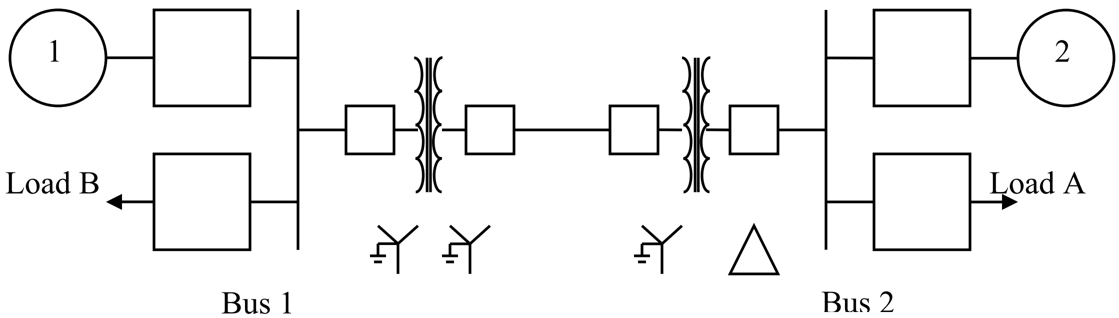
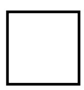
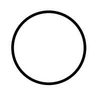
Power System Representation and Equations

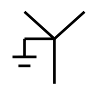


A one-line diagram of a simple power system


Oil or liquid circuit breaker


Rotating machine


Two-winding power transformer


Wye connection, neutral ground

Per-Phase, Per Unit System

$$I_{\text{base}} = \frac{S_{3\Phi,\text{base}}}{\sqrt{3}V_{\text{L,base}}}$$

$$Z_{\text{base}} = \frac{V_{\text{L,base}}}{\sqrt{3}I_{\text{base}}}$$

$$Z_{\text{base}} = \frac{\left(V_{\text{L,base}}\right)^2}{S_{3\Phi,\text{base}}}$$

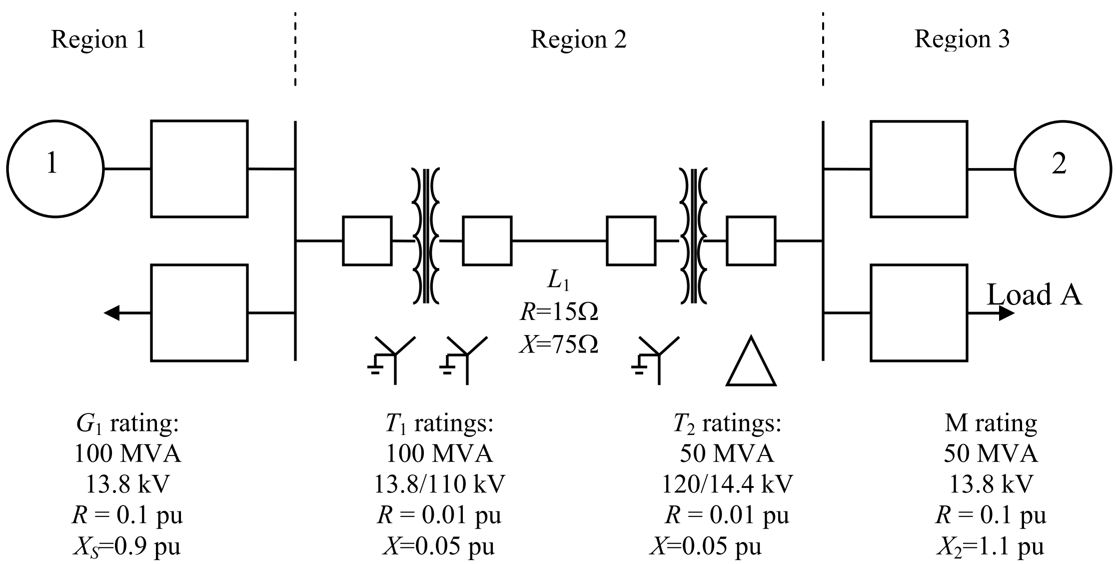
$$\text{Quantity in per unit} = \frac{\text{actual value}}{\text{base value of quantity}}$$

