CHAPTER 3
DC MACHINES

3.1 Introduction to Machines

3.1.1 Machine Terminology

Machine: A machine is a device which converts electrical power to mechanical power or vice versa. A machine can operate as either a motor or a generator and can change from one operating mode the other.

Motor: A motor is a machine that converts electrical power to mechanical power.

Generator: A generator is a machine that converts mechanical power to electrical power.

Mechanical Power: Mechanical power is produced by a rotating shaft, usually expressed as the product of torque and speed.

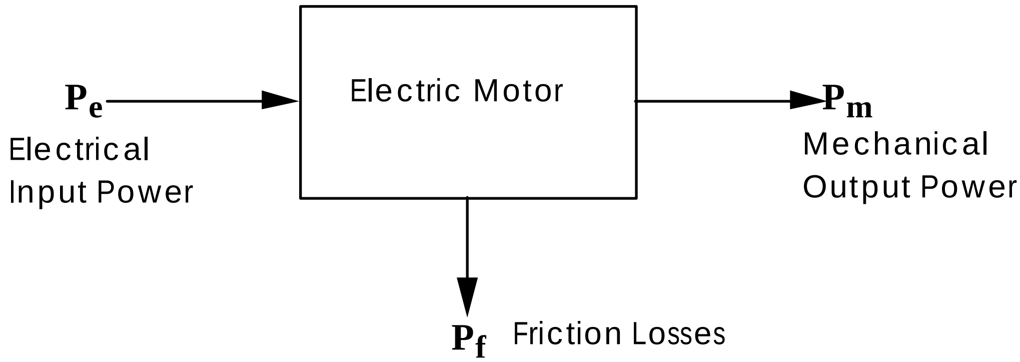
Electrical Power: Electrical power is produced by electricity, usually expressed as the product of voltage and current, (including power factor angle for AC circuits)

Load: The load is the ultimate destination of the power produced. For a generator, the load is electrical, dissipated in resistors. For a motor, the load is a torque on a rotating shaft.

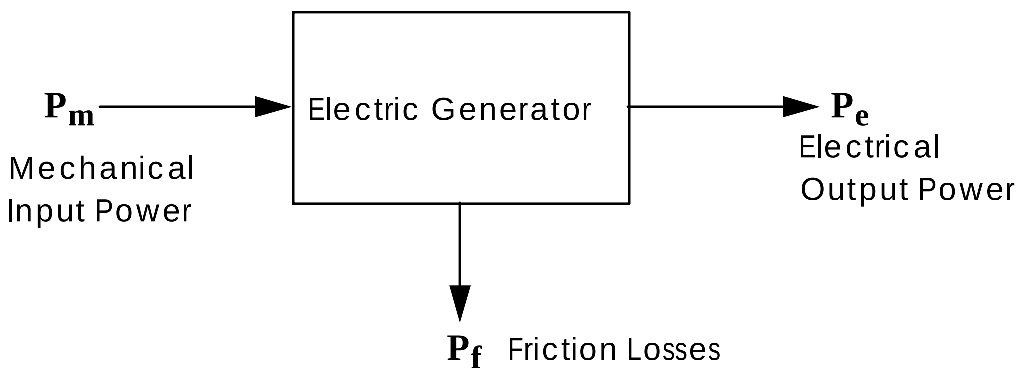
Prime Mover: A prime mover is the source of mechanical power that rotates the shaft in a generator, e.g. hydro turbines, steam turbines, axle of a vehicle.

3.1.2 Power Flow

In all motors there is a power flow from electrical to mechanical, as shown in **Figure 3.1a**. For generators the power flow is from mechanical to electrical as shown in **Figure 3.1b**.



a) Power flow in an electric motor



b) Power flow in an electric generator

Figure 3.1 Power Flow in an Electrical Machine Acting as a Motor or a Generator

In both cases there is mechanical movement and thus power lost due to friction. If there is also significant air movement, then there will also be power lost due to air friction known as "windage". As a first approximation we can lump the two together and refer to them interchangeably as "friction and windage" or just "friction" for which we will use the symbol P_f. Also we will use the symbol P_e to represent the power converted from electrical to mechanical in a motor, or from mechanical to electrical in a generator. We will use the symbol P_m to represent the mechanical power available from a motor or the mechanical power required to drive a generator.

