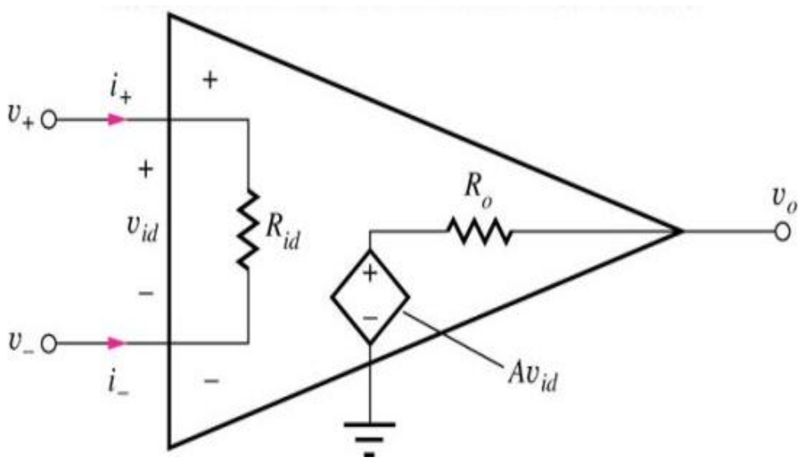
Operational Amplifiers

- Introduction of Operational Amplifier (Op-Amp)
- Characteristics of an Op-Amp
- Comparison of ideal and Non-ideal Op-Amp
- Feedback and Non-Feedback
- Configurations
- Gain; Input Impedance; Output Impedance
- Analysis of ideal Op-Amp Circuits.

What is an Operational Amplifier?

- Operational amplifier is an amplifier whose output voltage is proportional to the negative of its input voltage and that boosts the amplitude of an input signal, many times, i.e., has a very high gain.High-gain amplifiers.
- They were developed to be used in synthesizing mathematical operations in early analog computers, hence their name.
- Typified by the series 741 (The integrated circuit contains 8-pin mini-DIP, 20 transistors and 11 resistors).
- Used for amplifications, as switches, as filters, as rectifiers, and in digital circuits.
- Take advantage of large open-loop gain.
- It is usually connected so that part of the output is fed back to the input.
- Can be used with positive feedback to produce oscillation.

Basics of Differential Amplifier Model



Op Amp is represented by:

A = open-circuit voltage gain

$v_{id} = (v^+ - v^-)$ = differential input signal voltage

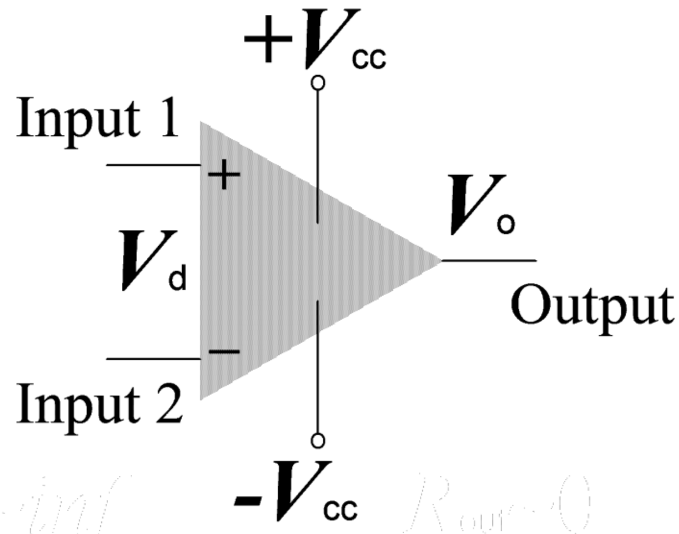
R_{id} = amplifier input resistance

R_o = amplifier output resistance

The output signal of the amplifier is in phase with the signal applied at the + input (non-inverting).
The output signal of the amplifier is 180° out of phase with the signal applied at the - input (inverting) terminal.

Characteristics of Operational Amplifier

- Very high differential gain
- High input impedance
- Low output impedance
- Wide range of applications: oscillators, filters and instrumentation
- Accumulate a very high gain by cascading multiple stages.

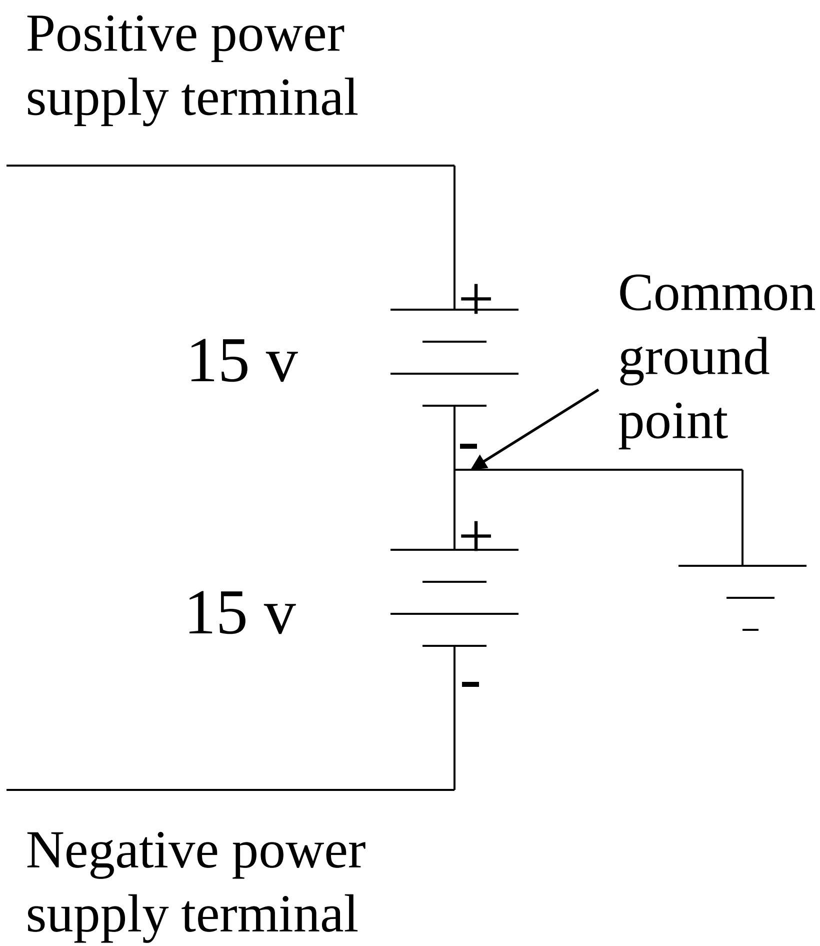


$$V_o = G_d V_d$$

G_d : differential gain normally very large, say more than 10^5

Op-Amp: Power Supply Connections

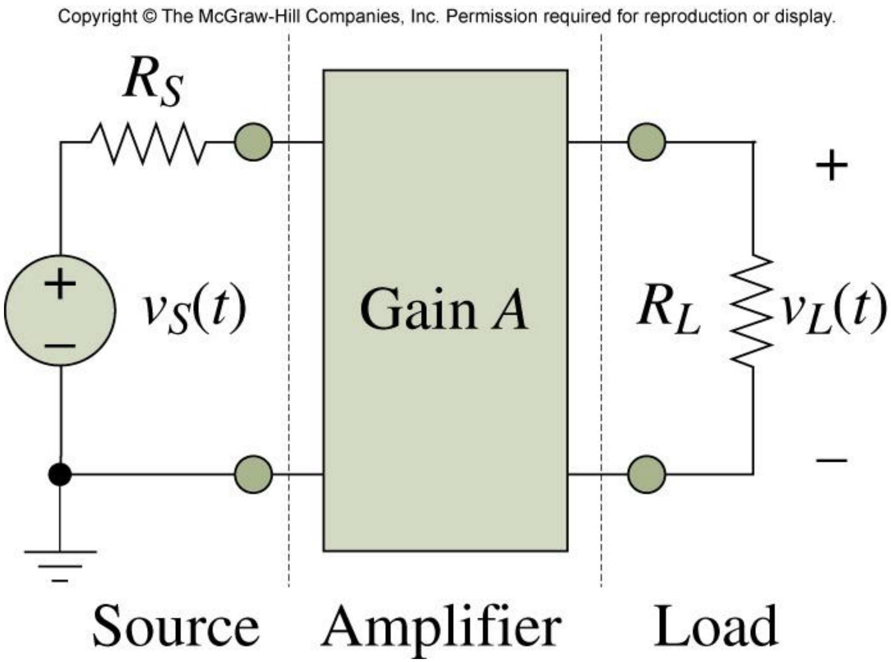
- Commonly used: dual power supplies
- Common values:
  - +15V (+V or V+)
  - -15V (-V or V-)
- All output loads connected between output terminal and common ground point
- Usually power supply connections are omitted for ease



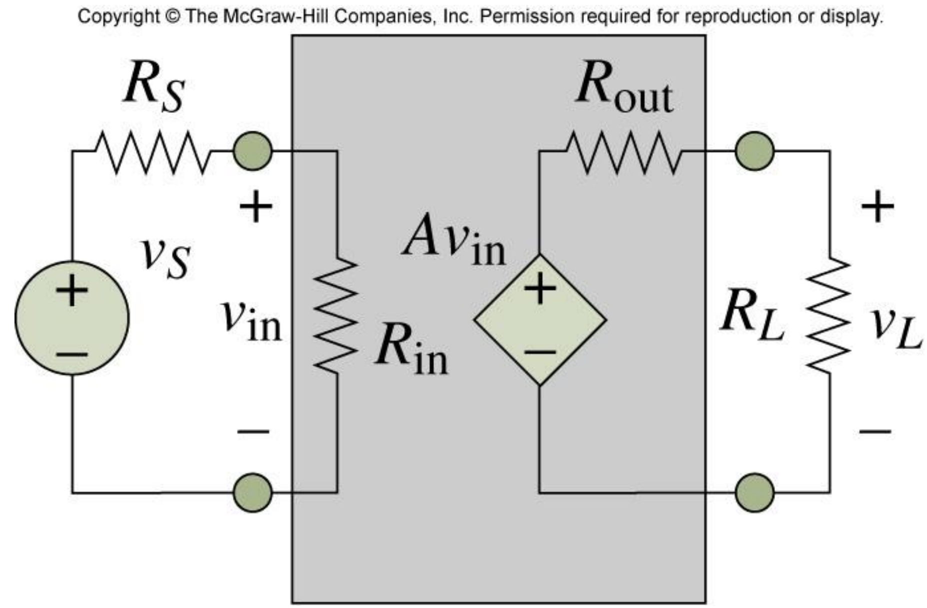
The ideal Op-Amp

- **Infinite Voltage Gain**
  - a voltage difference at the two inputs is magnified infinitely
  - something like 200,000, means difference between + terminal and – terminal is amplified by 200,000!
- **Infinite Input Impedance**
  - no current flows into both inputs
  - about 10^{12} Ω for FET input op-amps
- **Zero Output Impedance**
  - rock-solid independent of load
  - roughly to current maximum (usually 5–25 mA)
- **Infinitely Fast (Infinite Bandwidth)**
  - limited to few MHz range
  - slew rate limited to 0.5–20 V/μs

A Voltage Amplifier



Simple Voltage Amplifier Model



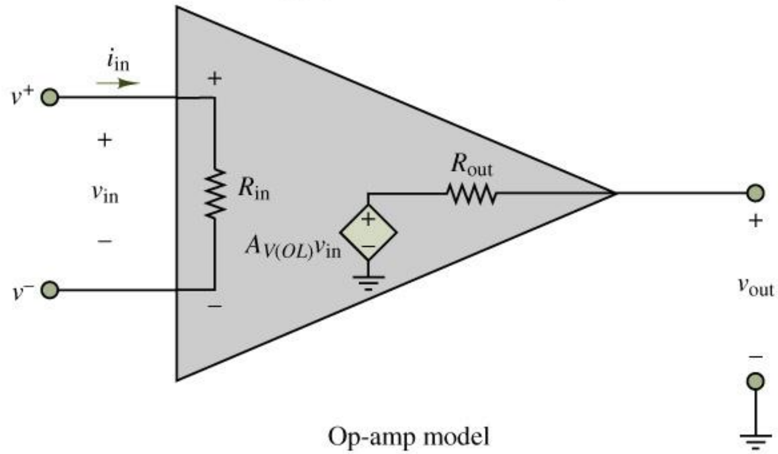
$$v_{in} = \frac{R_{in}}{R_S + R_{in}} v_S ; v_L = A v_{in} \frac{R_L}{R_{out} + R_L}$$

$$v_L = \left(A \frac{R_{in}}{R_S + R_{in}} \frac{R_L}{R_{out} + R_L} \right) v_S ; v_{in} \approx v_S ; v_L = A v_{in}$$

