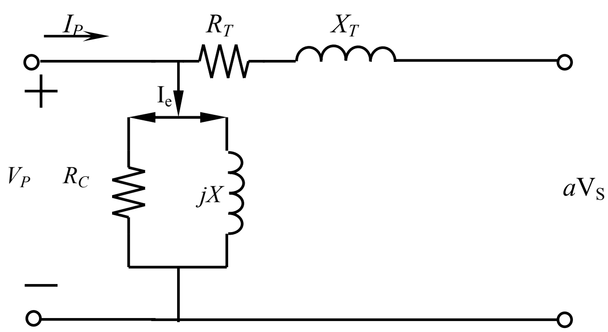
ELG3311: Tutorial 2

Problem 2-1:
The secondary winding of a transformer has a terminal voltage of $v_s\ (t)$ = 282.8 sin 377t V. The turns ratio of the transformer is 50:200 (a = 0.25). If the secondary current of the transformer is $i_S\ (t)$ = 7.07 sin (377t – 36.87°) A, what is the primary current of this transformer? What is the voltage regulation and efficiency? The impedance of this transformer referred to the primary side are:

$R_T = 0.005\ \Omega$	$R_C = 75\ \Omega$
$X_T = 0.225\ \Omega$	$X_M = 20\ \Omega$

Solution: The following equivalent circuit is referred to the primary.



The secondary voltage and current are

$$V_S = \frac{282.8}{\sqrt{2}}\angle 0^o\ V = 200\angle 0^o\ V$$
$$I_S = \frac{7.07}{\sqrt{2}}\angle -36.87^o\ A = 5\angle -36.87^o\ A$$

The secondary voltage referred to the primary side is

$$V_S^P = aV_S = 0.25 \times 200\angle 0^o = 50\angle 0^o$$

The secondary current referred to the primary side is

$$I_S^P = \frac{I_S}{a} = \frac{5\angle -36.87^o}{0.25} = 20\angle -36.87^o\ A$$

The primary circuit voltage is given by

$$V_P = V_S^P + I_S^P\left(R_T + jX_T\right)$$
$$V_P = 50\angle 0^o + \left(20\angle -36.87^o\right)\left(0.05 + j0.225\right) = 53.6\angle 3.2^o\ V$$

The excitation current of this transformer is

$$I_e = I_C + I_m = \frac{53.6\angle 3.2^o}{75} + \frac{53.6\angle 3.2^o}{j20} = 0.7145\angle 3.2^o + 2.675\angle -86.8^o = 2.77\angle -71.9^o$$

The total primary current is

$$I_P = I_S^P + I_e = 20\angle -36.87^o + 2.77\angle -71.9^o = 22.3\angle -41^o \text{ A}$$

The voltage regulation of the transformer

$$V_R = \frac{V_P - aV_S}{aV_S} \times 100\% = \frac{53.6 - 50}{50} \times 100\% = 7.2\%$$

The input power is

$$P_{in} = V_P\ I_P\ \cos\theta = 53.6 \times 22.3 \times \cos\left(3.2^o + 41^o\right) = 857 \text{ W}$$

The output power is

$$P_{out} = V_S\ I_S\ \cos\theta = 200 \times 5 \times \cos 36.87^o = 800 \text{ W}$$

The efficiency

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{800}{857} \times 100\% = 93.4\%$$

Problem 2-2:
A 20-kVA 8000/277-V distribution transformer has the following resistances and reactances:

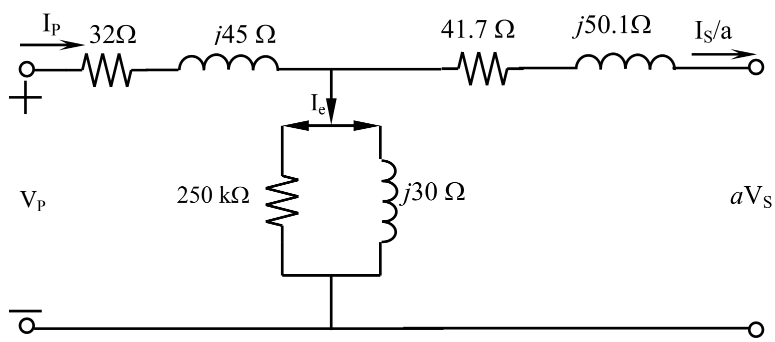
$R_P = 32\ \Omega$	$R_S = 0.05\ \Omega$
$X_P = 45\ \Omega$	$X_S = 0.06\ \Omega$
$R_C = 250\ \text{k}\Omega$	$X_m = 30\ \text{k}\Omega$

The excitation branch impedances are given referred to the high-voltage side of the transformer

a. Find the equivalent circuit of this transformer referred to the high-voltage side.
b. Find the per-unit equivalent circuit of this transformer.
c. Assume that this transformer is supplying rated load at 277 V and 0.8 PF lagging. What is the transformer's input voltage? What is its voltage regulation?
d. What is the transformer's efficiency under the conditions of part (c)?

