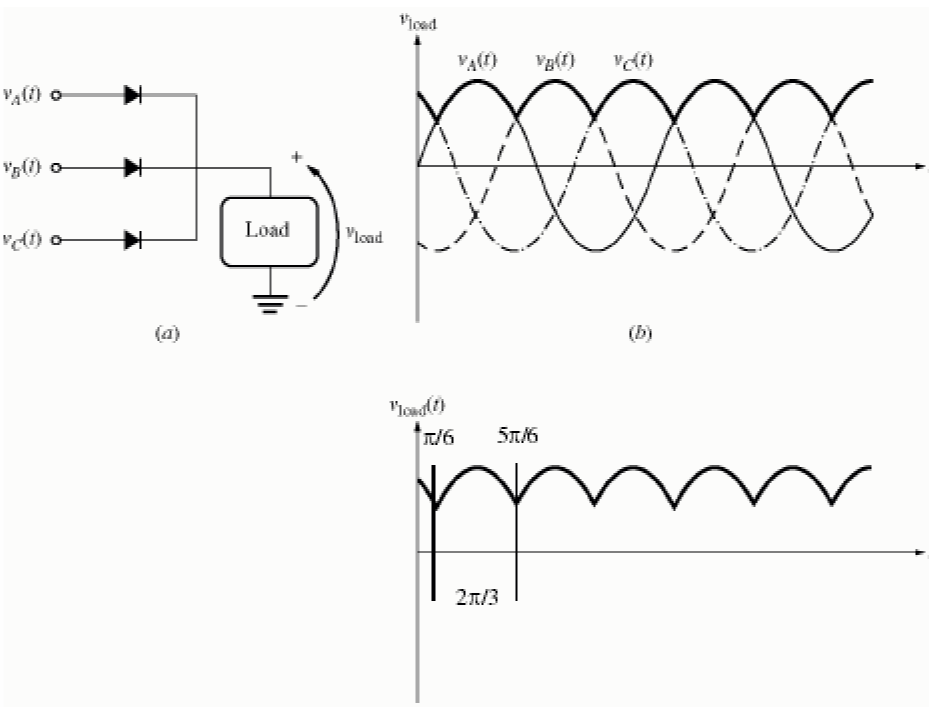
ELG3311: Tutorial for Chapter 3

Problem 3-1:

Calculate the ripple factor of a three-phase half-wave rectifier circuit, both analytically and using MATLAB.

Solution:

A three-phase half-wave rectifier and its output voltage are shown below:



$$v_A(t) = V_M \sin(\omega t)$$
$$v_B(t) = V_M \sin(\omega t - 2\pi/3)$$
$$v_C(t) = V_M \sin(\omega t + 2\pi/3)$$

The average voltage is

$$V_{DC} = \frac{1}{T}\int_0^T v(t)dt = \frac{3}{2\pi}\int_{\pi/6}^{5\pi/6} V_M \sin(\omega t)d(\omega t) = -\frac{3V_M}{2\pi}\cos\omega t\Big|_{\pi/6}^{5\pi/6}$$
$$V_{DC} = -\frac{3V_M}{2\pi}\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) = 0.8270V_M$$

The rms voltage is

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{3}{2\pi}\int_{\pi/6}^{5\pi/6} V_M{}^2 \sin^2(\omega t)d(\omega t)} = \sqrt{\frac{3V_M{}^2}{2\pi}\left(\frac{1}{2}\omega t - \frac{1}{4}\sin 2\omega t\right)\Bigg|_{\pi/6}^{5\pi/6}}$$

$$V_{rms} = \sqrt{\frac{3V_M{}^2}{2\pi}\left(\frac{1}{2}(\frac{5\pi}{6} - \frac{\pi}{6}) - \frac{1}{4}(\sin\frac{5\pi}{3} - \sin\frac{\pi}{3})\right)} = 0.8407V_M$$

