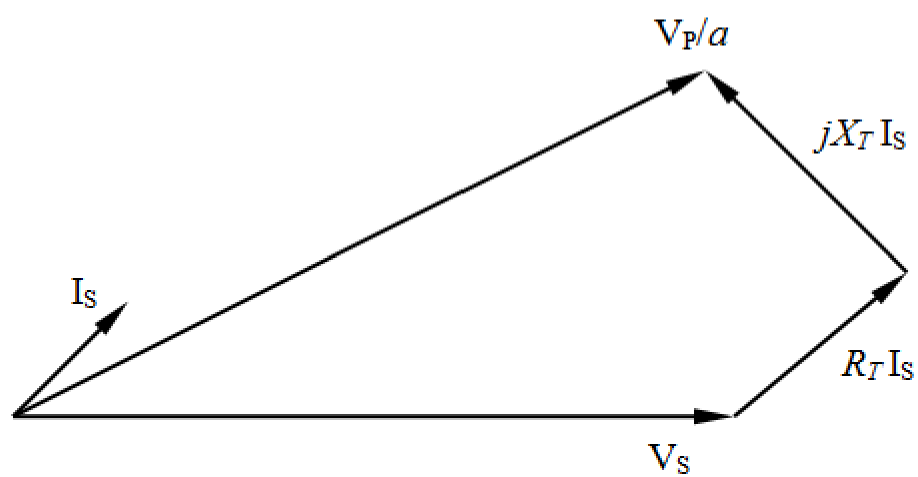
$$R_T^S = \frac{R_P}{a^2} + R_S$$
$$X_T^S = \frac{X_P}{a^2} + X_S$$



VP/a
VS
IS
jXT IS
RT IS



VP/a
jXT IS
IS
VS
RT IS



VP/a
jXT IS
IS
RT IS
VS

Transformer Tests

Test to Determine Transformer Parameters, we require to conduct

Open Circuit Test:

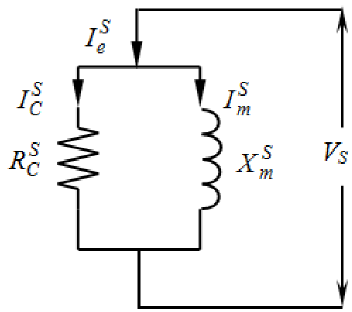
- Energize Low voltage winding at rated voltage, leaving other winding open
- Measure Current (I_{oc}) and Power (P_{oc}) into energized winding.
- Calculate parameters

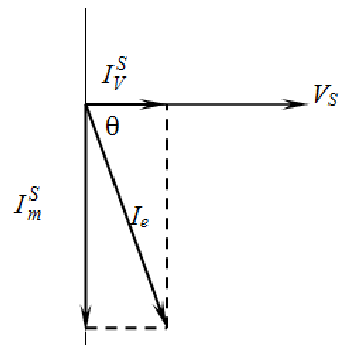
Short Circuit Test:

- Energize Low current (high voltage) winding at rated current with a solid short circuit applied across the other winding
- Measure Voltage and Power at terminals of energized winding
- Calculate parameters

Transformer Test

Open-Circuit Test





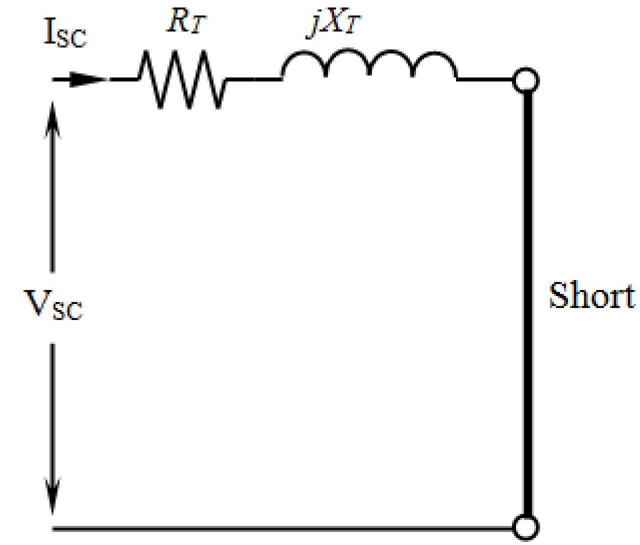
$$I_C^S = \frac{P_C^S}{V_S}, \text{ and}$$

$$I_C^S = \sqrt{\left(I_e^S\right)^2 - \left(I_C^S\right)^2}$$

$$R_C^S = \frac{V_S}{I_C^S}, \text{ and}$$

$$X_m^S = \frac{V_S}{I_m^S}$$

Short-Circuit Test



$$R_T = \frac{P_{SC}}{I_{SC}^2}$$

$$X_T = \sqrt{\left(\frac{V_{SC}}{I_{SC}}\right)^2 - R_T^2}$$

Example 1

An 8-kVA, 1600/240-V, 60-Hz, single-phase transformer has an open-circuit test excited on the secondary side of the circuit (240 V). The measured exciting current is 1.0 A and the measured power is 150 W. Find R_C^S and X_m^S.