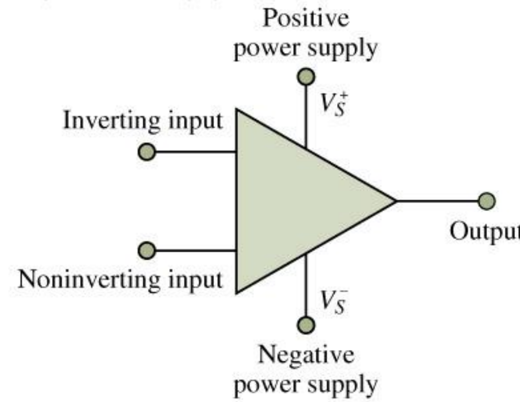
Op-Amp as an Integrated Circuit

- The integrated circuit operational amplifier evolved soon after development of the first bipolar integrated circuit.
- The μA-709 was introduced by Fairchild Semiconductor in 1965.
- Since then, a vast array of op-amps with improved characteristics, using both bipolar and MOS technologies, have been designed.
- Most op amps are inexpensive (less than a dollar) and available from a wide range of suppliers.
- There are usually 20 to 30 transistors that make up an op-amp circuit.
- From a signal point of view, the op-amp has two input terminals and one output terminal as shown in the following figures.
- The ideal op-amp senses the difference between two input signals and amplifies the difference to produce an output signal.
- Ideally, the input impedance is infinite, which means that the input current is zero. The output impedance is zero.

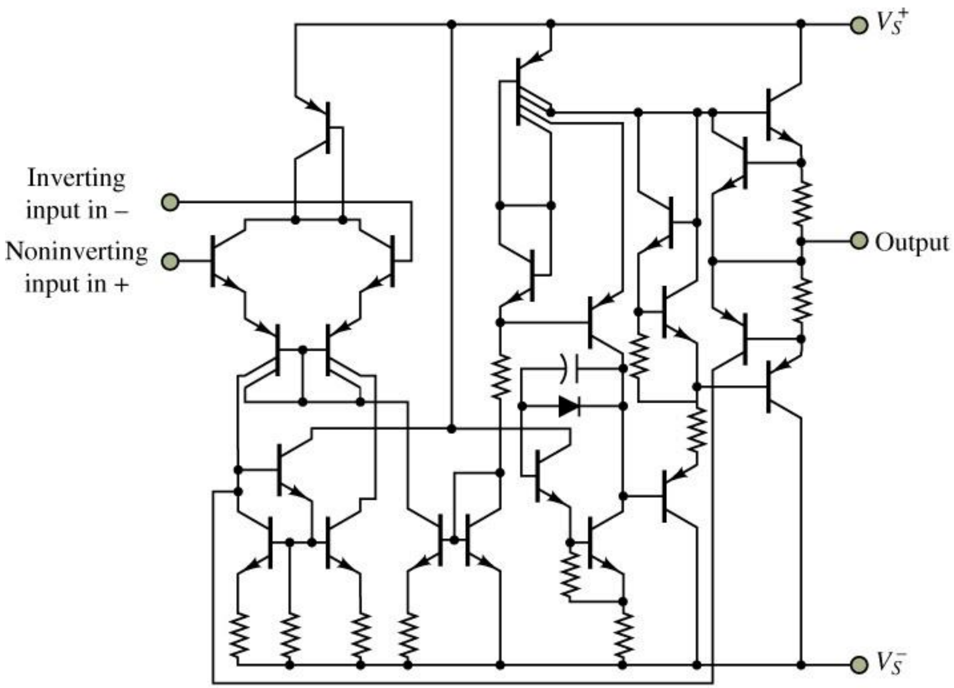
Operational Amplifier Model Symbols and Circuit Diagram



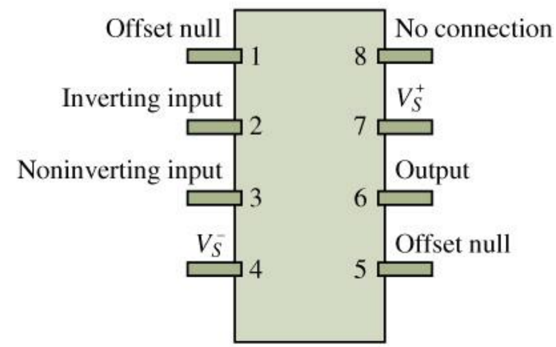
Op-amp model



Simplified circuit symbol

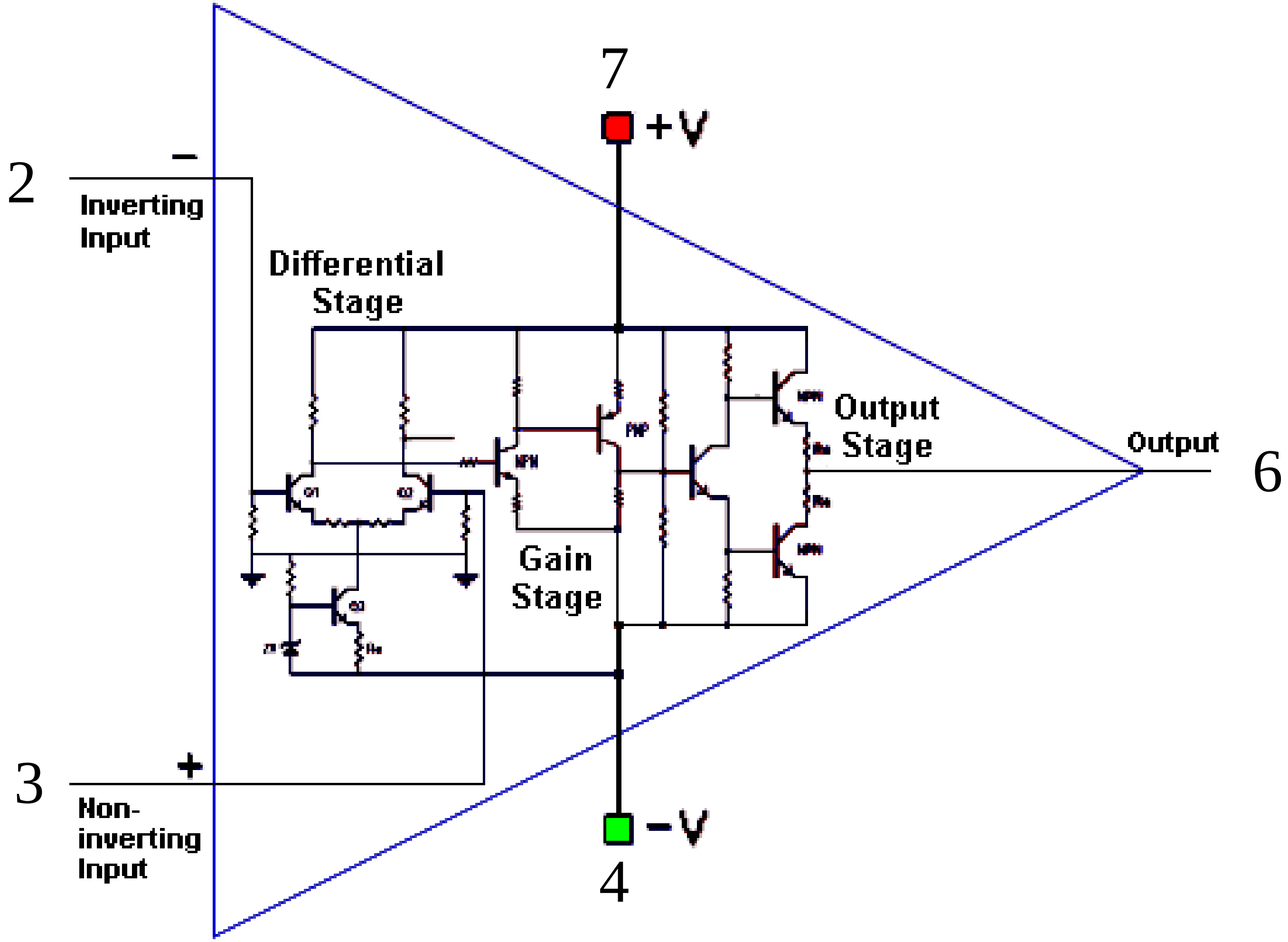


IC op-amp diagram

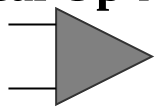


Integrated-circuit operational amplifier (IC op-amp)

Op-Amp Stages with Pin-outs of IC741

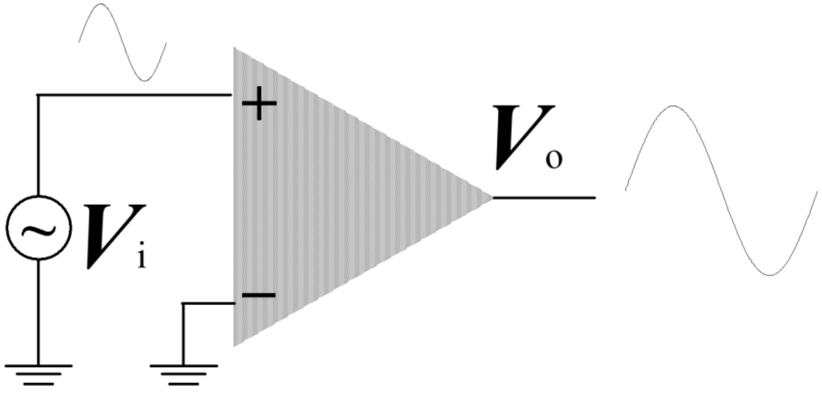


Ideal Versus Real Op-Amps

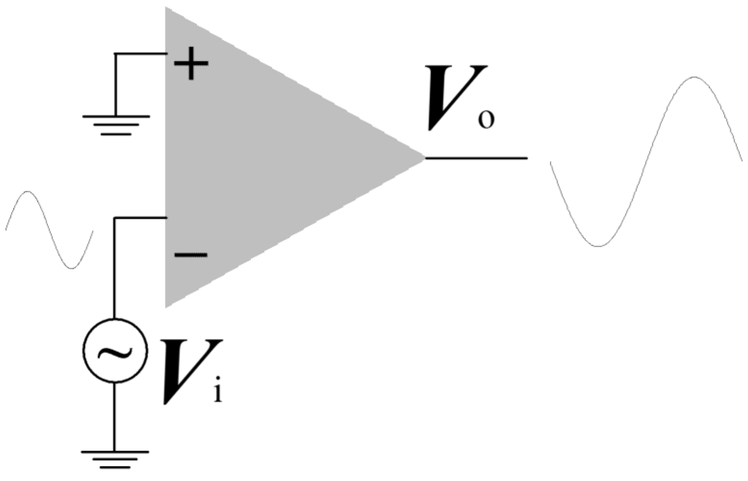
Characteristics	Ideal Op-Amp	Typical Op-Amp
		
