Transmission Lines: Example Problem

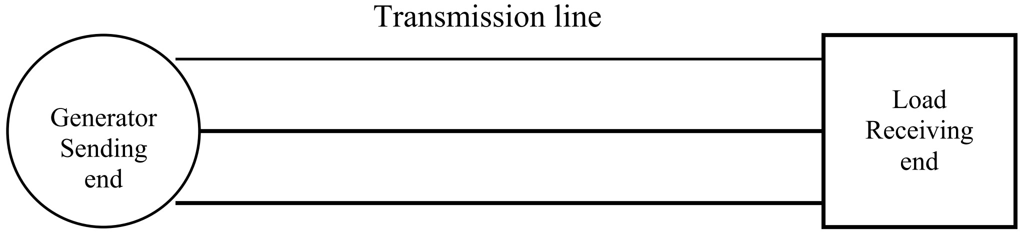
A 220-kV, 150 MVA, 60-Hz, three-phase transmission line is 140 km long. The characteristic parameters of the transmission line are:

$$\boldsymbol{r = 0.09\ \Omega/\text{km};\ x = 0.88\ \Omega/\text{km};\ y = 4.1\times 10^{-6}\ \text{S/km}}$$

where, r is the resistance per kilometer, x is the reactance per kilometer, y is the shunt admittance per kilometer.

The voltage at the receiving end of the transmission line is 210 kV. Although this transmission line would normally be considered a medium-length transmission line, we will treat it as short line:

a) What is series impedance and shunt impedance of the transmission line?
b) What is the sending end voltage if the line is supplying rated voltage and apparent power at 0.85 PF lagging? At unity PF? At 0.85 PF leading?
c) What is the voltage regulation of the transmission line for each of the cases in (b)?
d) What is the efficiency of the transmission line when it is supplying rated apparent power at 0.85 PF lagging?



Solution:

- The series resistance, series reactance, and shunt admittance of the transmission line:

$$R = rd = (0.12\ \Omega/\text{km})(140\ \text{km}) = 16.8\ \Omega$$
$$X = xd = (0.88\ \Omega/\text{km})(140\ \text{km}) = 123.2\ \Omega$$
$$Y = yd = (4.1\times10^{-6})(140\ \text{km}) = 5.74\times10^{-4}\ \text{S}$$

- The current out of this transmission line is given by. Note that the per-phase equivalent circuit implicitly assumes a wye (Y) connection, so the current is the same in phase or line configuration.

$$S_{out} = \sqrt{3}\, V_L I_R$$
$$I_R = \frac{S_{out}}{\sqrt{3} V_L} = \frac{150\,\text{MVA}}{\sqrt{3} \times 210\,\text{kV}} = 412\ \text{A}$$

The phase voltage of the transmission line is

$$V_R = \frac{210\,\text{kV}}{\sqrt{3}} = 121\,\text{kV}$$

Since the transmission line is considered as "short", the admittance (or shunt capacitance) may be ignored. This produces in a per phase transmission line model consisting of a series resistance and inductance only. The phase voltage at the sending end of the line when the power factor is 0.85 lagging will be

$$V_s = V_R + I_R R + I_R X_L$$
$$= 121\angle 0^o + (412\angle -31.8^o)(16.8 + j123.2) = 158.6\angle 14.4^\text{o}\ \text{kV}$$

The resulting line voltage at the sending end (0.85 PF lagging) is

$$V_L = \sqrt{3} \times 158.6 = 275\,\text{kV}$$

The phase voltage at the sending end of the line when the power factor is unity will be

$$V_s = VR + I_R R + I_R X_L$$
$$= 121\angle 0^o + (412\angle 0^o)(16.8 + j123.2) = 137.6\angle 21.6^\text{o}\ \text{kV}$$

The resulting line voltage at the sending end (unity PF) is

$$V_L = \sqrt{3} \times 137 = 238\,\text{kV}$$

The phase voltage at the sending end of the line when the power factor is 0.85 leading

$$V_s = VR + I_R R + I_R X_L$$
$$= 121\angle 0^o + (412\angle 31.8^o)(16.8 + j123.2) = 110.5\angle 25.0^\text{o}\ \text{kV}$$

The resulting line voltage at the sending end (0.85 leading) is

$$V_L = \sqrt{3} \times 110.5 = 191\,\text{kV}$$

The voltage regulation of a transmission line is given by

$$V_R = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100$$

- The voltage regulation at 0.85 PF lagging; PF unity; and 8.5 PF leading:

$$V_R = \frac{275 - 210}{210} \times 100 = 31.1\%$$
$$V_R = \frac{238 - 210}{210} \times 100 = 13.7\%$$
$$V_R = \frac{191 - 210}{210} \times 100 = -8.7\%$$

- The output power from the transmission line at 0.85 PF lagging

$$\begin{aligned} P_{out} &= 3 V_R I_R \cos\theta_R \\ &= 3 \times 121 \times 412 \times 0.85 = 127\ \text{kW} \end{aligned}$$

The input power from the transmission line

$$\begin{aligned} P_{in} &= 3 V_S I_S \cos\theta_S \\ &= 3 \times 158.6\ \text{kV} \times 412 \times \cos\left(14.4 - (-31.8)\right) = 135.7\ \text{kW} \end{aligned}$$

The transmission line efficiency at full load and 0.85 PF lagging is

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{127\ \text{kW}}{135.7\ \text{kW}} \times 100\% = 93.6\%$$