Solution: The turns ratio a = 8000/277 = 28.89. The secondary impedances referred to the primary side are

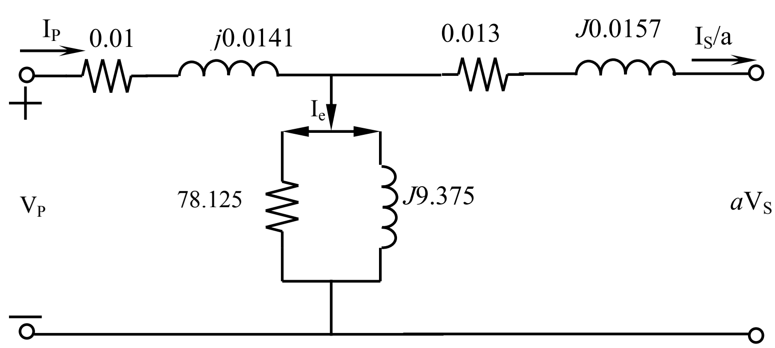
$$R_S^P = a^2\ R_S = (28.89)^2(0.05) = 41.7\ \Omega$$
$$X_S^P = a^2 X_S = (28.89)^2(0.06) = 50.1\ \Omega$$



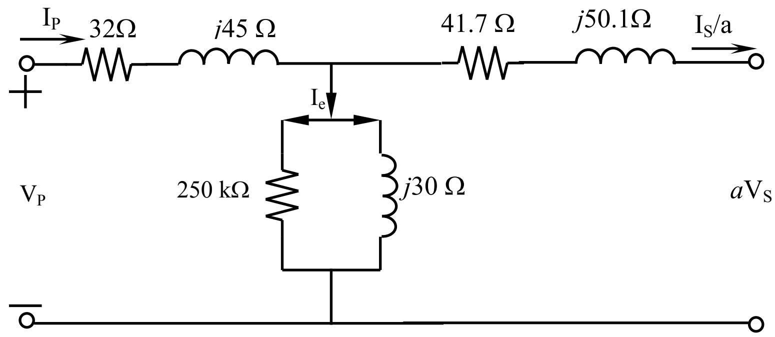
(b) The rated KVA of the transformer is 20 kVA, and the rated voltage on the primary side is 8000 V, therefore, the rated current in the primary side is 20 kVA/8000 V = 2.5 A. Accordingly, the base impedance on the primary side is

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{8000}{2.5} = 3200\ \Omega$$

Since $Z_{pu} = Z_{actual}/Z_{base}$, the resulting per unit equivalent circuit is



(c) The equivalent circuit referred to the primary side is shown in the following diagram:



$$I_S = \frac{20\text{kVA}}{277\text{V}}\angle -36.87^o = 72.2\angle\text{-}36.87^o$$

$$I_S^P = \frac{72.2\angle\text{-}36.87^o}{28.89} = 2.5\angle -36.87^o \text{ A}$$

The primary voltage is

$$V_P = V_S^P + (R_T + jX_T)I_S^P$$
$$= 8000\angle 0^o + (73.7 + j95.1)\left(2.5\angle\text{-}36.87^o\right) = 8290\angle 0.55^o$$

The voltage regulation is

$$V_R = \frac{8290\text{-}8000}{8000}\times 100\% = 3.63\%$$

(d) The losses are

$$P_{copper} = \left(I_S^P\right)^2 R_T = (2.5)^2(73.7) = 461 \text{ W}$$

$$P_{core} = \frac{(V_S^P)^2}{R_C} = \frac{82902}{250{,}000} = 275 \text{ W}$$

The efficiency is

$$\eta = \frac{P_{out}}{P_{out} + P_{copper} + P_{core}}\times 100\% = \frac{20\text{kVA}\times 0.8}{20\text{kVA}\times 0.8 + 461 + 275}\times 100\% = 95.6\%$$

Problem 2-3:
A 2000-VA, 230/115-V transformer has been tested to determine it equivalent circuit. The results of the test are shown below:

OC	**SC**
V_{OC} = 230 V	V_{SC} = 13.2 V
I_{OC} = 0.45 A	I_{SC} = 6 A
P_{OC} = 30 W	P_{SC} = 20.1 W

All data given were taken from the primary side of the transformer.

a. Find the equivalent circuit of the transformer referred to low-voltage side.
b. Find VR at rated conditions and (1) 0.8 PF lagging, (2) 1.0 PF, (3) 0.8 PF lagging.