Input Resistance	Infinity	$10^6\ \Omega$ (bipolar) $10^9\ \Omega$ - $10^{12}\ \Omega$ (FET)
Input Current	0	$10^{-12} – 10^{-8}$ A
Output Resistance	0	100 – 1000 Ω
Operational Gain	Infinity	10^5 - 10^9
Common Mode Gain	0	10^{-5}
Bandwidth	Infinity	Attenuates and phases at high frequencies (depends on slew rate)
Temperature	Independent	Bandwidth and gain

http://hyperphysics.phy-astr.gsu.edu/hbase/electronic/opampcon.html#c1

Single Inputs Op-Amp

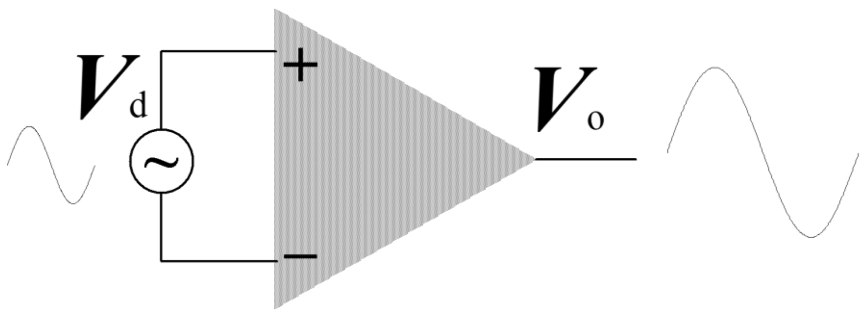


- + Terminal : Source
- – Terminal : Ground
- 0^o Phase change

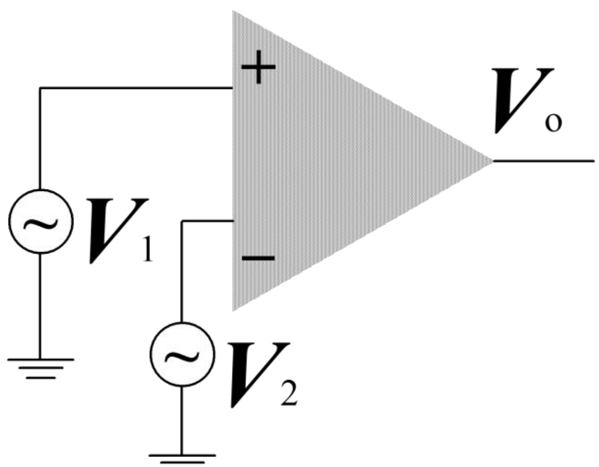


- + Terminal : Ground
- – Terminal : Source
- 180^o Phase change

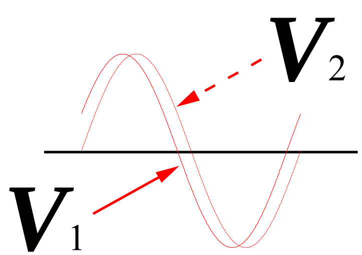
Double Inputs Op Amp



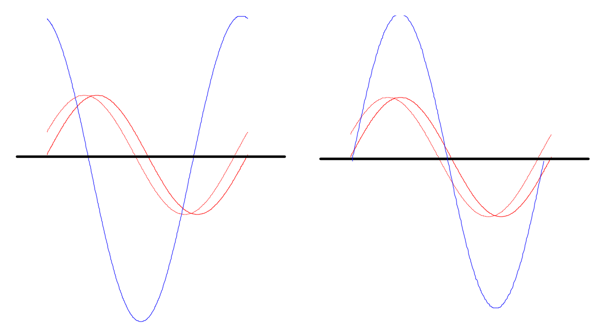
- Differential input
- $V_d = V_+ - V_-$
- 0^o phase shift change between V_o and V_d



Queation: What V_o should be if,

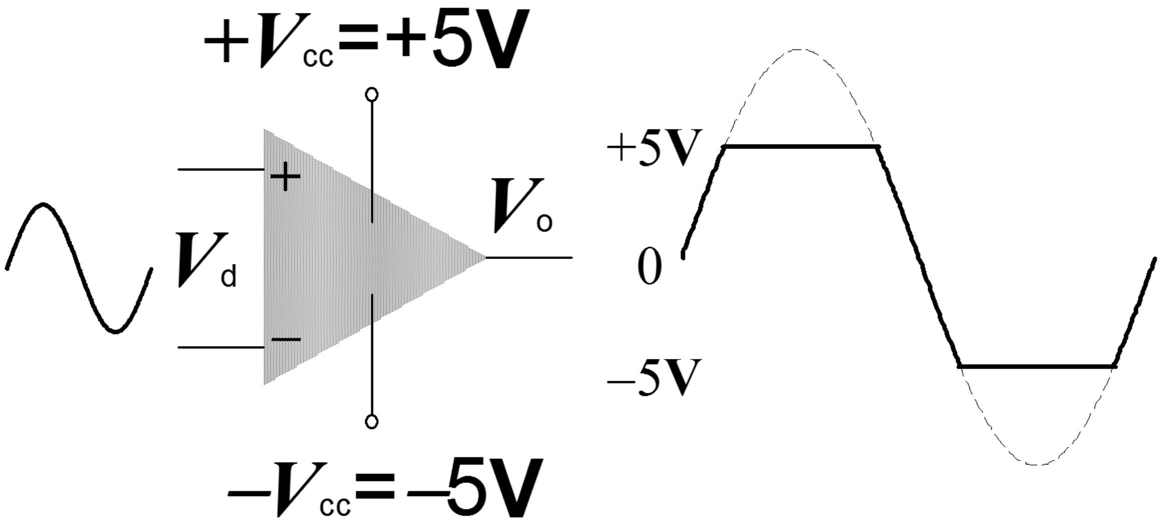


Ans: (A or B) ?



(A) (B)

Distortion / Saturation

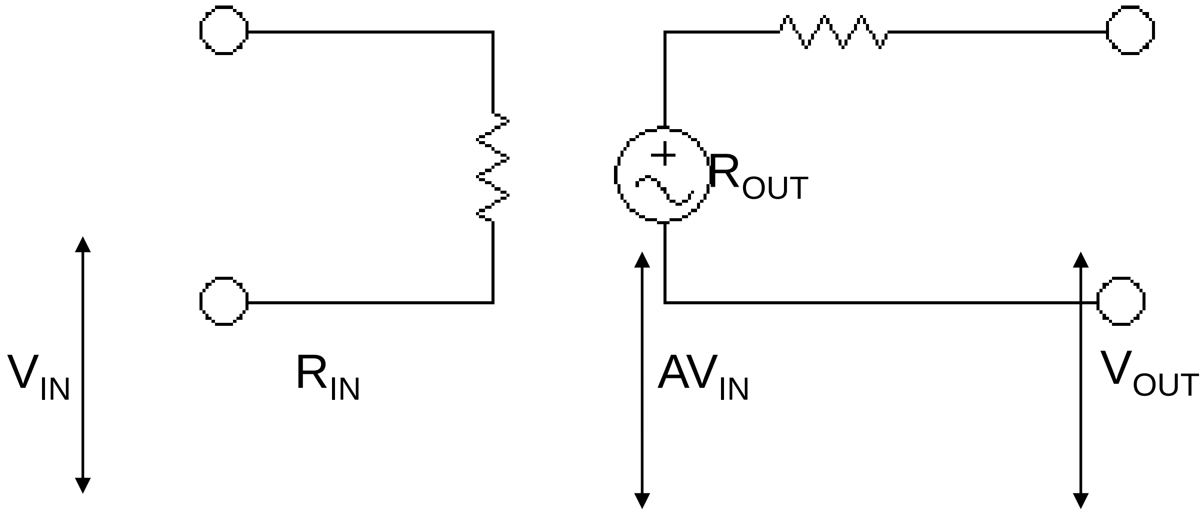


- The output voltage never exceeds the DC voltage supply of the Op-Amp.
- Practical Op Amps have limited output voltage and current ranges.
- **Voltage:** Usually limited to a few volts less than power supply span.
- **Current:** Limited by additional circuits (to limit power dissipation or protect against accidental short circuits).

A Practical Application: Why Feedback

- Self-balancing mechanism, which allows the amplifier to preserve zero potential difference between its input terminals.
- A practical example that illustrates a common application of negative feedback is the thermostat. This simple temperature control system operates by comparing the desired ambient temperature and the temperature measured by the thermometer and turning a heat source on and off to maintain the difference between actual and desired temperature as close to zero as possible.

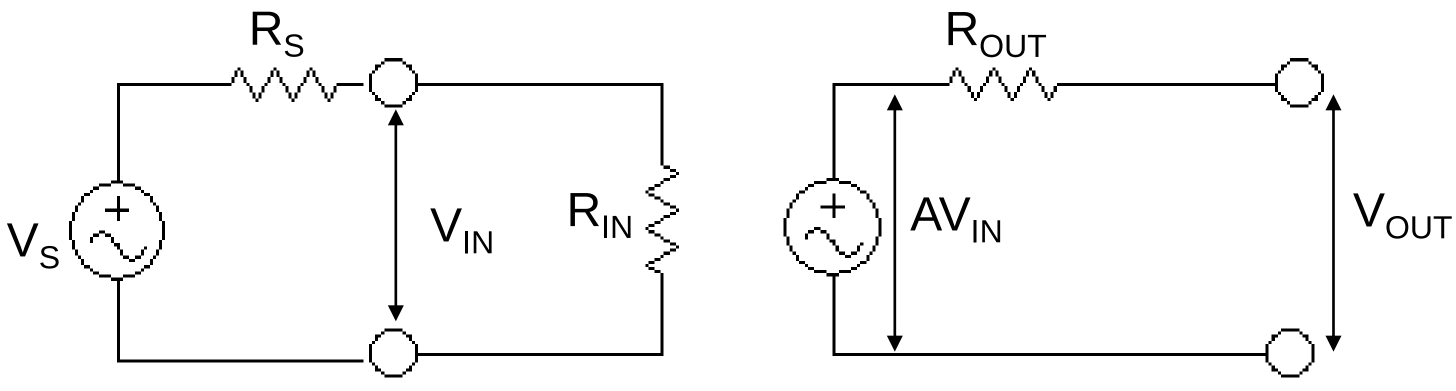
Impedances



- The amplifier measures voltage across R_{IN}, then generates a voltage which is larger by a factor **A**
- This voltage generator, in series with the output resistance R_{OUT}, is connected to the output port.
- **A** should be a constant (gain is linear)

Impedances

Add an input - a source voltage V_S plus source impedance R_S



Note the voltage divider $R_S + R_{IN}$.

$$V_{IN}=V_S(R_{IN}/(R_{IN}+R_S)$$

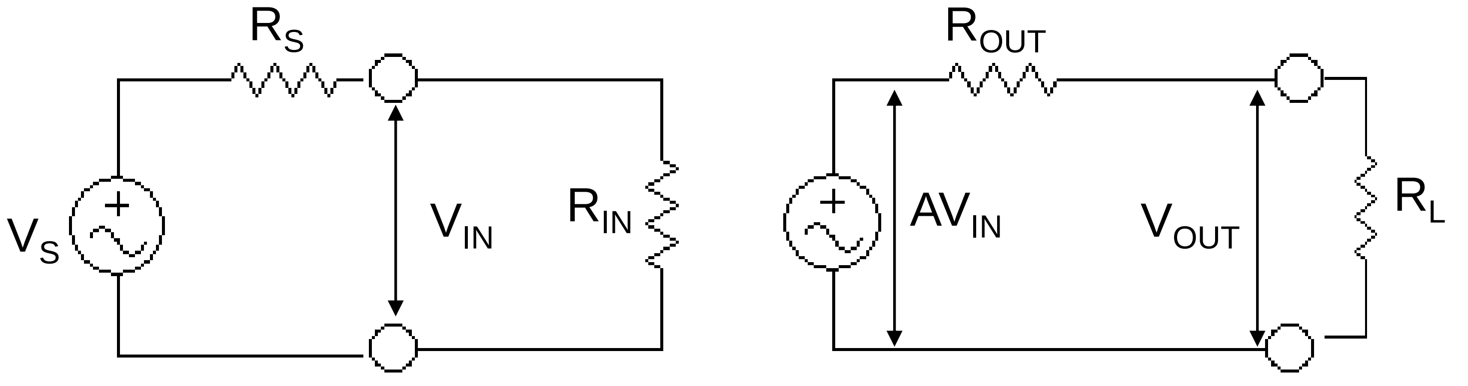
We want $V_{IN} = V_S$ regardless of source impedance

So want R_{IN} to be large.

The ideal amplifier has an infinite input impedance!