The resulting ripple factor is

$$r = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1} \times 100\% = \sqrt{\left(\frac{0.8407V_M}{0.8270V_M}\right)^2 - 1} \times 100\% = 18.3\%$$

The MATLAB function is shown below

```
function volts = halfwave3(wt)
% Function to simulate the output of a three-phase
% half-wave rectifier.
% wt = Phase in radians (=omega x time)

% Convert input to the range 0 <= wt < 2*pi
while wt >= 2*pi
      wt = wt - 2*pi;
end
while wt < 0
      wt = wt + 2*pi;
end

% Simulate the output of the rectifier.
a = sin(wt);
b = sin(wt - 2*pi/3);
c = sin(wt + 2*pi/3);

volts = max( [ a b c ] );
```

The function `ripple` is reproduced below. It is identical to the one in the textbook.

```
function r = ripple(waveform)
% Function to calculate the ripple on an input waveform.

% Calculate the average value of the waveform
nvals = size(waveform,2);
temp = 0;
for ii = 1:nvals
      temp = temp + waveform(ii);
end
average = temp/nvals;

% Calculate rms value of waveform
temp = 0;
for ii = 1:nvals
      temp = temp + waveform(ii)^2;
end
```

```
rms = sqrt(temp/nvals);

% Calculate ripple factor
r = sqrt((rms / average)^2 - 1) * 100;
```

Finally, the test driver program is shown below.

```
% M-file: test_halfwave3.m
% M-file to calculate the ripple on the output of a
% three phase half-wave rectifier.
% First, generate the output of a three-phase half-wave
% rectifier
waveform = zeros(1,128);
for ii = 1:128
      waveform(ii) = halfwave3(ii*pi/64);
end

% Now calculate the ripple factor
r = ripple(waveform);

% Print out the result
string = ['The ripple is ' num2str(r) '%.'];
disp(string);
```

When this program is executed, the results are

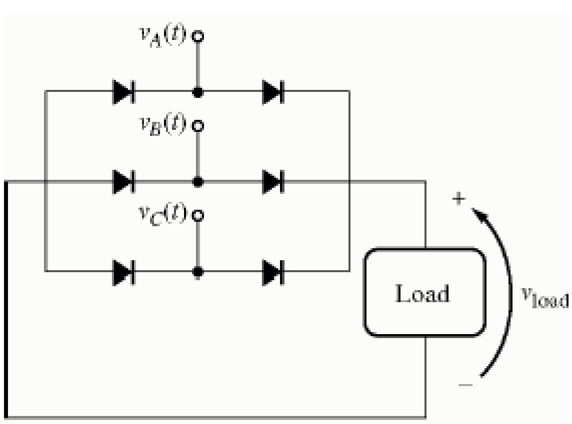
```
» test_halfwave3
The ripple is 18.2759%.
```