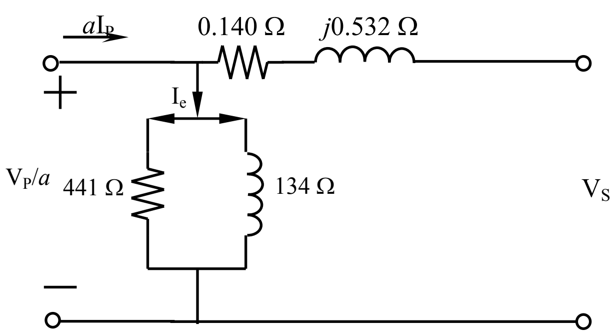
Solution: From OC Test, we get

$$R_C = 1763\,\Omega$$
$$X_m = 534\,\Omega$$

From SC, we get

$$RT = 0.588\,\Omega$$
$$X_T = j2.128\,\Omega$$

The equivalent circuit referred to the secondary side is shown in the following diagram



(b) The reated secondary current is

$$I_S = \frac{1000}{115} = 8.7\text{ A}$$

(1) When 0.8 PF lagging

$$V_P^S = V_S + Z_T I_S = 115\angle 0^o + (0.140 + j0.532)(8.7\angle -36.87^o)$$
$$= 118.8\angle 1.4^o$$

$$V_R = \frac{118.8 - 115}{115} \times 100\% = 3.3\%$$

(2) When 1.0 PF

$$V_P^S = V_S + Z_T I_S = 115\angle 0^o + (0.140 + j0.532)(8.7\angle 0^o)$$
$$= 116.3\angle 2.28^o\text{ V}$$

$$V_R = \frac{116.3 - 115}{115} \times 100\% = 1.1\%$$

(3) When 0.8 PF leading

$$V_P^S = V_S + Z_T I_S = 115\angle 0^o + (0.140 + j0.532)(8.7\angle 36.87^o)$$
$$= 113.3\angle 2.24^o$$

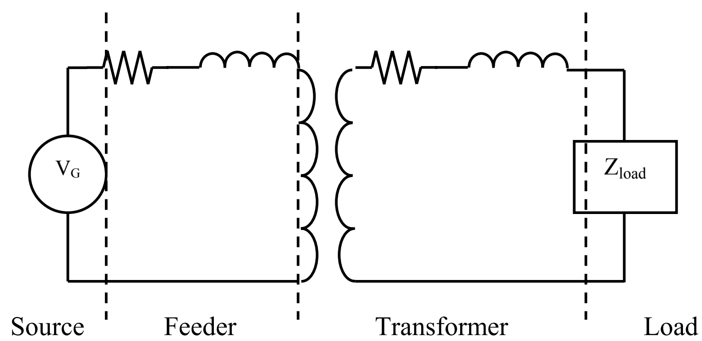
$$V_R = \frac{113.3 - 115}{115} \times 100\% = -1.5\%$$

The copper and core losses of the transformer are: 10.6 W and 32 W.

The efficiency is

$$\eta = \frac{115 \times 8.7 \times 0.8}{115 \times 8.7 \times 0.8 + 10.6 + 32} = 94.9\%$$

Problem 2-4:
A single-phase power system is shown in Figure P2-1. The power source feeds a 100-kVA 14/2.4-kV transformer through a feeder impedance of 38.2 + j140 Ω. The transformer's equivalent series impedance referred to its low-voltage side is 0.12 + j0.5 Ω. The load on the transformer is 90 kW at 0.85 PF lagging and 2300 V.



a. What is the voltage at the power source of the system?
b. What is the voltage regulation of the transformer?
c. How efficient is the overall power system?

Solution: We will refer this circuit to the secondary. The feeder's impedance referred to the secondary side is

$$Z_{\text{line}}^{S}=\left(\frac{2.4}{14}\right)(38.2+j140)=1.12+j4.11\Omega$$

The secondary current is given by

$$I_S=\frac{90}{2300\times 0.9}=43.48\text{ A}$$
$$I_S=43.48\angle -25.8^{\circ}$$

(a) The power at the power source of this system (referred to the secondary lab) is

$$\begin{aligned}V_{\text{source}}^{S}&=V_S+I_S\,Z_{\text{line}}^{S}+I_S\,Z_T\\&=2300\angle 0^{\circ}+\left(43.48\angle -25.8^{\circ}\right)+(1.12+j4.11)+\left(43.48\angle -25.8^{\circ}\right)(0.12+j0.5)\\&=2441\angle 3.7^{\circ}\text{ V}\end{aligned}$$

Now find the voltage at the power source

$$V_{\text{source}}=\left(2441\angle 3.7^{\circ}\right)\left(\frac{14}{2.4}\right)=14.24\angle 3.7^{\circ}\text{ kV}$$

(b) To find the VR, we must find the voltage at the primary side referred to the secondary under full load condition

$$\begin{aligned}V_P^S&=V_S+I_SZ_T\\&=2300\angle 0^{\circ}+\left(43.48\angle -25.8^{\circ}\right)(0.12+j0.5)=2314\angle 0.43^{\circ}\end{aligned}$$