Impedances

Add a load - an output circuit with a resistance R_L



Note the voltage divider R_{OUT} + R_L.

$$V_{OUT}=AV_{IN}(R_L/(R_L+R_{OUT}))$$

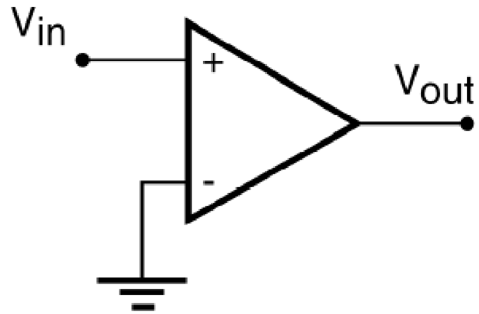
Want $V_{OUT}=AV_{IN}$ regardless of load

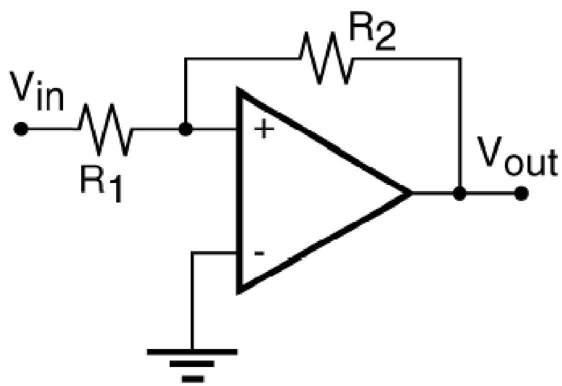
We want R_{OUT} to be small.

The ideal amplifier has zero output impedance!

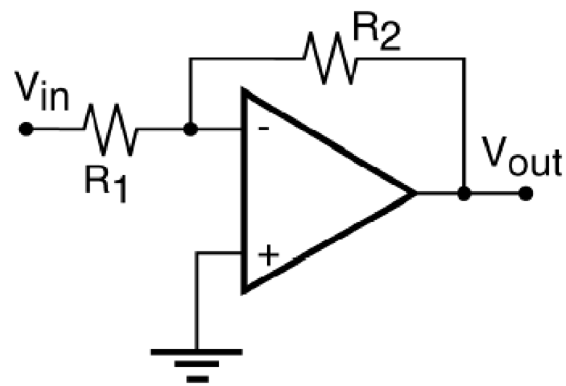
Feedback

Feedback refers to connecting the output of the amplifier to its input





Positive Feedback



Negative Feedback

Op-Amps for Math

- Inverting
- Non-Inverting
- Summing
- Differencing
- Integrating
- Differentiating

Common Op-Amp Configurations

Inverting Amplifier

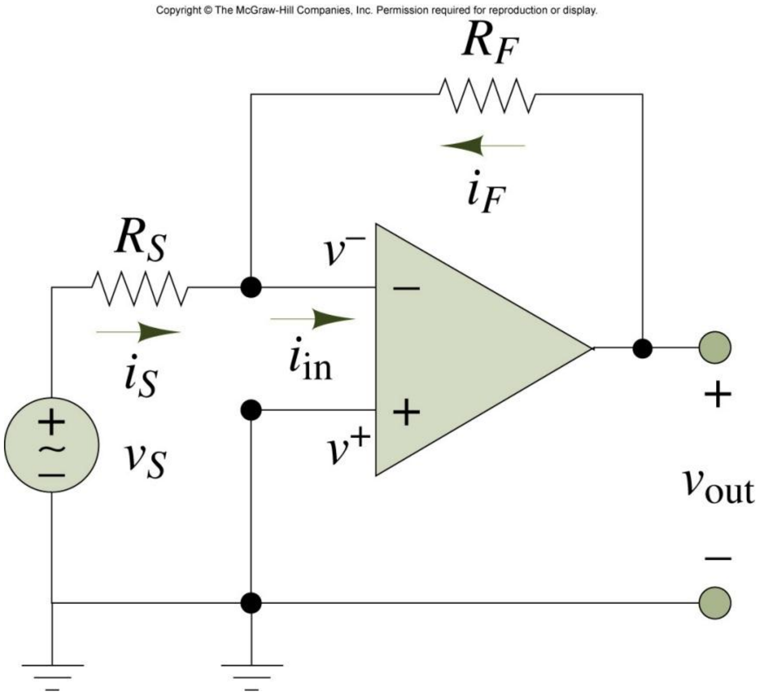
$$i_S + i_F = i_{in}$$

$$i_S = \frac{v_S - v^-}{R_S}; i_F = \frac{v_{out} - v^-}{R_F}; i_{in} = 0$$

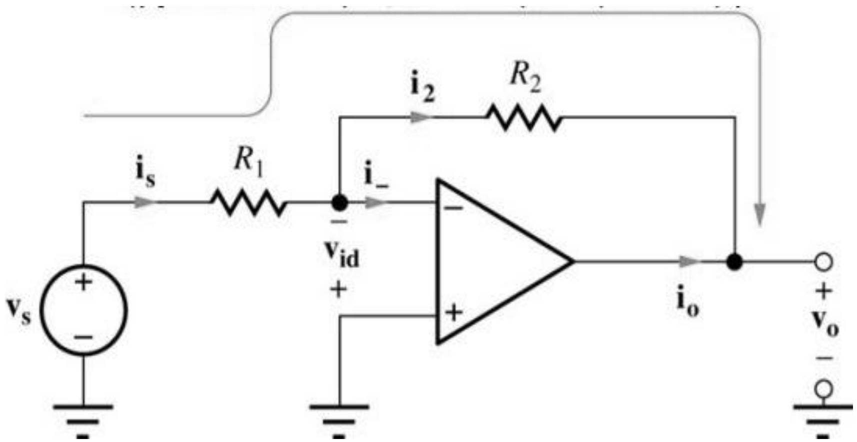
$$i_S = -i_F; i_{in} = 0; v^- = v^+$$

$$\frac{v_S}{R_S} + \frac{v_{out}}{A_v R_S} = -\frac{v_{out}}{R_F} - \frac{v_{out}}{A_v R_F}$$

$$v_{out} = -\frac{R_F}{R_S} v_S$$



Voltage Gain of Inverting Amplifier



$$v_s - i_s R_1 - i_2 R_2 - v_o = 0$$

But $i_s = i_2$ and $v_- = 0$ (since $v_{id} = v_+ - v_- = 0$)

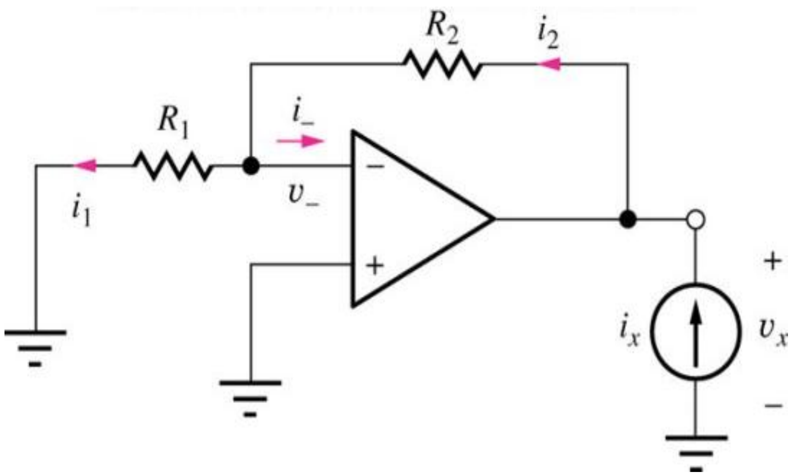
$$\therefore i_s = \frac{v_s}{R_1} \quad \text{and} \quad A_v = \frac{v_o}{v_s} = -\frac{R_2}{R_1}$$

- The negative voltage gain implies that there is a 180^0 phase shift between both dc and sinusoidal input and output signals.
- The gain magnitude can be greater than 1 if $R_2 > R_1$
- The gain magnitude can be less than 1 if $R_1 > R_2$

•The inverting input of the Op Amp is at ground potential (although it is physically connected to ground) and is said

to be at virtual ground.

Input and Output Resistances



R_{out} is found by applying a test current (or voltage) source to the amplifier output and determining the voltage (or current) after turning off all independent sources. Hence, $v_s = 0$

$$v_x = i_2 R_2 + i_1 R_1$$

But $i_1 = i_2$

$$\therefore v_x = i_1 (R_2 + R_1)$$

$$R_{in} = \frac{v_s}{i_s} = R_1 \text{ since } v_- = 0$$

Since $v_- = 0$, $i_1 = 0$. Therefore $v_x = 0$ irrespective of the value of i_x .

$$\therefore R_{out} = 0$$

Inverting Amplifier: An Example

- **Problem**: Design an inverting amplifier for
- **Given Data**: A_v= 20 dB, R_{in} = 20kΩ,
- **Assumptions:** Ideal op amp
- **Analysis**: Input resistance is controlled by R_1 and voltage gain is set by R_2 / R_1.

$$A_v(dB) = 20\log_{10}(|A_v|), \quad \therefore |A_v| = 10^{40\text{dB}/20\text{dB}} = 100 \quad \text{and } A_v = -100$$

A minus sign is added since the amplifier is inverting.

$$R_1 = R_{in} = 20\text{k}\Omega$$

$$A_v = -\frac{R_2}{R_1} \Rightarrow R_2 = 100R_1 = 2M\Omega$$

Design Example: Design an inverting amplifier with a closed loop voltage of A_v = -5. Assume the op-amp is driven by a sinusoidal soorce, v_s = 0.1 sin ωt volts, which has a source resistance of R_{sr} = 1 kΩ and which supply a maximum current of 5 μA. Assume that the frequency is low.

$$i_S = \frac{v_S}{R_S} \text{ (} R_S \text{ in this example means two resistances} : R_S = R_1 + R_{sr}.$$

$$R_{sr} \text{ represents the source resistance. Therefore } i_S = \frac{v_S}{R_{sr} + R1}$$

$$\text{If } i_S(\max) = 5\mu\text{A, then we write } R_1(\min) + R_{sr} = \frac{v_S(\max)}{i_S(\max)} = \frac{0.1}{5\times10^{-6}} = 20\text{ k}\Omega$$

$$R_1 \text{ should be } 19\text{ k}\Omega \quad \text{and } A_v = \frac{-R_2}{R_{sr} + R_1} = -5.$$

$$\text{Accordingly } R_2 = 5(R_{sr} + R_1) = 5\times20 = 100\ \text{k}\Omega$$

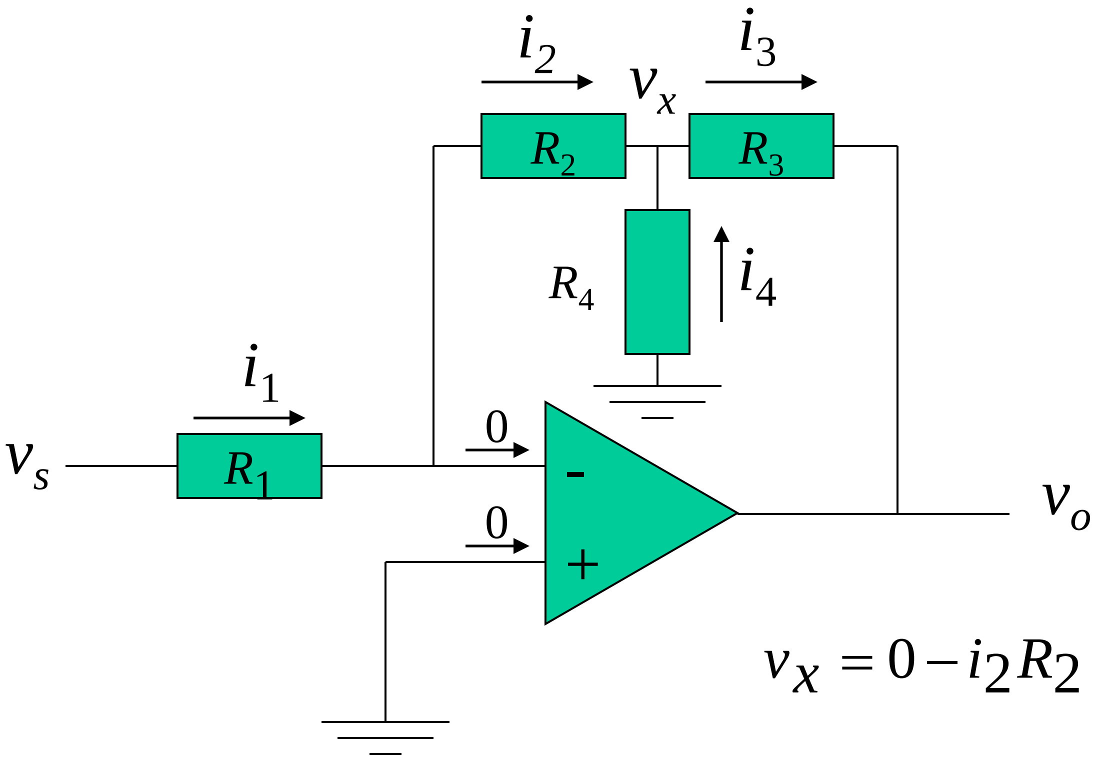
To Solve Ideal Op-Amp Circuit

- If the noninverting terminal of the op-amp is at ground potential, then the inverting terminal is at virtual ground. Sum currents at this point, assuming zero current enters the op-amp itself.