Thus in the case of a motor, the mechanical power available from the machine will be reduced by the power lost due to friction;

$$P_m = P_e - P_f \qquad \text{for a motor}$$

Whereas in the case of a generator, the mechanical power required to drive the machine will have to include the power lost due to friction;

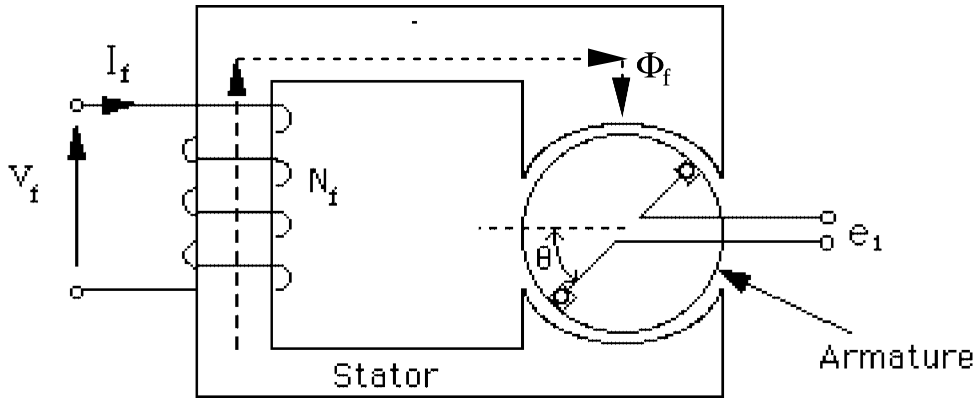
$$P_m = P_e + P_f \qquad \text{for a generator}$$

3.2 Separately Excited DC Machine

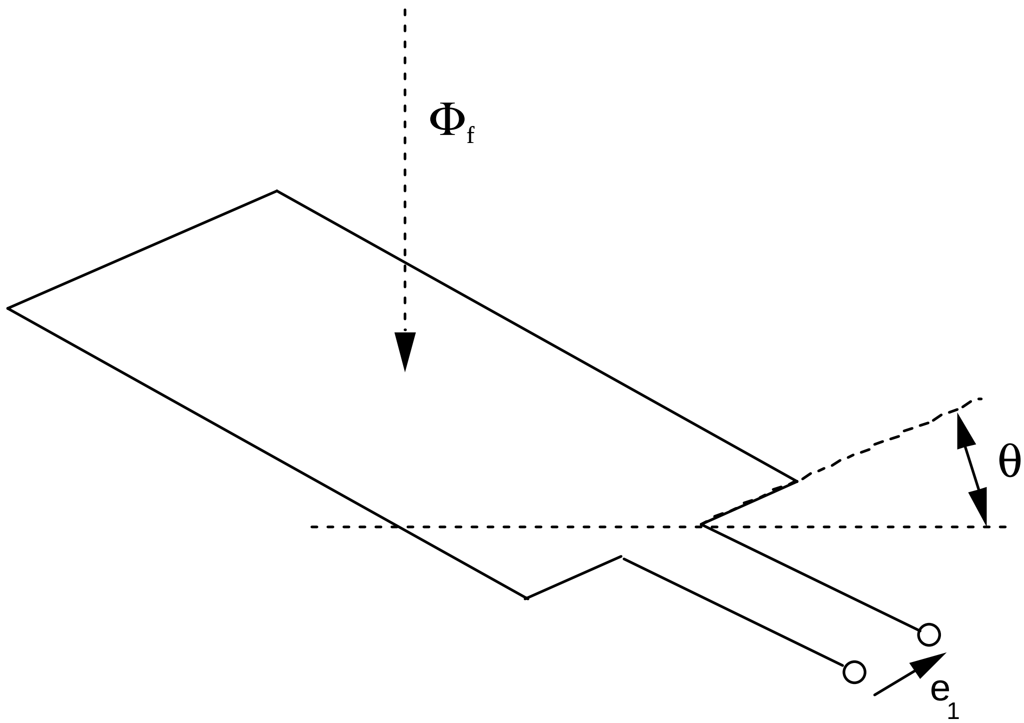
3.2.1 Derivation of Equivalent Circuit

A simplified magnetic circuit for a DC machine is shown in **Figure 3.2a**. The magnetic circuit consists of a stationary member called the stator and a rotating member called the armature, (sometimes called the rotor). The stator has a field winding of N_f turns and carries a current I_f. The armature has a winding of N_a turns and carries an armature current I_a. Only one turn of the armature winding is shown in **Figure 3.2a**. This type of machine configuration is called a separately excited machine because the field winding is not connected to the armature winding, in other words the field winding is separate from the armature winding.