Therefore

$$\boxed{\mathrm{VR} = \frac{2314 - 2300}{2300} \times 100\% = 0.6\%}$$

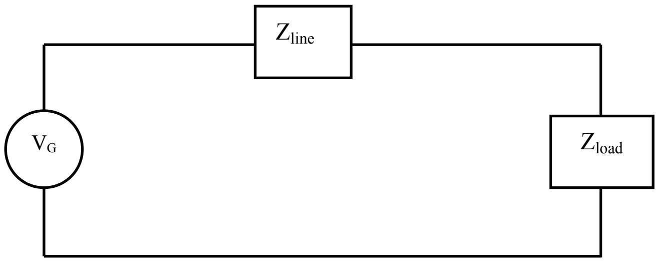
(c) The power supplied by the load is
(d)

$$\boxed{\mathrm{P_{in}} = \mathrm{V}_{\mathrm{source}}^{S}\ \mathrm{I_S} \cos\theta = 2441 \times 43.48 \times \cos 29.5^{\mathrm{o}} = 92.37\ \mathrm{kW}}$$

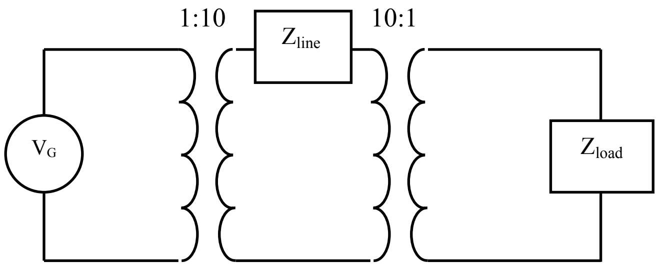
The efficiency is

$$\boxed{\eta = \frac{\mathrm{P_{out}}}{\mathrm{P_{in}}} \times 100\% = \frac{90}{92.37} \times 100\% = 97.4\%}$$

Problem 2-14:
A 12.4-kV single-phase generator supplies power to the load through a transmission line. The load's impedance is $Z_{load} = 500\angle 36.87^o\ \Omega$, and the transmission line's impedance is $Z_{line} = 60\angle 60^o\ \Omega$.



(a)



(b)

a. If the generator is directly connected to the load (Figure 2-3a), what is the ratio of the load voltage to the generator voltage? What are the transmission losses of the system?
b. If a 1:10 step-up transformer is placed at the output of the generator and a 10:1 transformer is placed at the load end of the transmission line, what is the new ratio of the load voltage to the generated voltage? What are the transmission losses of the system now?

Solution: In case of the directly-connected load, the line current is

$$I_{line} = I_{load} = \frac{12\angle 0^o}{60\angle 60^o + 500\angle 36.87^o} = 22.32\angle -39.3^o \text{ A}$$

The load voltage is

$$V_{load} = I_{load}\, Z_{load} = \left(22.32\angle -39.3^o\right)\left(500\angle 36.87^o\right) = 11.16\angle -2.43^o \text{ kV}$$

The ratio of the load voltage to the generated voltage is 11.16/12.4 = 0.9. The transmission losses are

$$P_{loss} = I_{line}^2 Z_{load} = (22.32)^2(30) = 14.9 \text{ kW}$$

(b) Two transformers are used. The impedance of the transmission line becomes

$$Z_{\text{line}}^{\text{load}} = \left(\frac{1}{10}\right)^2 Z_{\text{line}} = \left(\frac{1}{10}\right)^2 \left(60\angle 60^{\text{o}}\right) = 0.6\angle 60^{\text{o}}\ \Omega$$

$$I_{\text{line}}^{\text{load}} = I_{\text{load}} = \frac{12.4\angle 0^{\text{o}}}{0.6\angle 60^{\text{o}} + 500\angle 36.87^{\text{o}}} = 24.773\angle -3690^{\text{o}}\ \text{A}$$

The load voltage is

$$V_{\text{load}} = I_{\text{load}}\ Z_{\text{load}} = \left(24.773\angle\text{ - }36.90^{\text{o}}\right)\left(500\angle 36.87^{\text{o}}\right) = 12.386\angle -0.03^{\text{o}}\ \text{kV}$$

The ratio of load voltage to generated voltage is 12.386/12.4=0.999. The current in the transmission line is

$$I_{\text{line}} = \left(\frac{1}{10}\right) I_{\text{load}} = \left(\frac{1}{10}\right)(24.77) = 2.477\ \text{A}$$

The losses in the transmission line are

$$P_{\text{loss}} = I_{\text{line}}^2 R_{line} = (2.477)^2 (30) = 184\ \text{W}$$

Transmission losses have decreased by a factor of more than 80.