	$I_C^S = \frac{150}{240} = 0.625\ \text{A}$ $I_m^S = \sqrt{(1.0)^2 - (0.625)^2} = 0.78\ \text{A}$	

	$R_C^S = \frac{240\ \text{V}}{0.625\ \text{A}} = 384\ \Omega$, and $X_m^S = \frac{240\ \text{V}}{0.78\ \text{A}} = 307.7\ \Omega$	

Example 2

An 8-kVA, 1600/240-V, 60-Hz, single-phase transformer has the secondary side shorted. It has been found that 60.0 V on the 1600-V winding produces the rated current of 4.0 A with an input power of 150 W. Find R_T and X_T.

$$R_T = \frac{150\ \text{W}}{(4.0)^2} = 9.375\ \Omega$$

$$X_T = \sqrt{\left(\frac{60.0}{4.0}\right)^2 - (9.375)^2} = 11.7\ \Omega$$

The Per unit system

- In the per-unit system, the voltages, currents, powers, impedances, and other electrical quantities are expressed on a per-unit basis by the equation:

$$\textit{Quantity per unit} = \frac{\textit{Actual value}}{\textit{Base value of quantity}}$$

- It is customary to select two base quantities to define a given per-unit system. The ones usually selected are **voltage** and **power**.

$$V_b = V_{rated} \qquad S_b = S_{rated}$$

Then compute base values for currents and impedances

$$I_b = \frac{S_b}{V_b} \qquad Z_b = \frac{V_b}{I_b} = \frac{V_b^2}{S_b}$$

$$V_{p.u.} = \frac{V_{actual}}{V_b}$$

$$I_{p.u.} = \frac{I_{actual}}{I_b}$$

$$S_{p.u.} = \frac{S_{actual}}{S_b}$$

$$Z_{p.u.} = \frac{Z_{actual}}{Z_b}$$

$$Z\% = Z_{p.u.} \times 100\%$$

Example 1

A single-phase, 75-kVA, 2,400:240-volt, 60 Hz distribution transformer has the following parameters: $R_P = 1.0\ \Omega$, $R_S = 0.005\ \Omega$, $X_P = 1\ \Omega$, and $X_S = 0.01\ \Omega$.

Solution: The base quantities for primary and secondary sides are given as

	Primary Side	**Secondary Side**
VA_{base}	75,000 VA	75,000 VA
V_{base}	2,400 V	240 V
I_{base}	$\frac{75000\text{ VA}}{2400\text{ V}} = 31.25\text{ A}$	$\frac{75000\text{ VA}}{240\text{ V}} = 312.5\text{ A}$
R_{base}	$\frac{2400\text{ V}}{31.25} = 76.8\ \Omega$	$\frac{240\text{ V}}{312.5\text{ A}} = 0.768\ \Omega$

Example 2

An electrical load is rated 120 volts, 500 watts. Compute the per-unit and percent impedance of the load. Give the per unit equivalent circuit.

$$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(120)^2}{500} = 28.8\Omega$$

Power factor = 1.0

$$Z = 28.8\angle 0\Omega$$

Select base quantities

$$S_b = 500VA$$

$$V_b = 120V$$

Compute base impedance

$$Z_b = \frac{V_b^{\,2}}{S_b} = \frac{(120)^2}{500} = 28.8\Omega$$

The per-unit impedance is

$$Z_{p.u.} = \frac{Z}{Z_b} = \frac{28.8\angle 0}{28.8} = 1\angle 0 \text{p.u.}$$

Percent impedance:

$$Z\% = 100\%$$

Per-unit equivalent circuit:

Per-unit System for Single Phase Circuits

One-phase circuits

$$S_b = S_{1-\phi} = V_\phi I_\phi$$

where

$$V_\phi = V_{line-to-neutral}$$

$$I_\phi = I_{line-current}$$

$$V_{bLV} = V_{\phi LV} \qquad V_{bHV} = V_{\phi HV}$$

$$I_{bLV} = \frac{S_b}{V_{bLV}} \qquad I_{bHV} = \frac{S_b}{V_{bHV}}$$

$$Z_{bLV} = \frac{V_{bLV}}{I_{bLV}} = \frac{(V_{bLV})^2}{S_b} \qquad Z_{bHV} = \frac{V_{bHV}}{I_{bHV}} = \frac{(V_{bHV})^2}{S_b}$$

$$S_{pu} = \frac{S}{S_b} = V_{pu} I_{pu}^*$$

$$P_{pu} = \frac{P}{S_b} = V_{pu} I_{pu} \cos\theta$$

$$Q_{pu} = \frac{Q}{S_b} = V_{pu} I_{pu} \sin\theta$$

Transformation Between Bases

$$S_{b1} = S_A \qquad V_{b1} = V_A$$

$$Z_{b1} = \frac{V_{b1}^2}{S_{b1}} \qquad Z_{pu1} = \frac{Z_L}{Z_{b1}}$$

And second

$$S_{b2} = S_B \qquad V_{b2} = V_B$$

$$Z_{b2} = \frac{V_{b2}^2}{S_{b2}} \qquad Z_{pu2} = \frac{Z_L}{Z_{b2}}$$

Transformation Between Bases

$$\frac{Z_{pu2}}{Z_{pu1}} = \frac{Z_L}{Z_{b2}} \times \frac{Z_{b1}}{Z_L} = \frac{Z_{b1}}{Z_{b2}} = \frac{V_{b1}^2}{S_{b1}} \times \frac{S_{b2}}{V_{b2}^2}$$

$$Z_{pu2} = Z_{pu1}\left(\frac{V_{b1}}{V_{b2}}\right)^2 \times \left(\frac{S_{b2}}{S_{b1}}\right)$$