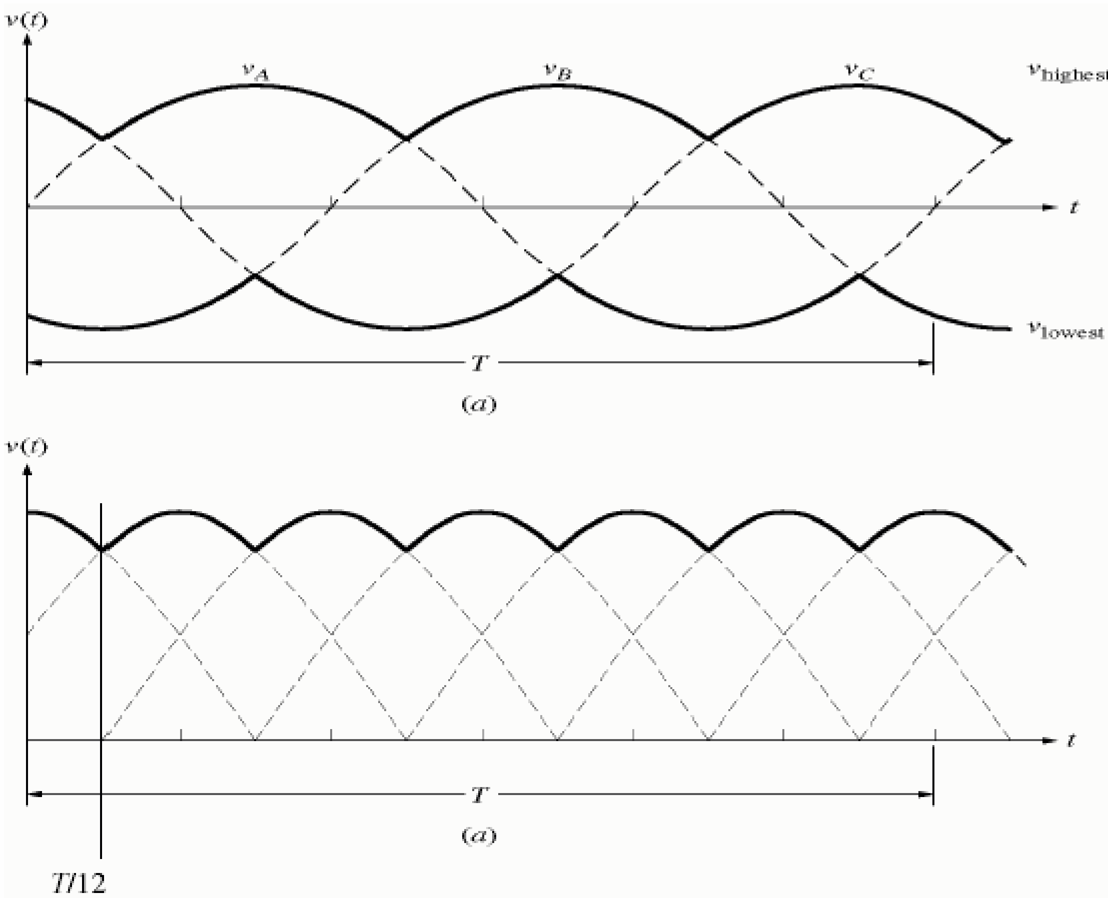
This answer agrees with the analytical solution above.

Problem 3-2:

Calculate the ripple factor of a three-phase full-wave rectifier circuit, both analytically and using MATLAB.

Solution:





$$v_A(t) = V_M \sin(\omega t)$$
$$v_B(t) = V_M \sin(\omega t - 2\pi/3)$$
$$v_C(t) = V_M \sin(\omega t + 2\pi/3)$$

Over the interval from 0 to T/12, the output voltage is

$$v(t) = v_C(t) - v_B(t) = V_M(\omega t + 2\pi/3) - V_M(\omega t - 2\pi/3)$$
$$v(t) = V_M(\sin\omega t\cos 2\pi/3 + \cos\omega t\sin 2\pi/3) - V_M(\sin\omega t\cos 2\pi/3 - \cos\omega t\sin 2\pi/3)$$
$$v(t) = \sqrt{3}V_M\cos(\omega t)$$

The period of the waveform is $T = 2\pi / \omega$, so $T/12 = \pi / 6\omega$, the average voltage over the interval from 0 to T/12 is

$$V_{DC} = \frac{1}{T}\int_0^T v(t)dt = \frac{6\omega}{\pi}\int_0^{\pi/6\omega}\sqrt{3}V_M\cos(\omega t)dt = \frac{6\sqrt{3}V_M}{\pi}\sin\omega t\Big|_0^{\pi/6\omega}$$
$$V_{DC} = \frac{6\sqrt{3}V_M}{\pi}(\frac{1}{2} - 0) = 1.6540V_M$$