- If the noninverting terminal of the op-amp is not at ground potential, then the inverting terminal voltage is equal to that at the noninverting terminal. Sum currents at the inverting terminal node, assuming zero current enters the op-amp itself.

- For an ideal op-amp circuit, the output voltage is determined from either step 1 or step 2 above and is independent of any load connected to the output terminal.

Inverting Amplifier with a T-Network



$$v_x = 0 - i_2 R_2 = -v_s \left(\frac{R_2}{R_1}\right)$$

$$i_2 + i_4 = i_3; -\frac{v_x}{R_2} - \frac{v_x}{R_4} = \frac{v_x - v_o}{R_3}$$

Combing the above equations we get

$$A_v = \frac{v_o}{v_s} = -\frac{R_2}{R_1}\left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2}\right)$$

Design Example: An op-amp with a T-network is to be used as a preamplifier for a microphone. The maximum microphone output voltage is 12 mV (rms) and the microphone has an output resistance of 1 kΩ. The op-amp circuit is to be designed such that the maximum output voltage is 1.2 V (rms). The input amplifier resistance should be fairly large but all resistance values should be less than 500 kΩ.

$$|A_v| = \frac{1.2}{0.012} = 100$$

$$A_v = -\frac{R_2}{R_1}\left(1 + \frac{R_3}{R_4}\right) - \frac{R_3}{R_1}$$

If we choose $\frac{R_2}{R_1} = \frac{R_3}{R_1} = 8$

$$-100 = -8\left(1 + \frac{R_3}{R_4}\right) - 8; \quad \frac{R_3}{R_4} = 10.5$$

We should include the value of the source resistance in the calculation

If we set $R_1 = 49\ \text{k}\Omega$ and $R_{sr} = 1\ \text{k}\Omega$ then the total resistance ($R_{1\,\text{effective}}$) will be 50 kΩ

$R_2 = R_3 = 400\ \text{k}\Omega$ and $R_4 = 38.1\ \text{k}\Omega$

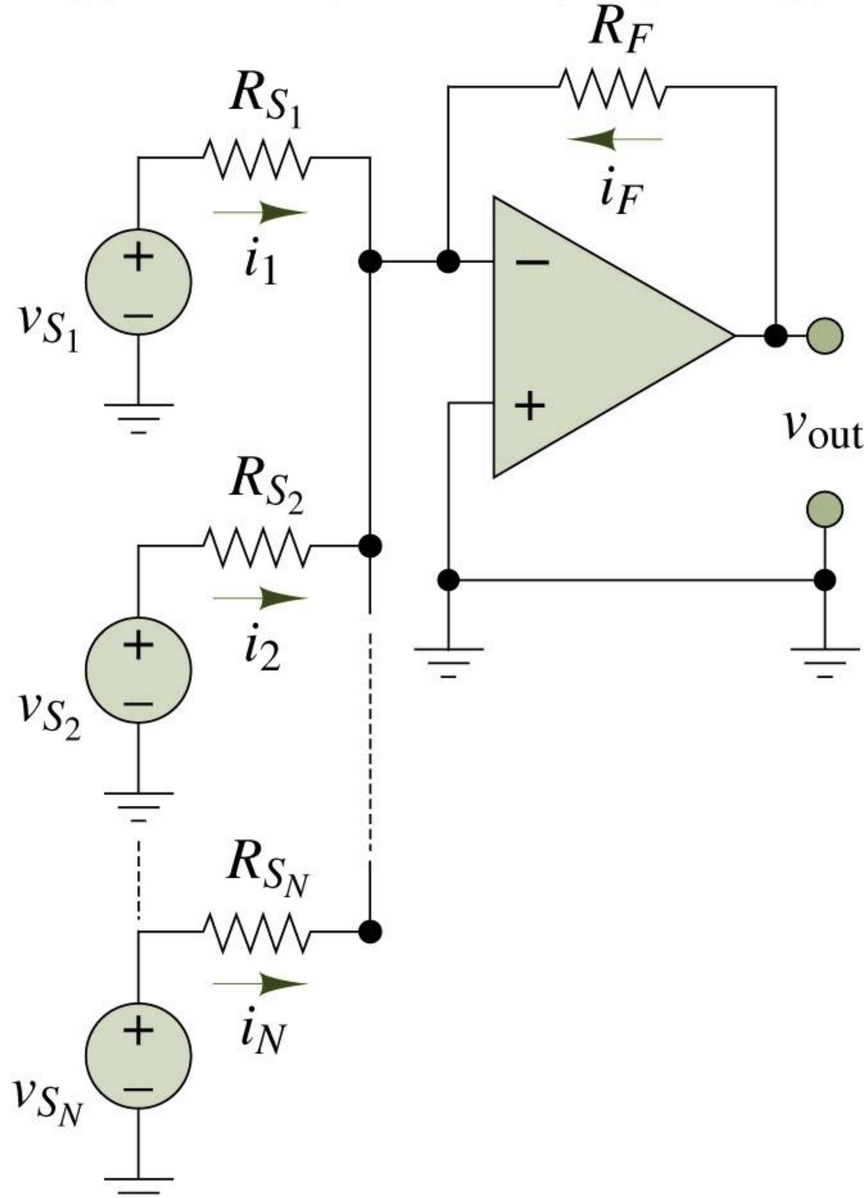
Summing Amplifier

$$i_1 + i_2 + \ldots + i_N = -i_F$$

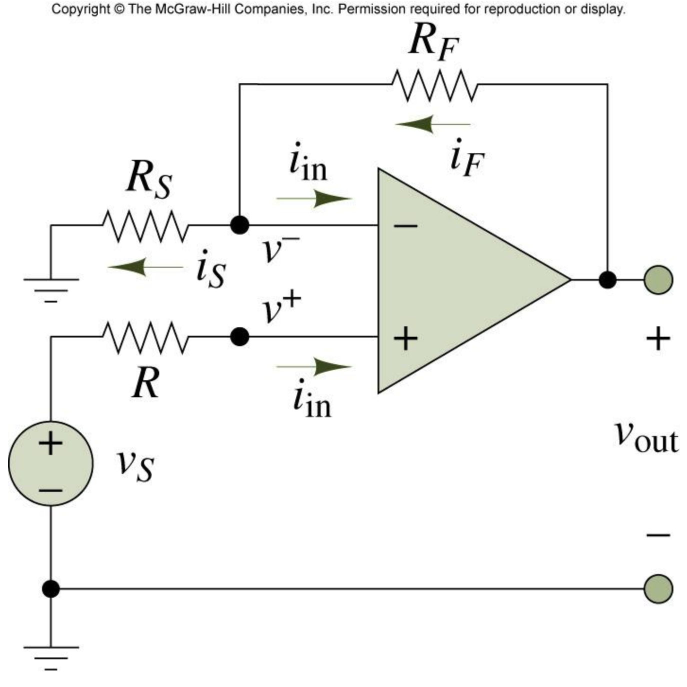
$$i_n = \frac{v_{S_n}}{R_{S_n}} \ldots\ldots n = 1,2,\ldots N$$

$$i_F = \frac{v_{out}}{R_F}$$

$$v_{out} = -\sum_{n=1}^{N} \frac{R_F}{R_{S_n}} v_{S_n}$$

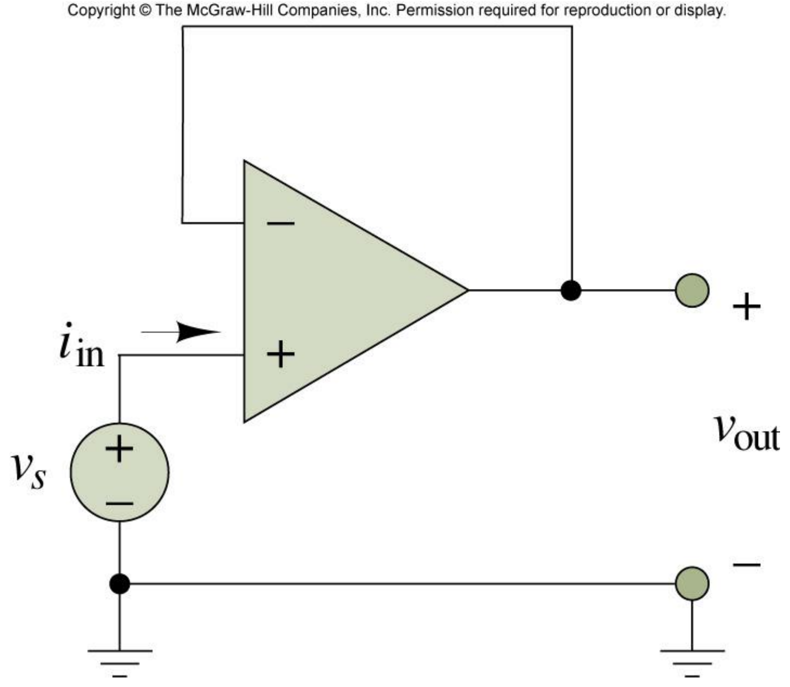


Non-inverting Amplifier



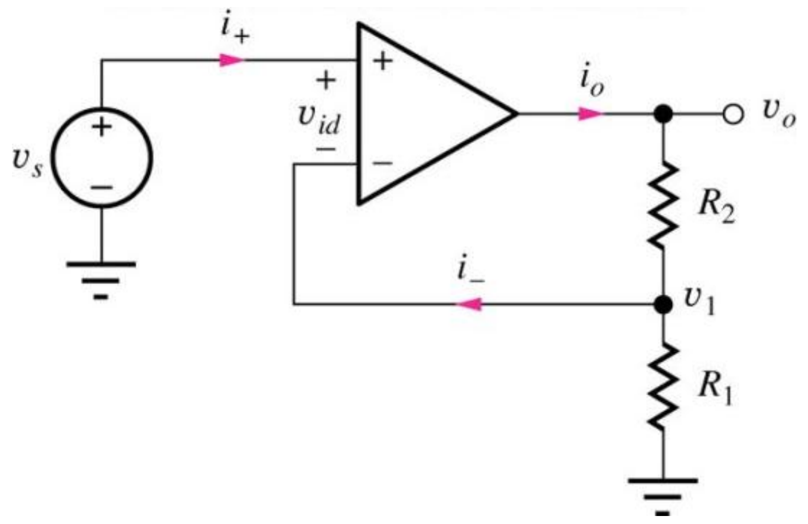
$$\frac{v_{out}}{v_S} = 1 + \frac{R_F}{R_S}$$

Voltage Follower



$$v_S = v_{out}$$

Non-inverting Amplifier



The input signal is applied to the non-inverting input terminal.

A portion of the output signal is fed back to the negative input terminal.

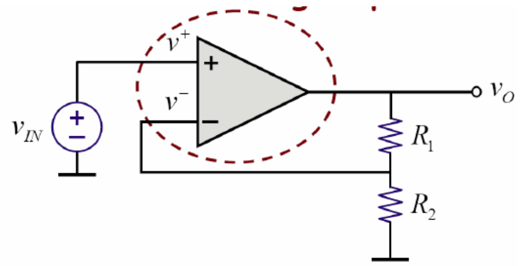
Analysis is conducted by relating the voltage at v_1 to input voltage v_s and output voltage v_o.