"1" – old
"2" - new

$$Z_{pu,new} = Z_{pu,old}\left(\frac{V_{b,old}}{V_{b,new}}\right)^2 \times \left(\frac{S_{b,new}}{S_{b,old}}\right)$$

Transformation Between Bases

Generally per-unit values given to another base can be converted to new base by the equations:

$$(P, Q, S)_{pu\_on\_base\_2} = (P, Q, S)_{pu\_on\_base\_1} \frac{S_{base1}}{S_{base2}}$$

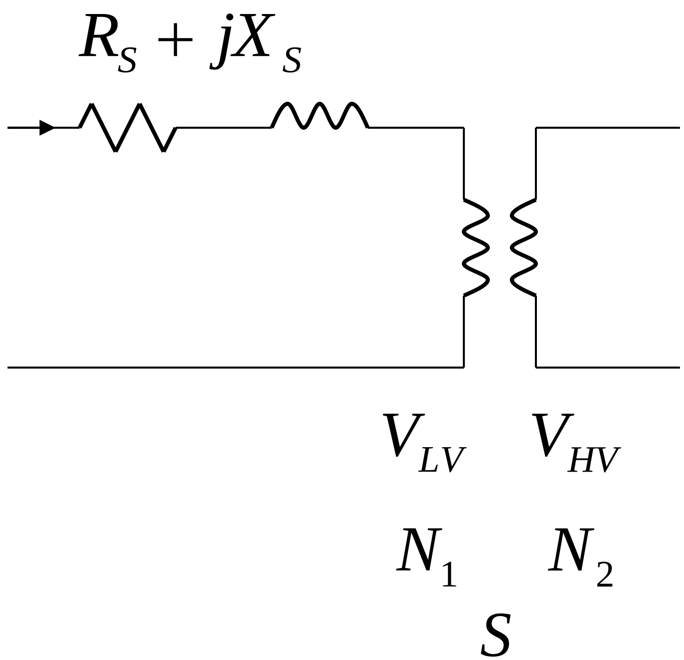
$$V_{pu\_on\_base\_2} = V_{pu\_on\_base\_1} \frac{V_{base1}}{V_{base2}}$$

$$(R, X, Z)_{pu\_on\_base\_2} = (R, X, Z)_{pu\_on\_base\_1} \frac{(V_{base1})^2 S_{base2}}{(V_{base2})^2 S_{base1}}$$

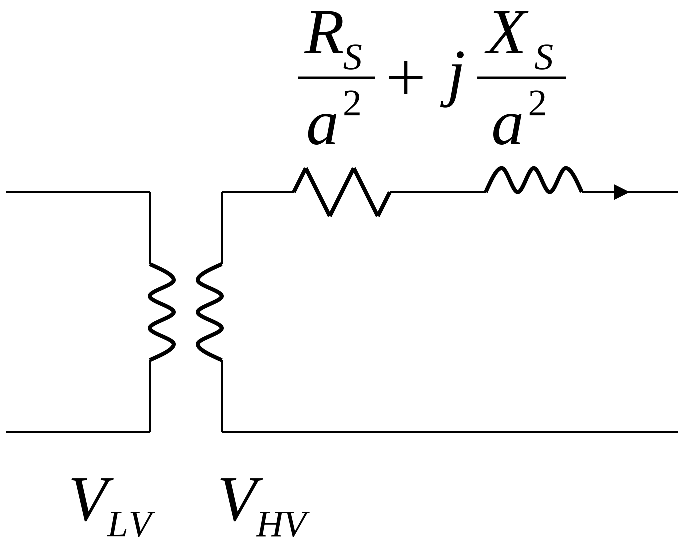
When performing calculations in a power system, every per-unit value must be converted to the same base.

Per-unit System for Single Phase Transformer

Consider the equivalent circuit of transformer referred to LV side and HV side shown below:



Referred to LV side



Define $a = \frac{V_{LV}}{V_{HV}} = \frac{N_1}{N_2} < 1$

Referred to HV side

Per-unit System for Single Phase Transformer

Choose

$$V_{b1} = V_{LV,rated}$$

$$S_b = S_{rated}$$

Compute

$$V_{b2} = \frac{V_{HV}}{V_{LV}} V_{b1} = \frac{1}{a} V_{b1}$$

$$Z_{b1} = \frac{V_{b1}^2}{S_b} \quad Z_{b2} = \frac{V_{b2}^2}{S_b}$$

$$\frac{Z_{b1}}{Z_{b2}} = \frac{V_{b1}^2}{V_{b2}^2} = \frac{V_{b1}^2}{(\frac{1}{a} V_{b1})^2} = a^2$$

Per-unit System for Single Phase Transformer

Per-unit impedances are

$$Z_{p.u.1} = \frac{R_S + jX_S}{Z_{b1}}$$

$$Z_{p.u.2} = \frac{\frac{R_S}{a^2} + \frac{jX_S}{a^2}}{Z_{b2}} = \frac{\frac{R_S}{a^2} + \frac{jX_S}{a^2}}{\frac{Z_{b1}}{a^2}} = \frac{R_S + jX_S}{Z_{b1}}$$

$$Z_{p.u.1} = Z_{p.u.2}$$

Voltage Regulation

Voltage regulation is defined as

$$VR = \frac{\left|V_{no-load}\right| - \left|V_{full-load}\right|}{\left|V_{full-load}\right|} \times 100\%$$

In per-unit system

$$VR = \frac{\left|V_{pu,no-load}\right| - \left|V_{pu,\,full-load}\right|}{\left|V_{pu,\,full-load}\right|} \times 100\%$$

$V_{full\text{-}load}$: Desired load voltage at full load. It may be equal to, above, or below rated voltage.