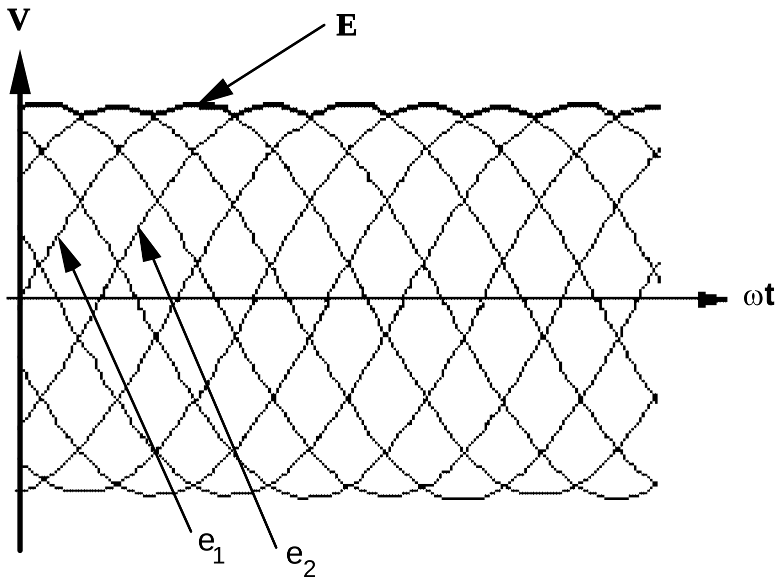
The field current, I_f, produces a flux, ϕ_f, which is called the field flux. The field flux, ϕ_f, passes through a small air gap separating the armature from the stator and then goes through the armature. As a first approximation one can assume that there is negligible fringing or leakage and thus all of ϕ_f goes through the armature. The armature has an armature winding of N_a turns, only one of which is shown in **Figure 3.2a**.



a) Simplified winding configuration,
only one turn of the armature winding is shown.



b) expanded view of one turn from the armature winding



c) Voltages produced be each turn and
net voltage, **E**, ’picked off’ at the commutator

Figure 3.2 Winding Configuration for a Basic DC Machine and Associated Voltage Waveforms

The voltage induced in this turn, e_1 , is given by the expression;

$$e_1 = \frac{\partial \Phi}{\partial t}$$

$$e_1 = \frac{\partial \left[\Phi_f \cos(\theta) \right]}{\partial t}$$

Assume that;

$$\theta = \omega t$$

Substitute for θ into the equation for e_1 to obtain;

$$e_1 = -\,\omega \phi_f \sin(\omega t)$$

The above expression is for the output voltage from one turn and is shown in more detail in **Figure 3.2b**. The actual armature consist of many turns each at a slightly different angle θ. Each of these turns will also produce a sinusoidal voltage but at a slightly different angle as shown in **Figure 3.2c**. The output voltage from each turn is brought out to corresponding segments of a commutator as shown in **Figure 3.3**.

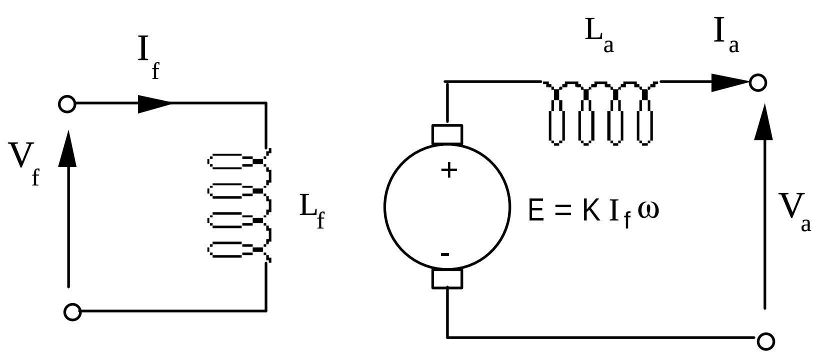


Figure 3.3 Commutator for a DC Machine

The commutator is an extension of the armature and thus also rotates. The voltage is 'picked off' the commutator by two brushes which are placed diametrically opposite each other. This results in each turn reaching its peak voltage as it passes the brushes. Thus the voltage, E, picked off by the commutator brushes will be:

$$E = \omega \phi_f$$

However, ignoring saturation;

$\phi_f = KI_f$

Substitute for ϕ_f into the equation for E to obtain:

$E = KI_f\omega$

Thus the basic equivalent circuit for a DC machine can be derived as shown in **Figure 3.4**.