As we have seen in Chapter 2, many transformers and machines have their internal impedances specified as per unit resistances and reactances, using the voltage and apparent power ratings of the device itself as the base quantity. If these impedances are expressed in per unit to a base other than the one selected as a base for a power system, we must convert the impedances to per unit on the new base. This conversion could be done by using the original base impedance to convert the impedances back into ohms, and then using the power system base impedance to convert the value on ohms to per unit on the new base. Alternatively, we may combine the two steps into a single equation:

$$\text{Per}-\text{unit } Z_{\text{new}} = \text{per}-\text{unit } Z_{\text{given}}\left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2\left(\frac{S_{\text{new}}}{S_{\text{given}}}\right)$$

Example

A simple power system consisting of one synchronous generator and one synchronous motor connected by two transformers and a transmission line is shown in the following Figure. Develop a per-phase, per unit equivalent system for this power system using a base apparent power of 100 MVA and a base line voltage at generator G_1 of 13.8 kV.



Solution:

$V_{\text{base},1} = 13.8\,\text{kV}$	Region 1
$V_{\text{base},2} = V_{\text{base},1}\left(\frac{110\,\text{kV}}{13.8\,\text{kV}}\right) = 110\,\text{kV}$	Region 2
$V_{\text{base},3} = V_{\text{base},2}\left(\frac{14.4\,\text{kV}}{120\,\text{kV}}\right) = 13.2\,\text{kV}$	Region 3

The corresponding base impedances in each region are:

$Z_{\text{base},1} = \frac{(V_{\text{L,base}})^2}{S_{3\phi\text{base}}} = \frac{(13.8\,\text{kV})^2}{100\,\text{MVA}} = 1.904\,\Omega$	Region 1
$Z_{\text{base},2} = \frac{(V_{\text{L,base}})^2}{S_{3\phi\text{base}}} = \frac{(110\,\text{kV})^2}{100\,\text{MVA}} = 121\,\Omega$	Region 2

$Z_{\text{base},3} = \dfrac{(V_{\text{L,base}})^2}{S_{3\phi\text{base}}} = \dfrac{(13.2\text{ kV})^2}{100\text{ MVA}} = 1.743\,\Omega$	Region 3

$$R_{\text{G1,pu}} = 0.1 \text{ per unit}$$
$$X_{\text{G1,pu}} = 0.9 \text{ per unit}$$
$$R_{\text{T1,pu}} = 0.01 \text{ per unit}$$
$$X_{\text{T1,pu}} = 0.05 \text{ per unit}$$

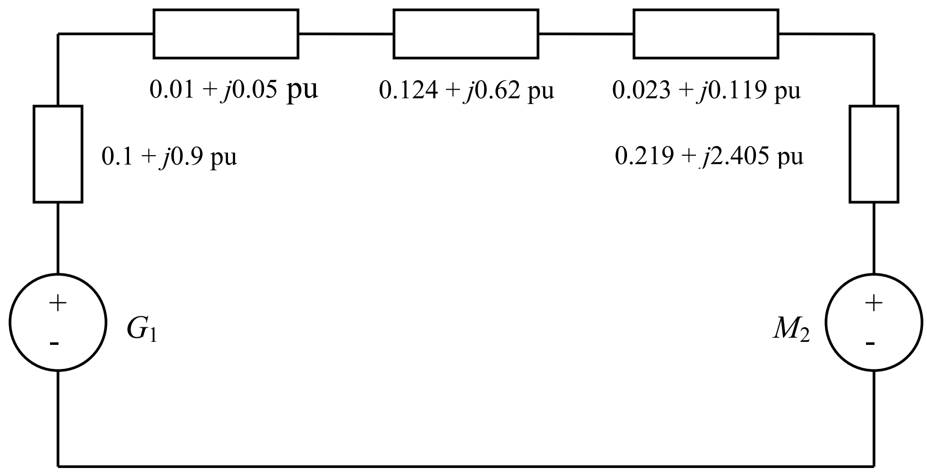
$$R_{\text{line, system}} = \left(\frac{15\,\Omega}{121\,\Omega}\right) = 0.124 \text{ per unit}$$
$$X_{\text{line, system}} = \left(\frac{75\,\Omega}{121\,\Omega}\right) = 0.620 \text{ per unit}$$

$$\text{Per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}}\left(\frac{\text{V}_{\text{given}}}{\text{V}_{\text{new}}}\right)^2\left(\frac{S_{\text{new}}}{S_{\text{given}}}\right)$$