$V_{no\text{-}load}$: The no load voltage when the primary voltage is the desired voltage in order the secondary voltage be at its desired value at full load.

Example

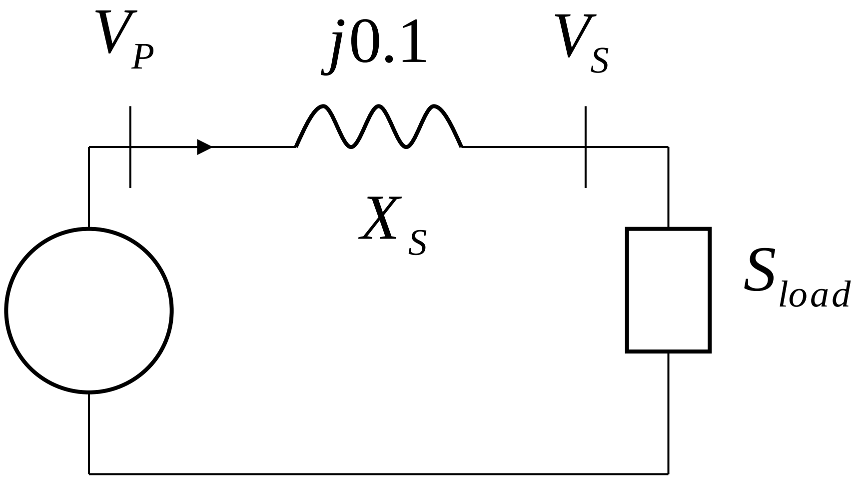
A single-phase transformer rated 200-kVA, 200/400-V, and 10% short circuit reactance. Compute the voltage regulation when the transformer is fully loaded at unity power factor and rated voltage 400-V.

$$V_{b2} = 400V$$

$$S_b = 200kVA$$

$$S_{load,pu} = 1\angle 0\, pu$$

$$X_{S,pu} = j0.1pu$$



Rated voltage

$$V_{S,pu} = 1.0\angle 0\, pu$$

$$I_{load,pu} = \left(\frac{S_{load,pu}}{V_{S,pu}}\right)^* = \left(\frac{1.0\angle 0}{1.0\angle 0}\right)^* = 1.0\angle 0\, pu$$

$$V_{P,pu} = V_{S,pu} + I_{pu} X_{S,pu}$$

$$= 1.0\angle 0 + 1.0\angle 0 \times j0.1 = 1 + j0.1$$

$$= \boxed{1.001\angle 5.7^o\, pu}$$

Secondary side

$$V_{pu,\,full-load} = V_{S,\,pu} = 1.0\angle 0\,pu$$

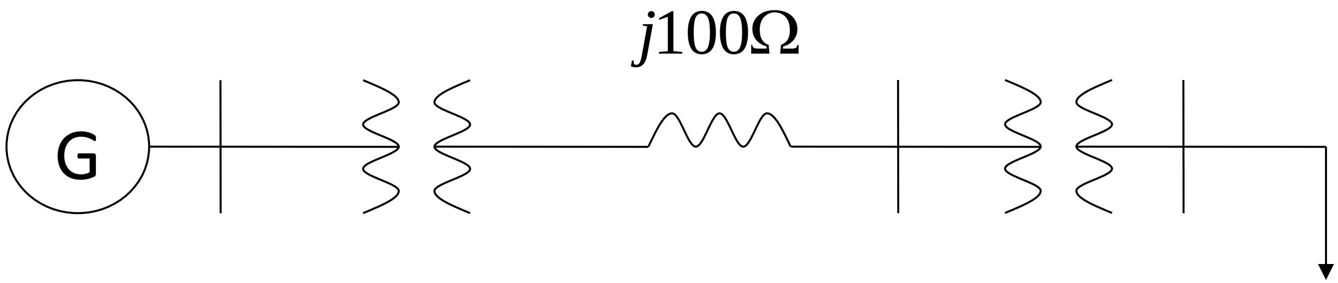
$$V_{pu,no-load} = V_{P,\,pu} = 1.001\angle 5.7^{o}\ pu$$

Voltage regulation

$$VR = \frac{\left|V_{pu,no-load}\right| - \left|V_{pu,\,full-load}\right|}{\left|V_{pu,\,full-load}\right|} \times 100\%$$