Non-inverting Amplifier

Voltage Gain, Input Resistance and Output Resistance

Since $i_- = 0$ and $v_s - v_{id} = v_1$

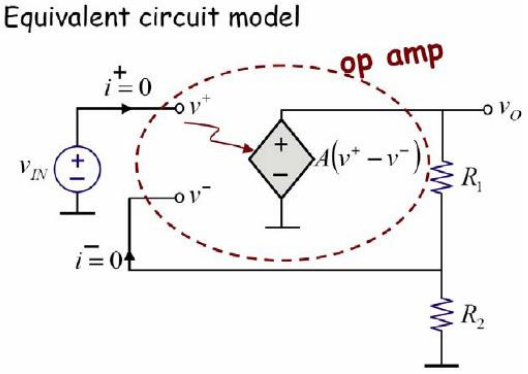
$$v_1 = v_o \frac{R_1}{R_1 + R_2}$$



But $v_{id} = 0$

$$\therefore v_s = v_1$$

$$v_o = v_s \frac{R_1 + R_2}{R_1}$$



$$\therefore A_v = \frac{v_o}{v_s} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

Since $i_+ = 0$

$$R_{in} = \frac{v_s}{i_+} = \infty$$

R_{out} is found by applying a test current source to the amplifier output after setting $v_s = 0$. It is identical to the output resistance of the inverting amplifier i.e. $R_{out} = 0$.

Non-inverting Amplifier: An Example

- **Problem**: Determine the output voltage and current for the given non-inverting amplifier.
- **Given Data**: R_1= 3kΩ, R_2 = 43kΩ, v_s= +0.1 V
- **Assumptions:** Ideal op amp
- **Analysis**:

$$A_v = 1 + \frac{R_2}{R_1} = 1 + \frac{43\text{k}\Omega}{3\text{k}\Omega} = 15.3$$

$$\text{v}_\text{O} = A_v \text{v}_\text{S} = (15.3)(0.1\text{V}) = 1.53\text{V}$$

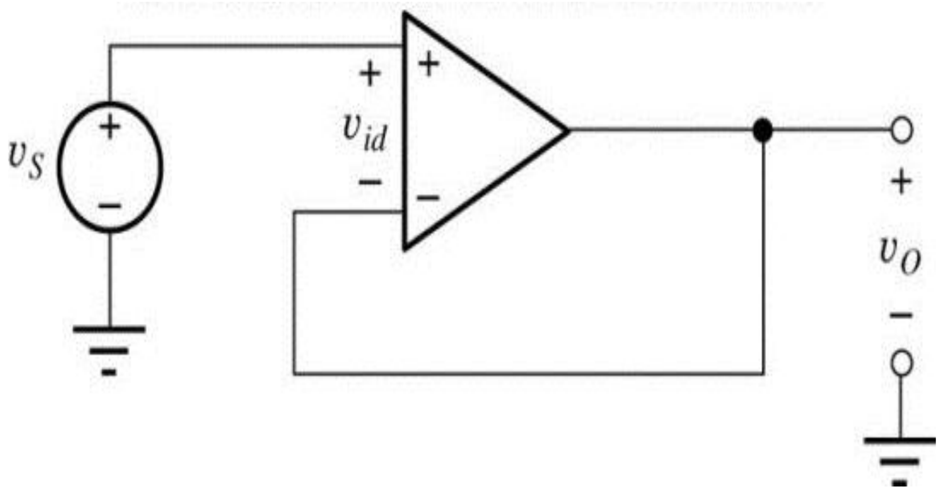
Since $i_- = 0$,

$$\text{i}_\text{O} = \frac{\text{v}_\text{O}}{R_2 + R_1} = \frac{1.53\text{V}}{43\text{k}\Omega + 3\text{k}\Omega} = 33.3\mu\text{A}$$

Design Example: Design a noninverting amplifier with a closed loop gain of $A_v = 5$. The output voltage is limited to $-10 \text{ V} \leq v_o \leq +10 \text{ V}$ and the maximum current in any resistor is limited to 50 μA

Answer: $R_1 = 40 \text{ k}\Omega$, $R_2 = 160 \text{ k}\Omega$

The Unity Gain Amplifier or “The Buffer”

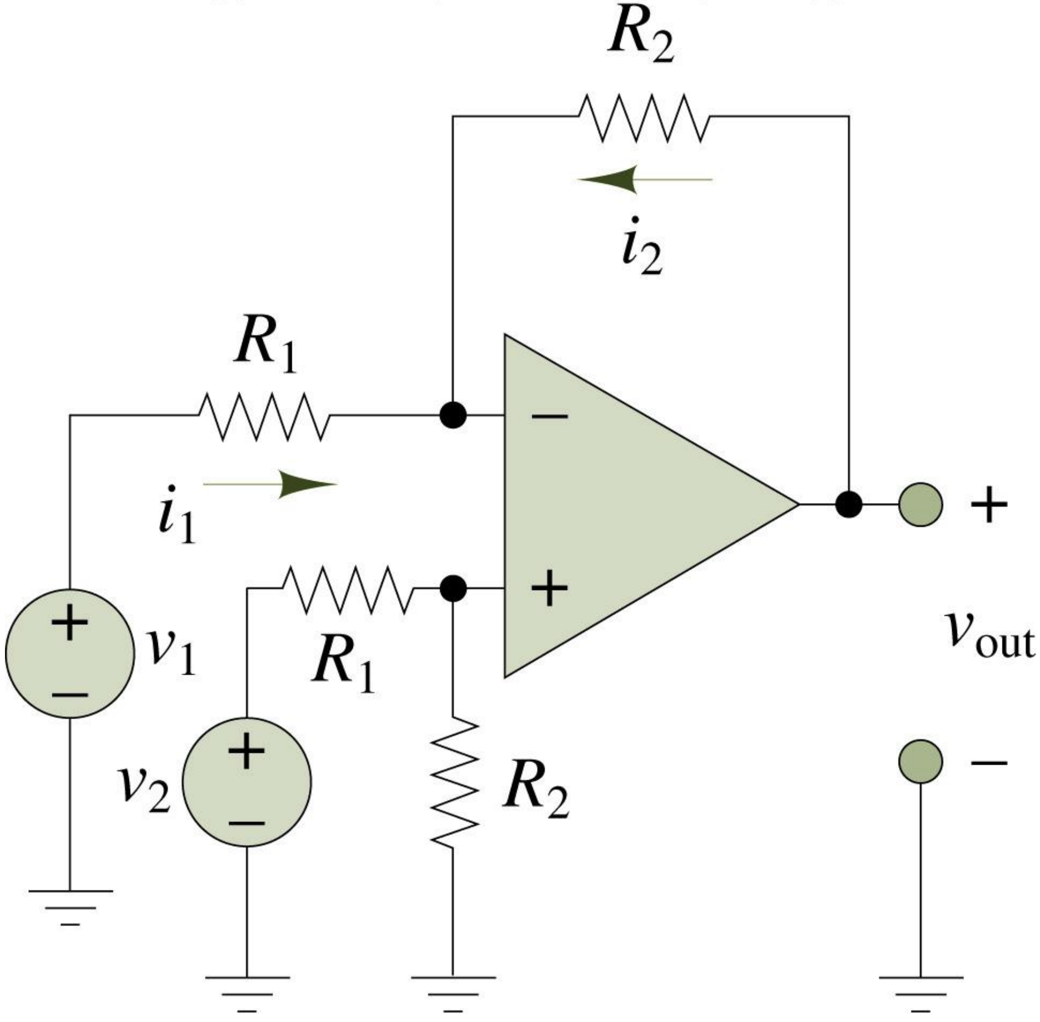


Buffer is a special case of the non-inverting amplifier with infinite R_1 and zero R_2. Hence $A_v = 1$.

It provides an excellent electrical isolation while maintaining the signal voltage level. The ideal buffer requires no input current and can drive any desired load resistance without loss of signal voltage.

Buffer is used in many data acquisition system applications.

Differential Amplifier



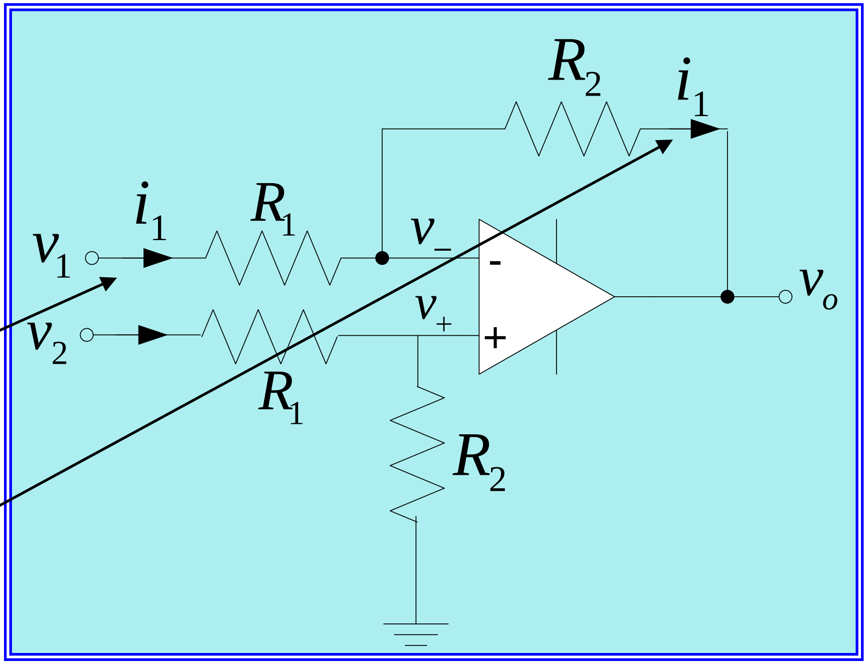
$$\frac{v_1 - v^-}{R_1} = -\frac{v_{out} - v^-}{R_2}$$

$$v^- = v^+$$

$$v^+ = \frac{R_2}{R_1 + R_2} v_2 = v^-$$

$$v_{out} = \frac{R_2}{R_1}(v_2 - v_1)$$

Differential Amplifier Using Op-Amp



$$v_- = v_+$$

$$i_1 = \frac{v_1 - v_-}{R_1}$$

$$i_1 = \frac{v_- - v_0}{R_2}$$

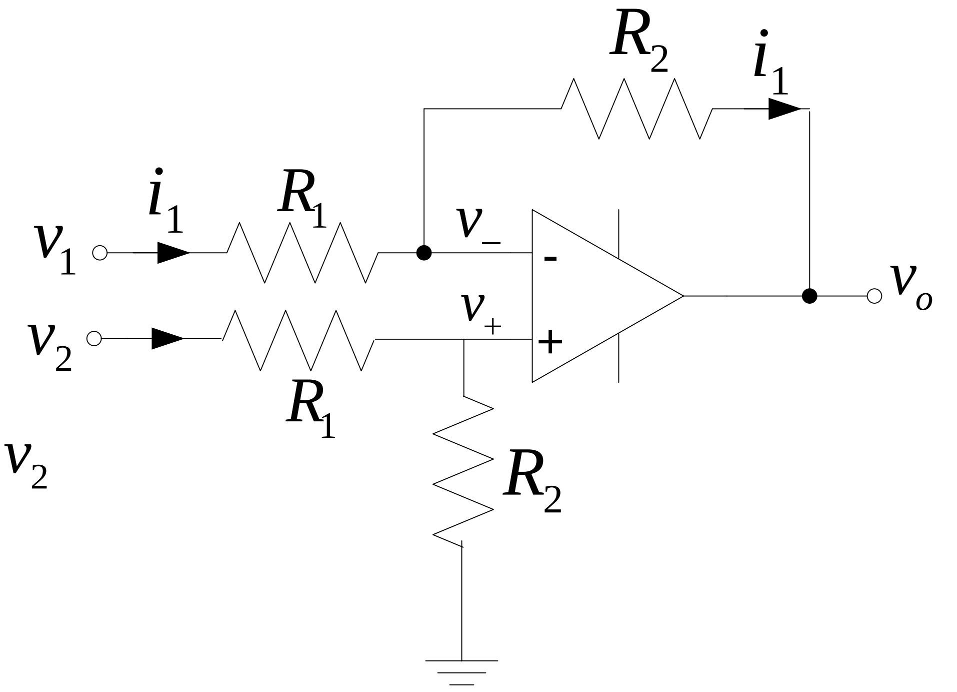
$$v_+ = \frac{R_2}{R_1 + R_2} v_2$$

$$\frac{v_1 - v_+}{R_1} = \frac{v_+ - v_0}{R_2}$$

$$\frac{v_1 - \frac{R_2}{R_1 + R_2} v_2}{R_1} = \frac{\frac{R_2}{R_1 + R_2} v_2 - v_0}{R_2}$$