$$R_{\text{T2,pu}} = (0.01)\left(\frac{14.4\text{ kV}}{13.2\text{ kV}}\right)^2\left(\frac{100\text{ MA}}{50\text{ MA}}\right) = 0.238 \text{ per unit}$$
$$X_{\text{T2,pu}} = (0.05)\left(\frac{14.4\text{ kV}}{13.2\text{ kV}}\right)^2\left(\frac{100\text{ MA}}{50\text{ MA}}\right) = 0.119 \text{ per unit}$$

$$\text{Per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}}\left(\frac{\text{V}_{\text{given}}}{\text{V}_{\text{new}}}\right)^2\left(\frac{S_{\text{new}}}{S_{\text{given}}}\right)$$

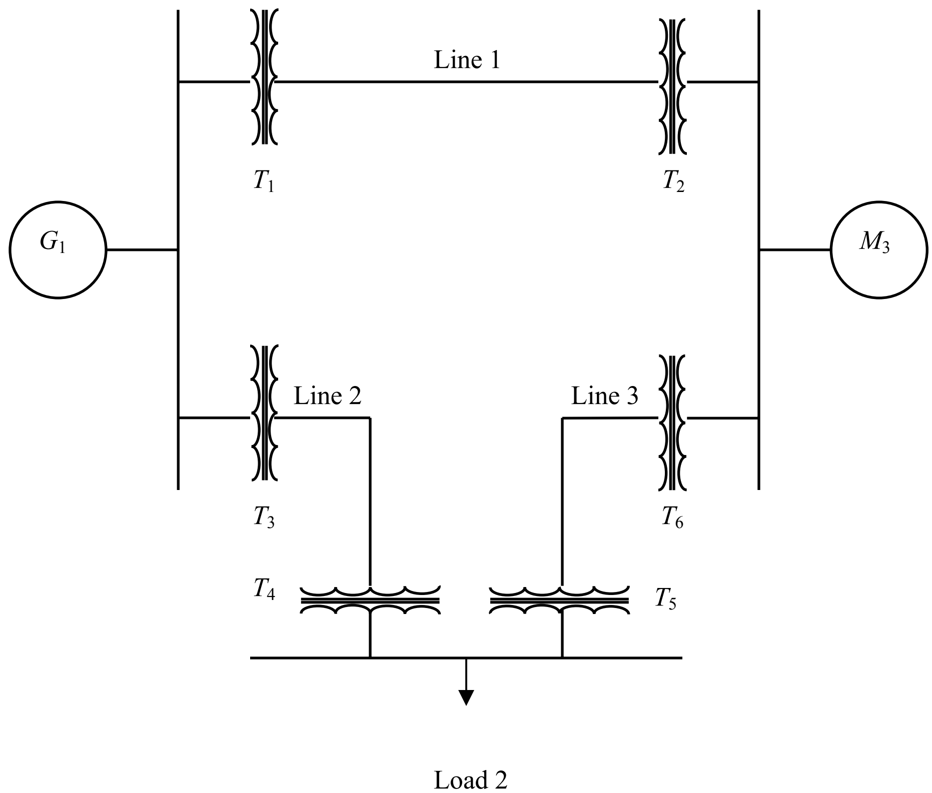
$$R_{\text{M2,pu}} = (0.01)\left(\frac{13.8\text{ kV}}{13.2\text{ kV}}\right)^2\left(\frac{100\text{ MA}}{50\text{ MA}}\right) = 0.219 \text{ per unit}$$
$$X_{\text{M2,pu}} = (0.05)\left(\frac{13.8\text{ kV}}{13.2\text{ kV}}\right)^2\left(\frac{100\text{ MA}}{50\text{ MA}}\right) = 2.405 \text{ per unit}$$