$$= \frac{1.001 - 1.0}{1.0} \times 100\% = 0.1\%$$

Problem



20 kV

22kV/220kV
80MVA
14%

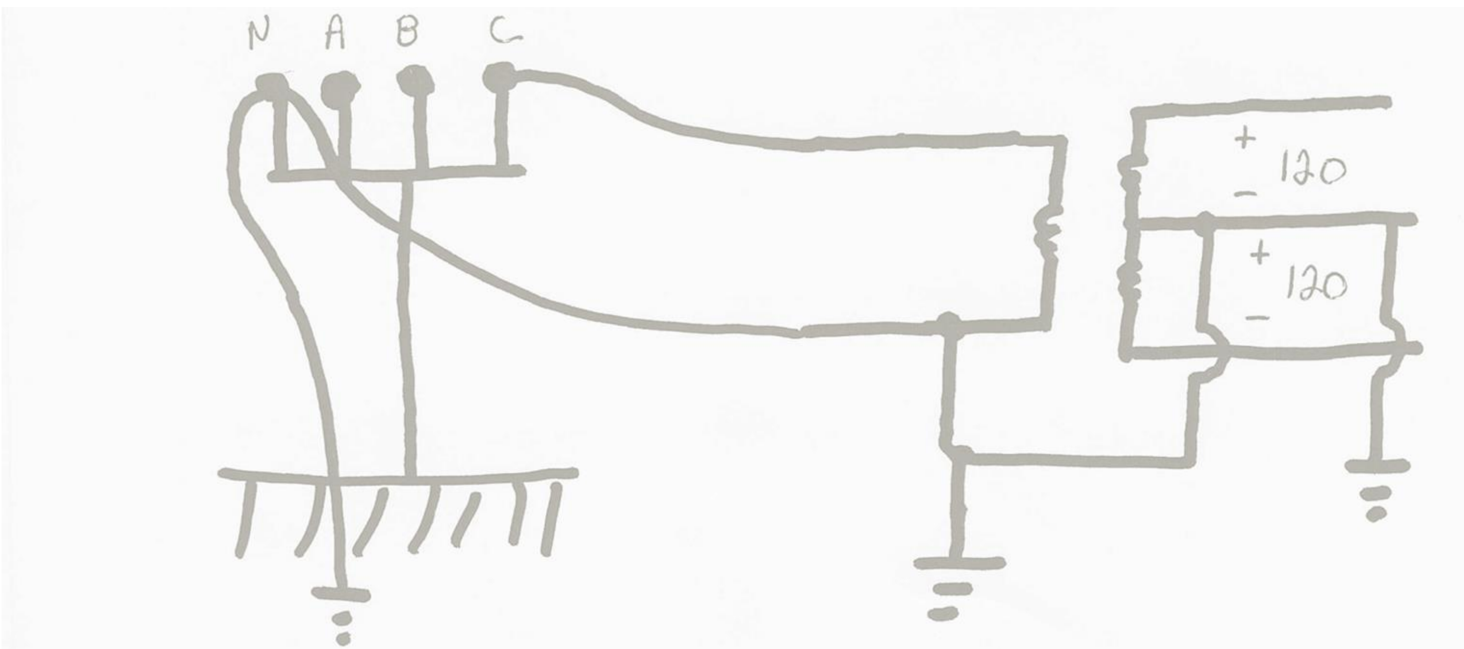
220kV/20kV
50MVA
10%

50MVA
0.8 PF
lagging

Select V_{base} in generator circuit and S_b=100MVA, compute the per unit equivalent circuit.

Residential Distribution Transformers

Single phase transformers are commonly used in residential distribution systems. Most distribution systems are 4 wire, with a multi-grounded, common neutral.

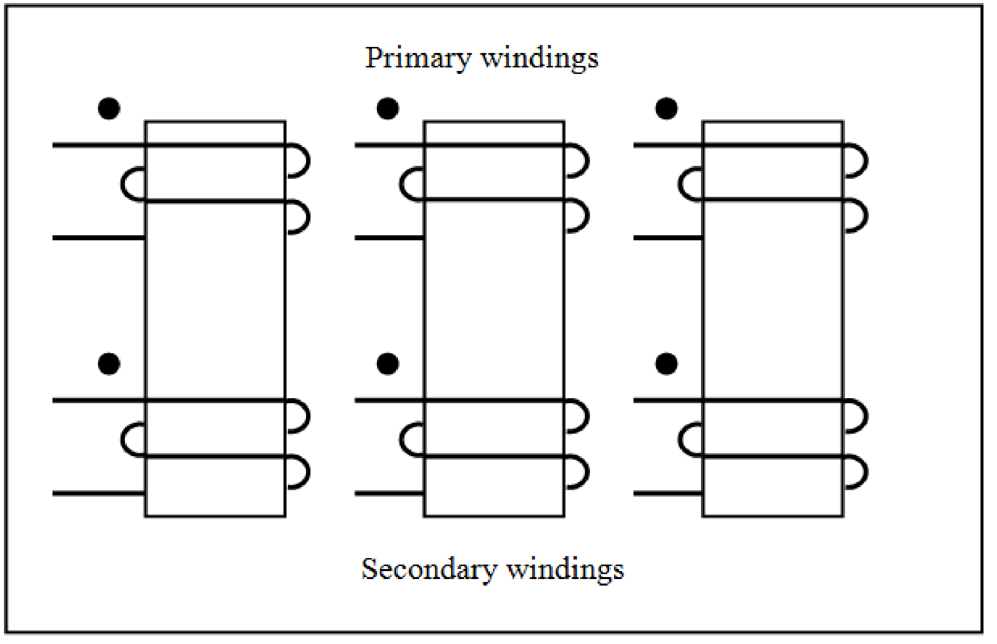


Three-Phase Transformers

Y-Δ	Δ-Y	Δ-Δ	Y-Y

The current and voltage relationships between phase and line values for the Y connection are