The rms voltage is

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{6\omega}{\pi}\int_0^{\pi/6\omega} 3V_M{}^2 \cos^2(\omega t)dt} = \sqrt{\frac{18\omega V_M{}^2}{\pi}\left(\frac{1}{2}t + \frac{1}{4\omega}\sin 2\omega t\right)\Bigg|_0^{\pi/6\omega}}$$

$$V_{rms} = V_M\sqrt{\frac{18\omega}{\pi}\left(\frac{\pi}{12\omega} + \frac{1}{4\omega}\sin\frac{\pi}{3}\right)} = 1.6554V_M$$

The resulting ripple factor is

$$r = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1} \times 100\% = \sqrt{\left(\frac{1.6554V_M}{1.6540V_M}\right)^2 - 1} \times 100\% = 4.2\%$$

The MATLAB function is shown below

```
function volts = fullwave3(wt)
% Function to simulate the output of a three-phase
% full-wave rectifier.
% wt = Phase in radians (=omega x time)

% Convert input to the range 0 <= wt < 2*pi
while wt >= 2*pi
      wt = wt - 2*pi;
end
while wt < 0
      wt = wt + 2*pi;
end

% Simulate the output of the rectifier.
a = sin(wt);
b = sin(wt - 2*pi/3);
c = sin(wt + 2*pi/3);

volts = max( [ a b c ] ) - min( [ a b c ] );
```

The test driver program is shown below.

```
% M-file: test_fullwave3.m
% M-file to calculate the ripple on the output of a
% three phase full-wave rectifier.

% First, generate the output of a three-phase full-wave
% rectifier
waveform = zeros(1,128);
for ii = 1:128
      waveform(ii) = fullwave3(ii*pi/64);
end

% Now calculate the ripple factor
r = ripple(waveform);

% Print out the result
```

```
string = ['The ripple is ' num2str(r) '%.'];
disp(string);
```

When this program is executed, the results are

```
» test_fullwave3
The ripple is 4.2017%.
```

This answer agrees with the analytical solution above.

Problem 3-4:

What would the rms voltage on the load in the circuit in Figure P3-1 be if the firing angle of the SCR were (a) 0º, (b) 30º, (c) 90º?

Solution:

The input voltage to the circuit of Figure P3-1 is

$$v_{ac}(t) = 339 \sin \omega t \text{ , where } \omega = 377 \, rad/s$$

Therefore, the voltage on the secondary of the transformer will be

$$v_{ac}(t) = \frac{339}{a} \sin \omega t = \frac{339}{2} \sin \omega t = 169.5 \sin \omega t$$

The average voltage applied to the load will be the integral over the conducting portion of the half cycle divided by π / ω, the period of a half cycle.

(a) For a firing angle of 0º, the average voltage will be

$$V_{avg} = \frac{1}{T}\int_0^T v(t)dt = \frac{\omega}{\pi}\int_0^{\pi/\omega} V_M \sin(\omega t)dt = -\frac{1}{\pi} V_M \cos \omega t \Big|_0^{\pi/\omega}$$

$$V_{avg} = -\frac{1}{\pi} V_M (-1-1) = 108V$$

(b) For a firing angle of 30º, the average voltage will be

$$V_{avg} = \frac{1}{T}\int_{\pi/6}^T v(t)dt = \frac{\omega}{\pi}\int_{\pi/6\omega}^{\pi/\omega} V_M \sin(\omega t)dt = -\frac{1}{\pi} V_M \cos \omega t \Big|_{\pi/6\omega}^{\pi/\omega}$$

$$V_{avg} = -\frac{1}{\pi} V_M (-1-\frac{\sqrt{3}}{2}) = 101V$$

(c) For a firing angle of 90º, the average voltage will be

$$V_{avg} = \frac{1}{T}\int_{\pi/2}^{T} v(t)dt = \frac{\omega}{\pi}\int_{\pi/2\omega}^{\pi/\omega} V_M \sin(\omega t)dt = -\frac{1}{\pi}V_M \cos\omega t\Big|_{\pi/2\omega}^{\pi/\omega}$$

$$V_{avg} = -\frac{1}{\pi}V_M(-1-0) = 54V$$