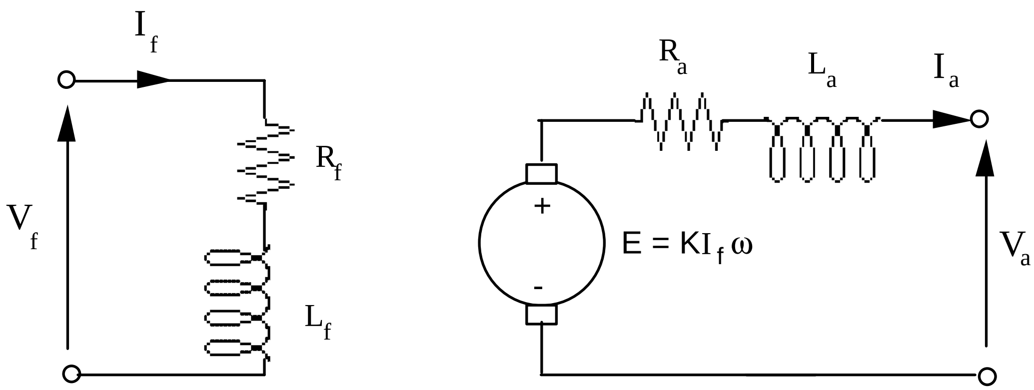


Figure 3.4 Basic Equivalent Circuit for a Separately Excited DC Machine

The armature voltage is represented by a voltage source, E, (often referred to as the back EMF), which is proportional to the field current and the shaft speed. The field windings are electrically isolated from the armature circuit and can be represented by an inductor, L_f.

All real windings have effective resistance and thus the equivalent circuits for the field and armature windings have to include series resistances, R_a and R_f, respectively. In practice, the field current, I_f, is sometimes controlled by an additional external series resistance. However it is convenient to lump both internal wiring resistance and external resistor and include them both in R_f. Furthermore, the armature winding is in reality a coil with some inductance L_a, thus the complete equivalent circuit for the armature side would include this inductance as well, as shown in **Figure 3.5**.

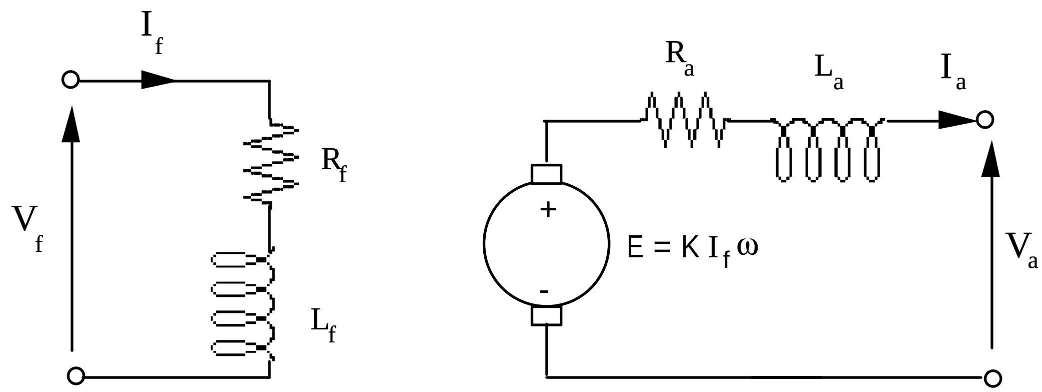


Figure 3.5 Equivalent Circuit for a Separately Excited DC Machine Including Winding Resistances

It is important to note that the inductances, L_f and L_a, are necessary for modeling the transient behaviour of a DC machine. However, for steady state conditions, L_f and L_a can be ignored.