	$V_{line} = \sqrt{3}\,.V_{phase}$ $I_{line} = I_{phase}$	



Per-unit System for Three Phase Circuits

$$S_b = S_{3-\phi} = 3S_{1-\phi} = 3V_\phi I_\phi$$

$$V_\phi = V_{line-to-neutral} = V_{L(line)} / \sqrt{3}$$

$$I_\phi = I_{line-current} = I_L$$

$$S_b = \sqrt{3} V_L I_L$$

$$V_{bLV} = V_{L,LV} \qquad V_{bHV} = V_{L,HV}$$

$$S_b = \sqrt{3} V_{bLV} I_{bLV} = \sqrt{3} V_{bHV} I_{bHV}$$

Per-unit System for Three Phase Circuits

$$I_{bLV} = \frac{S_b}{\sqrt{3}V_{bLV}} \qquad I_{bHV} = \frac{S_b}{\sqrt{3}V_{bHV}}$$

$$Z_{bLV} = \frac{V_{\phi LV}}{I_{\phi LV}} = \frac{V_{bLV}}{\sqrt{3}} \frac{\sqrt{3}V_{bLV}}{S_b} = \frac{(V_{bLV})^2}{S_b}$$

$$Z_{bHV} = \frac{(V_{bHV})^2}{S_b}$$

$$S_{pu} = \frac{S_{3\phi}}{S_b} = \frac{\sqrt{3}V_L I_L^*}{\sqrt{3}V_b I_b} = V_{pu} I_{pu}^*$$

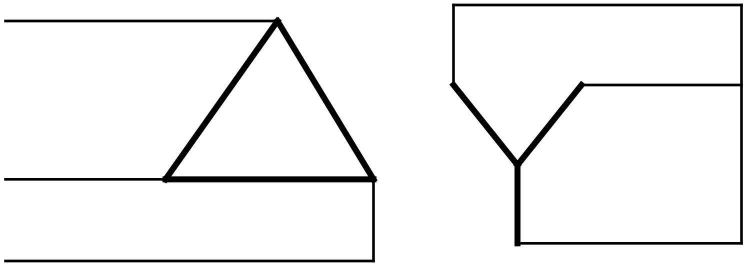
Per-unit System for Three Phase Transformer

Three 25-kVA, 34500/277-V transformers connected in Δ-Y. Short-circuit test on high voltage side:

$$V_{Line,SC} = 2010V$$

$$I_{Line,SC} = 1.26A$$

$$P_{3\phi,SC} = 912W$$



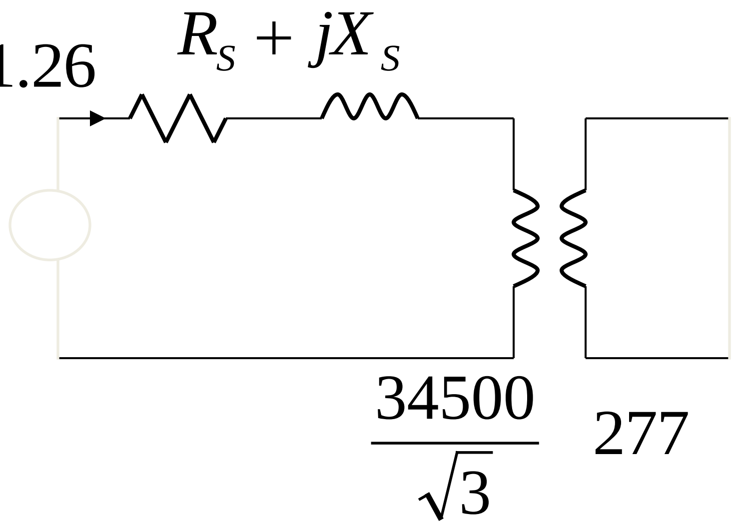
Determine the per-unit equivalent circuit of the transformer.

Per-unit System for Three Phase Transformer

Using Y-connection equivalent circuit

$$I_{SC} = 1.26$$

$$V_{SC} = \frac{2010}{\sqrt{3}}$$



$$S_b = 25000VA$$

$$|V_{SC}| = \frac{2010}{\sqrt{3}} = 1160.47V$$

$$|Z_{SC}| = \left|\frac{1160.47}{1.26}\right| = 921.00\Omega$$

Per-unit System for Three Phase Transformer

$$P_{\phi} = \frac{912}{3} = 304W \qquad R_S = \frac{P_{\phi}}{I_{SC}^2} = \frac{304}{1.26^2} = 191.48\Omega$$

$$X_S = \sqrt{\left|Z_{SC}\right|^2 - R_S^2} = \sqrt{921^2 - 191.48^2} = 900.86\Omega$$

$$Z_{SC} = 191.48 + j900.86\Omega$$

$$S_b = 25000VA \qquad V_{b,HV} = \frac{34500}{\sqrt{3}} = 19918.58V$$

$$Z_{b,HV} = \frac{19918.58^2}{25000} = 15869.99\Omega$$

$$Z_{SC,pu,Y} = \frac{191.48 + j900.86}{15869.99} = 0.012 + j0.0568\,pu$$

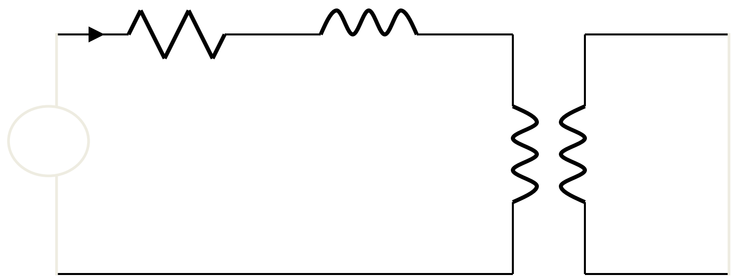
Per-unit System for Three Phase Transformer