Differential Amplifier Using Op-Amp

$$\frac{v_1 - \dfrac{R_2}{R_1 + R_2} v_2}{R_1} = \frac{\dfrac{R_2}{R_1 + R_2} v_2 - v_0}{R_2}$$



$$v_0 = -\frac{R_2}{R_1} v_1 + \frac{R_2}{R_1 + R_2} v_2 + \frac{R_2^2}{R_1 \left(R_1 + R_2 \right)} v_2$$

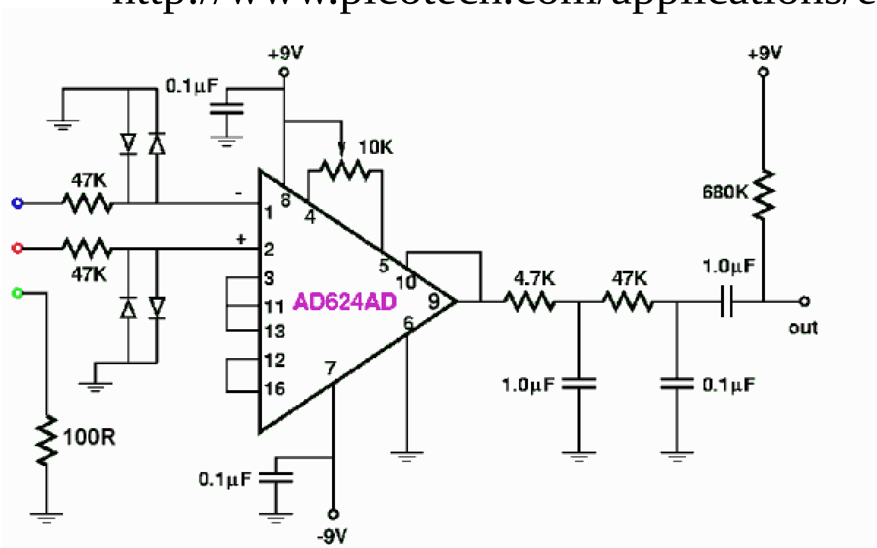
$$v_0 = -\frac{R_2}{R_1} v_1 + \frac{R_2}{R_1 + R_2} \left(1 + \frac{R_2}{R_1} \right) v_2$$

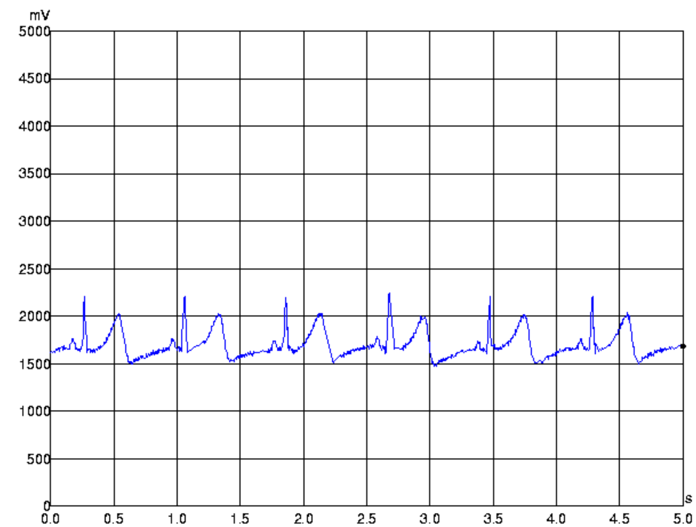
$$v_0 = \frac{R_2}{R_1} \left(v_2 - v_1 \right)$$

Differential Amplifier Applications

- Very useful if you have two inputs corrupted with the same noise
- Subtract one from the other to remove noise, remainder is signal
- Applications: Electrocardiagram to measure the potential difference between two points on the body

http://www.picotech.com/applications/ecg.html





The AD624AD is an **instrumentation** amplifier - this is a high gain, dc coupled differential amplifier with a high input impedance and high CMRR (the chip actually contains a few Op Amps)

Difference Amplifier: An Example

- **Problem**: Determine v_o
- **Given Data**: R_1= 10kΩ, R_2 =100kΩ, v_1=5 V, v_2=3 V
- **Assumptions:** Ideal op amp. Hence, $v_- = v_+$ and $i_- = i_+ = 0$.
- **Analysis:** Using dc values,

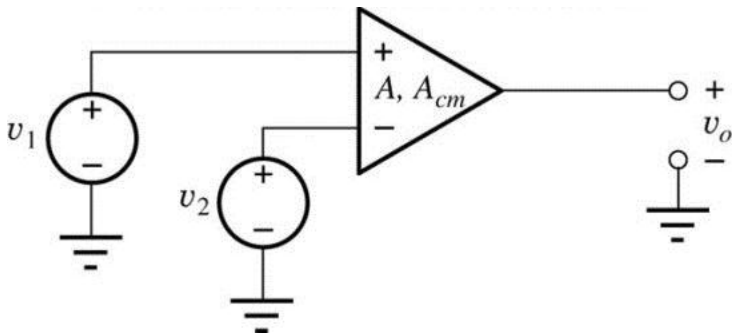
$$A_{dm} = -\frac{R_2}{R_1} = -\frac{100\text{k}\Omega}{10\text{k}\Omega} = -10$$

$$V_o = A_{dm}\left(V_1 - V_2\right) = -10(5-3)$$

$$V_o = -20.0 \text{ V}$$

Here A_{dm} is called the "differential mode voltage gain" of the difference amplifier.

Finite Common-Mode Rejection Ratio (CMRR)



A(or A_{dm}) = differential-mode gain

A_{cm} = common-mode gain

v_{id} = differential-mode input voltage

v_{ic} = common-mode input voltage

$$v_1 = v_{ic} + \frac{v_{id}}{2} \qquad v_2 = v_{ic} - \frac{v_{id}}{2}$$

A real amplifier responds to signal common to both inputs, called the common-mode input voltage (v_{ic}). In general,

$$v_o = A_{dm}(v_1 - v_2) + A_{cm}\left(\frac{v_1 + v_2}{2}\right)$$

$$v_o = A_{dm}(v_{id}) + A_{cm}(v_{ic})$$

An ideal amplifier has $A_{cm} = 0$, but for a real amplifier,

$$v_o = A_{dm}\left(v_{id} + \frac{A_{cm} v_{ic}}{A_{dm}}\right) = A_{dm}\left(v_{id} + \frac{v_{ic}}{\text{CMRR}}\right)$$

$$\text{CMRR} = \left|\frac{A_{dm}}{A_{cm}}\right|$$

and $\text{CMRR(dB)} = 20\log_{10}(\text{CMRR})$

Finite CMRR: An Example

- **Problem**: Find output voltage error introduced by finite CMRR.
- **Given Data**: A_{dm}= 2500, CMRR = 80 dB, v_1 = 5.001 V, v_2 = 4.999 V
- **Assumptions:** Op amp is ideal, except for CMRR. A CMRR in dB of 80 dB corresponds to a CMRR of 10^4.
- **Analysis:** $v_{id} = 5.001\text{V} - 4.999\text{V}$

$$v_{ic} = \frac{5.001\text{V} + 4.999\text{V}}{2} = 5.000\text{V}$$

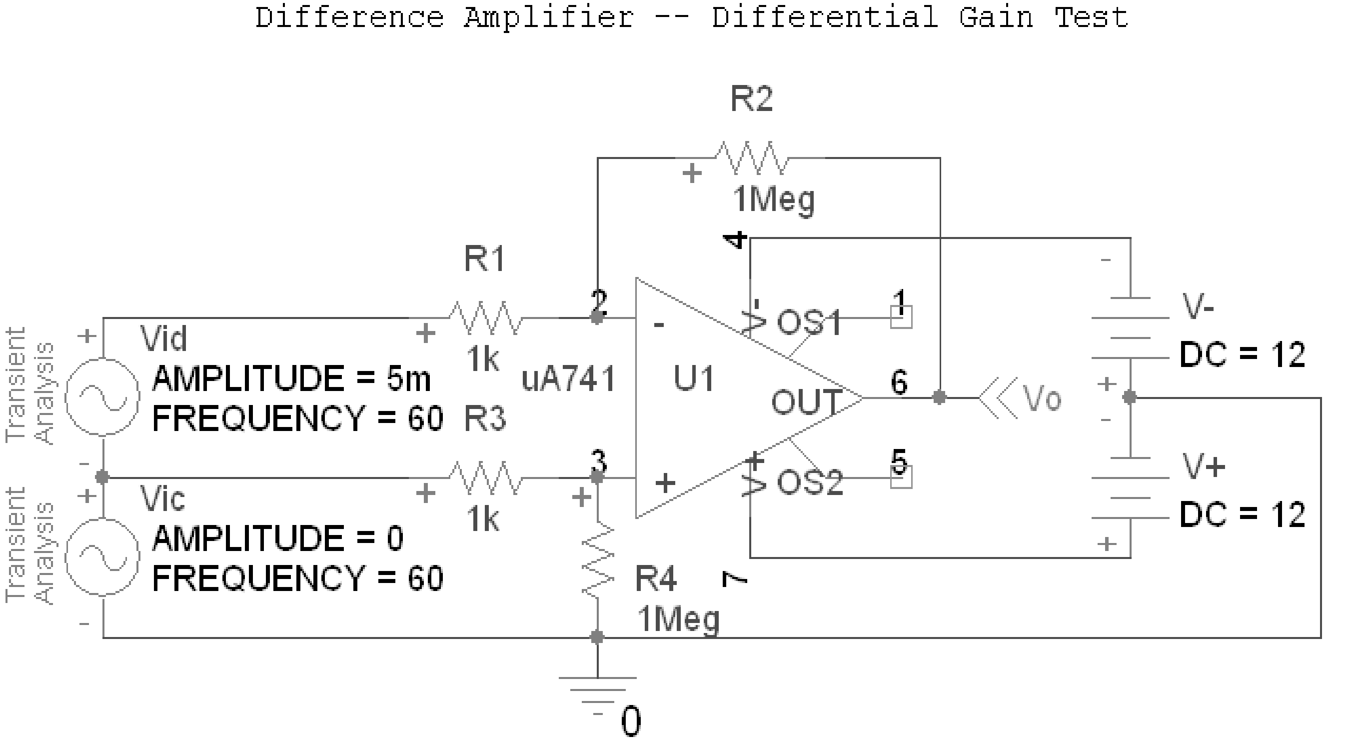
$$v_o = A_{dm}\left(v_{id} + \frac{v_{ic}}{\text{CMRR}}\right) = 2500\left(0.002 + \frac{5.000}{10^4}\right)\text{V} = 6.25\text{V}$$