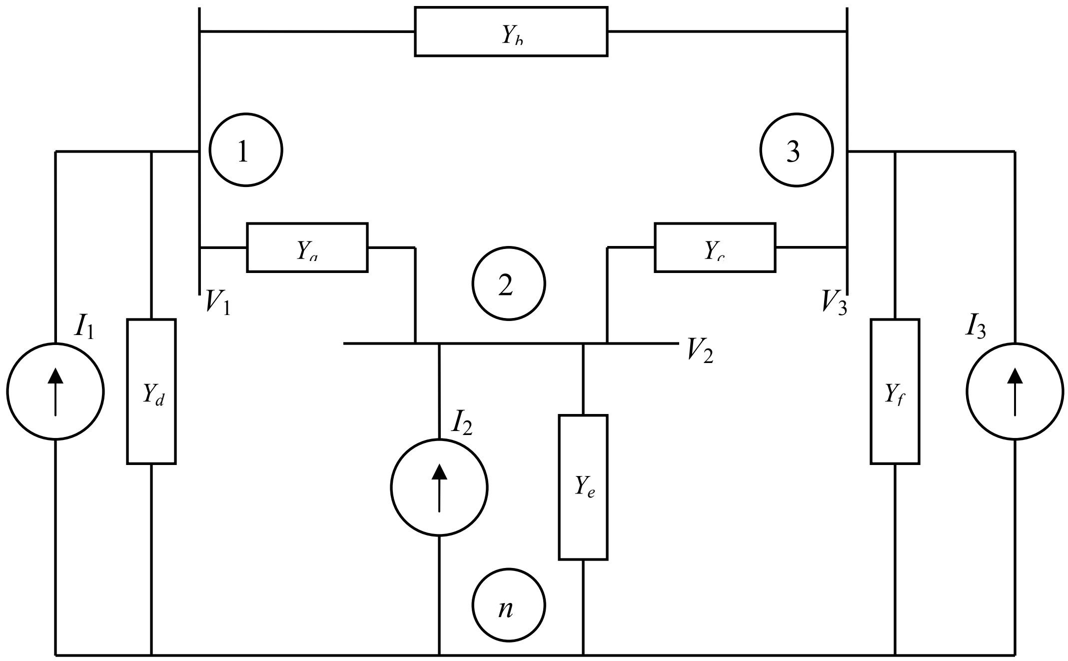
Per-phase, per unit equivalent circuit of the simple power system.

Writing Node Equations for Equivalent Circuit

Once the per-phase, per unit equivalent circuit of a power system is created, it may be used to find the voltages, currents, and powers present at various points in a power system. The most common technique used to solve such circuits is nodal analysis. In nodal analysis, we use Kirchhoff's current law equations to determine the voltages at each node (each bus) in the power system, and then using the resulting voltages to calculate the currents and power flows at various points in the power system, and then use the resulting voltages to calculate the currents and power flows at various points in the system.Consider the following simple three-phase power system containing three busses connected by three transmission lines. The system includes a generator connected to bus 1, a load connected to bus 2, and a motor connected to bus 3.



A simple three-phase power system



Sum of currents out of a node = sum of currents into the node

Apply KCL on node 1

$$(V_1 - V_2)Y_a + (V_1 - V_3)Y_b + V_1 Y_d = I_1$$

Apply KCL on node 2

$$(V_2 - V_1)Y_a + (V_2 - V_3)Y_c + V_2 Y_e = I_2$$

Apply KCL on node 3

$$(V_3 - V_1)Y_b + (V_3 - V_2)Y_c + V_3 Y_f = I_3$$

Rearrange these equations to collect the terms in each voltage

$$(Y_a + Y_b + Y_d)V_1 - Y_a V_2 - Y_b V_3 = I_1$$
$$-Y_a V_1 + (Y_a + Y_c + Y_e)V_2 - Y_c V_3 = I_2$$
$$-Y_b V_1 - Y_c V_2 + (Y_b + Y_c + Y_f)V_3 = I_3$$

The above equation can be expressed in matrix form

$$\begin{bmatrix} Y_a + Y_b + Y_d & -Y_a & -Y_b \\ -Y_a & Y_a + Y_c + Y_e & -Y_c \\ -Y_a & -Y_c & Y_b + Y_c + Y_f \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

This equation is of the form

$$Y_{bus} V = I$$

Where Y_{bus} is the self admittance of a system.

There are many ways for solving systems of simultaneous linear equations, such as substitution, Gaussian elimination, LU factorization, and so forth! For us, MATLAB has very efficient equation solvers built directly into it! If a system of *n* simultaneous linear equations in *n* unknowns can be expressed in the form:

$$Ax = b$$

Where A is $n \times n$ matrix, and b is a n-element column vector. It may be expressed as

$$x = A^{-1}b$$

Where A^{-1} is the inverse of a matrix!