Using Δ-connection equivalent circuit

$$I_{SC} = \frac{1.26}{\sqrt{3}}$$

$$Z_{SC,\Delta}$$

$$V_{SC} = 2010$$



$$S_b = 25000VA$$

34500 277

$$|V_{SC}| = 2010V$$

$$|I_{SC}| = \frac{1.26}{\sqrt{3}} = 0.727A$$

$$|Z_{SC,\Delta}| = \left|\frac{2010}{0.727}\right| = 2764.79\Omega$$

Per-unit System for Three Phase Transformer

$$P_{\phi} = \frac{912}{3} = 304W \qquad R_{S,\Delta} = \frac{P_{\phi}}{I_{SC}^2} = \frac{304}{0.727^2} = 575.18\Omega$$

$$X_{S,\Delta} = \sqrt{\left|Z_{SC,\Delta}\right|^2 - R_{S,\Delta}^2} = \sqrt{2764.79^2 - 575.18^2} = 2704.30\Omega$$

$$Z_{SC} = 191.48 + j900.86\Omega$$

$$S_b = 25000VA \qquad V_{b,HV} = 34500V$$

$$Z_{b,HV} = \frac{34500^2}{25000} = 47610\Omega$$

$$Z_{SC,pu,\Delta} = \frac{575.18 + j1704.30}{47610} = 0.012 + j0.0568\,pu$$

Transformer Efficiency

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}}$$

$$P_{out} = V_S \; I_S \; \cos\;\theta$$

$$\eta = \frac{V_S \; I_S \; \cos\;\theta}{V_S \; I_S \; \cos\;\theta + P_{Cu} + P_C} \times 100\,\%$$

Load Tap Changing Transformers

- LTC transformers have tap ratios that can be varied to regulate bus voltages.

- The typical range of variation is $\pm$10% from the nominal values, usually in 33 discrete steps (0.0625% per step).

- Because tap changing is a mechanical process, LTC transformers usually have a 30 second dead-band to avoid repeated changes to minimize wear and tear.

- Unbalanced tap positions can cause “circulating VARs;” that is, reactive power flowing from one winding to the next in a three phase transformer.

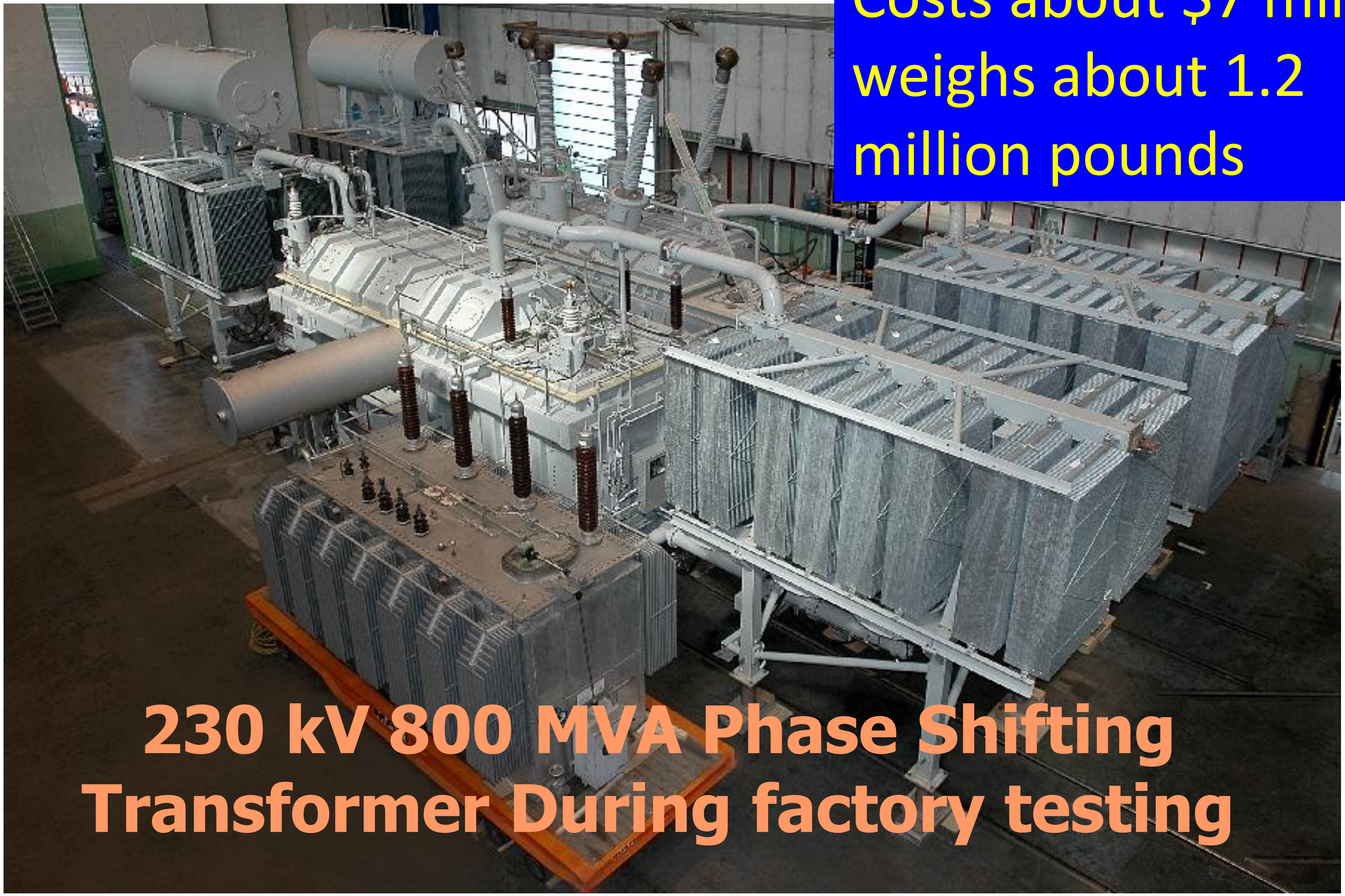
Phase Shifting Transformers

- Phase shifting transformers are used to control the phase angle across the transformer.