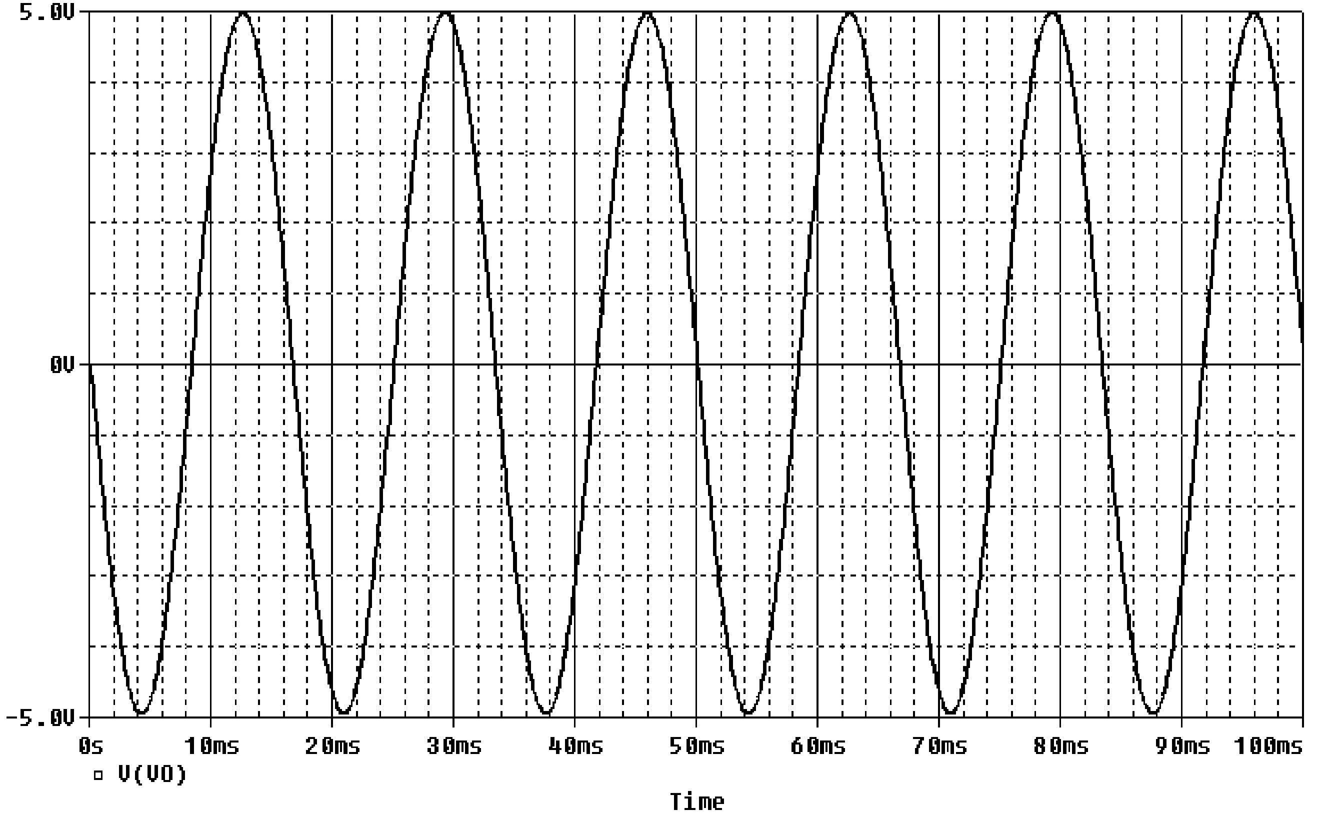
$$\text{In the "ideal" case, } v_o = A_{dm} v_{id} = 5.00 \text{ V}$$

$$\%\text{ output error} = \frac{6.25 - 5.00}{5.00} \times 100\% = 25\%$$

The output error introduced by finite CMRR is 25% of the expected ideal output.

uA741 CMRR Test
Differential Gain

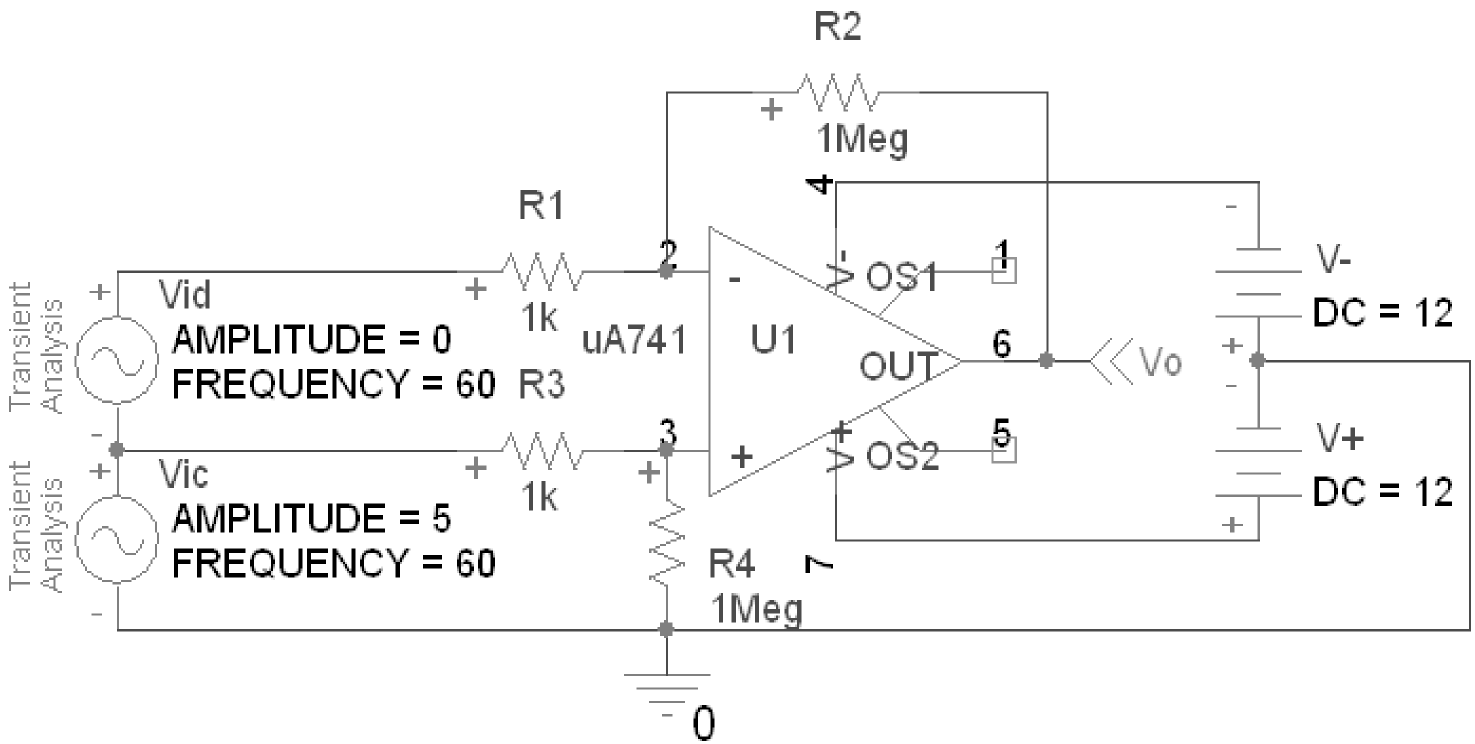


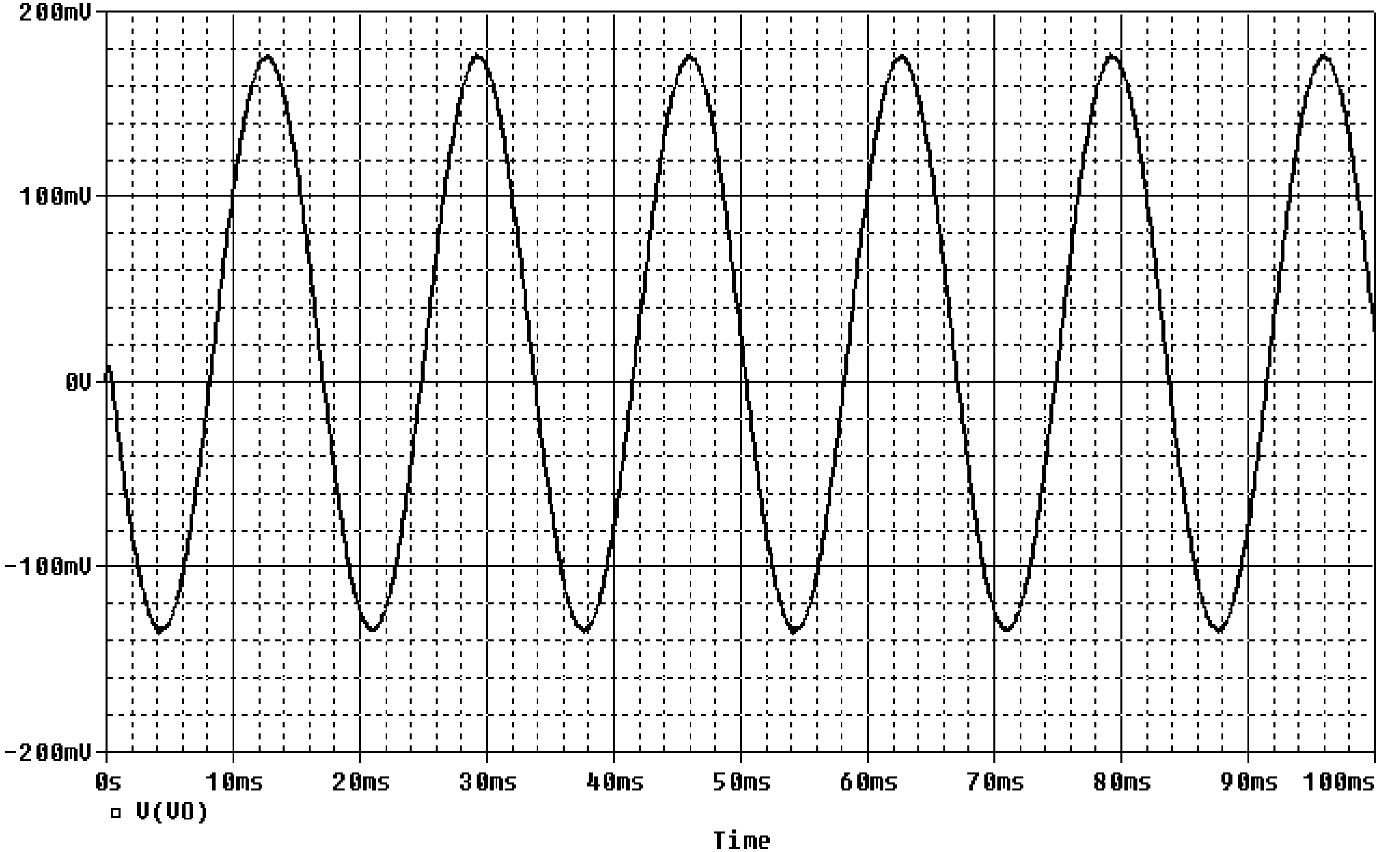


Differential Gain A_{dm} = 5 V/5 mV = 1000

uA741 CMRR Test
Common Mode Gain

Difference Amplifier -- Common Mode Gain Test





Common Mode Gain A_{cm} = 160 mV/5 V = 0.032

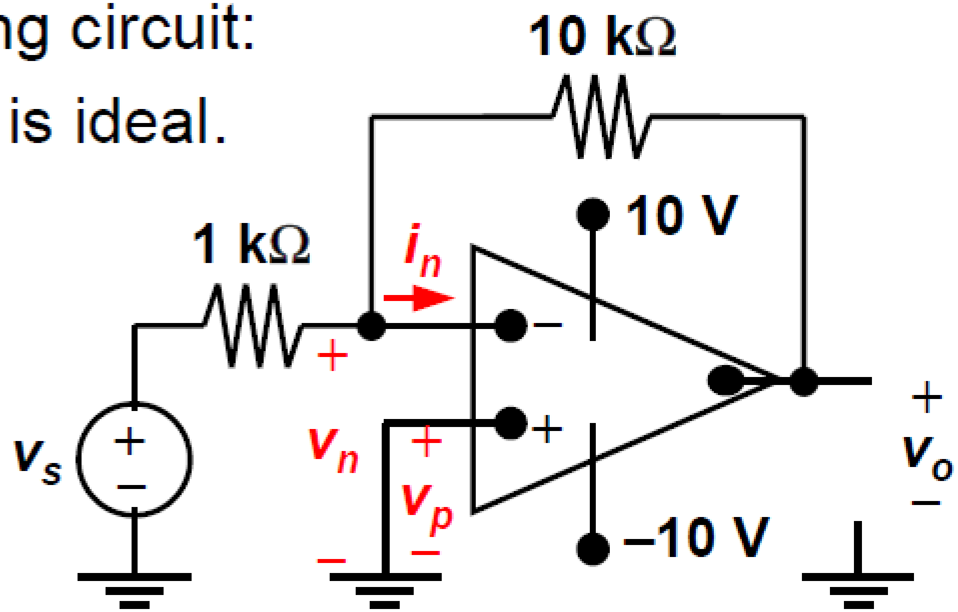
$$\text{CMRR} = \frac{|A_{dm}|}{|A_{cm}|} = \frac{1000}{.032} = 3.125x10^4$$

$$\text{CMRR(dB)} = 20\log_{10}(\text{CMRR}) = 89.9\ \text{dB}$$

Op-Amp circuit Analysis

Consider the following circuit:

Assume the op amp is ideal.



a) Calculate v_o if v_s = 100 mV

b) What is the ***voltage gain*** v_o/v_s of this amplifier?

c) Specify the range of values of v_s for which the op amp operates in a linear mode

What if the op amp is not ideal?

R_i = 10 kΩ

R_o = 1 kΩ

$A = 10^3$