- Since power flow through the transformer depends upon phase angle, this allows the transformer to regulate the power flow through the transformer.

- Phase shifters can be used to prevent inadvertent "loop flow" and to prevent line overloads.

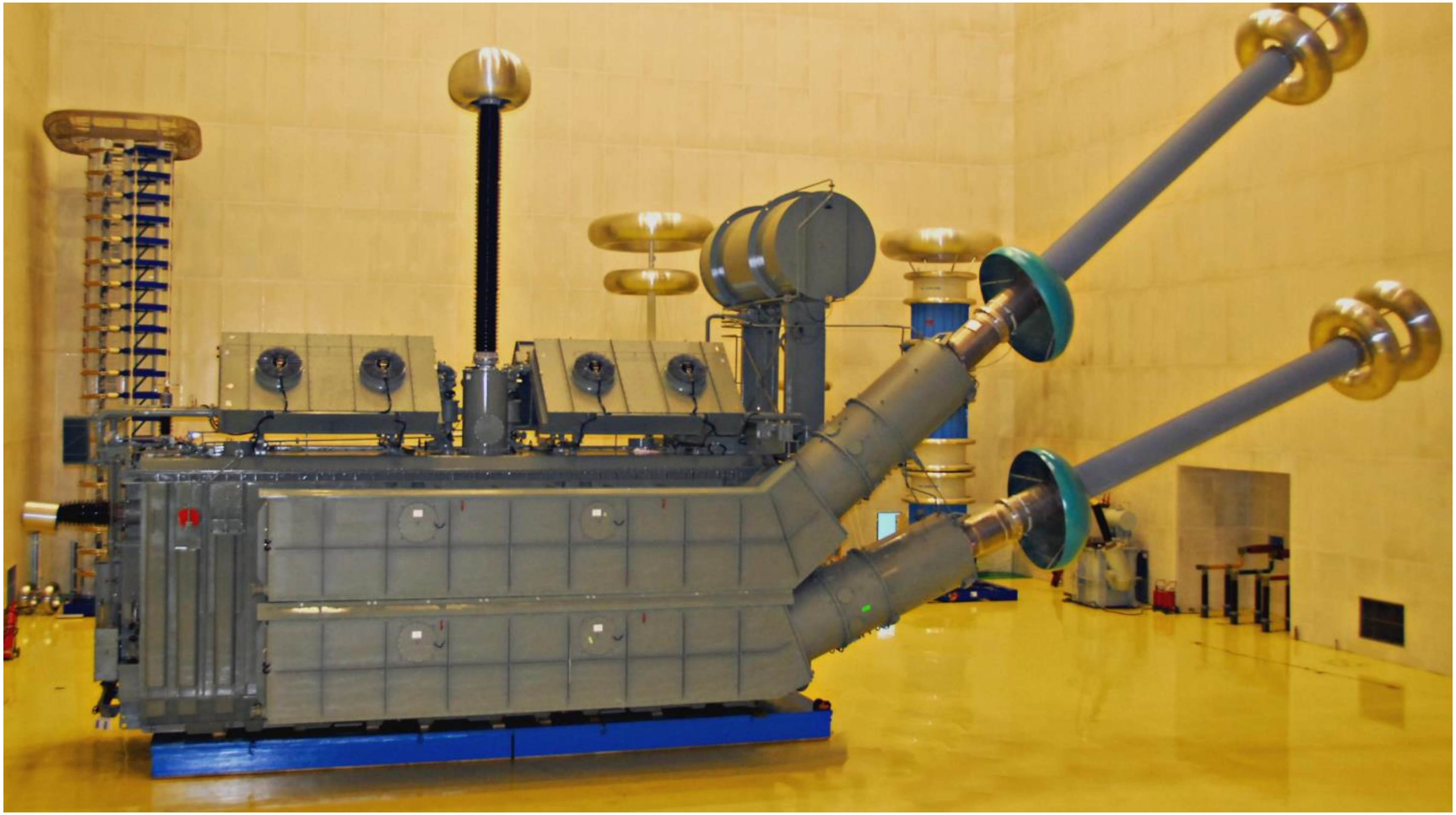
Phase Shifting Transformer Picture



Costs about $7 million, weighs about 1.2 million pounds

Source: Tom Ernst, Minnesota Power

Power Transformers from China



Autotransformers

- Autotransformers are transformers in which the primary and secondary windings are coupled magnetically and electrically.
- This results in lower cost, and smaller size and weight.
- The key disadvantage is loss of electrical isolation between the voltage levels. This can be an important safety consideration when a is large. For example in stepping down 7160/240 V we do not ever want 7160 on the low side!