For generators the current, I_a, is usually shown coming out of the machine, and thus the equation for the armature voltage, V_a, as measured at the armature terminals is:

$$V_a = E - I_aR_a = KI_f\omega - I_aR_a \qquad \text{for a generator}$$

For motors the current, I_a, is usually shown going into the machine, and thus the equation for the armature voltage, V_a, as measured at the armature terminals is:

$$V_a = E + I_aR_a = KI_f\omega + I_aR_a \qquad \text{for a motor}$$

The above equations are equally valid for positive and negative values of E, I_a, I_f, V_a, and ω, and thus a motor can change to a generator and vice versa merely by an appropriate change in polarity of one or more of these variables. Regardless of which of the above equations is used, the actual operating mode of the machine (motoring or generating) is determined by the actual polarity of the power flow into or out of the machine. The resultant voltage vs current characteristics represented by the above equations are shown graphically in **Figure 3.6**.

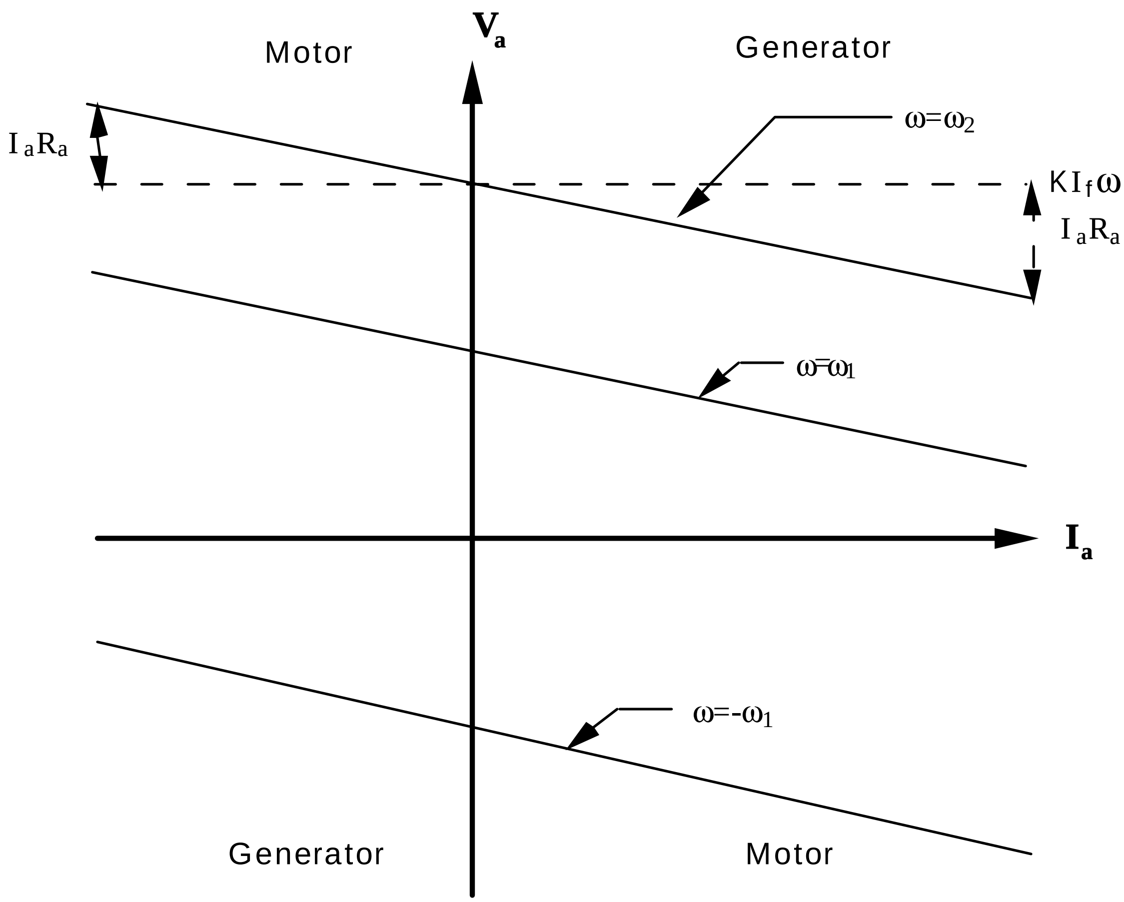


Figure 3.6 Voltage vs. Current Curves for a Separately Excited Machine

Example 3.1

A 250V, 15A, separately excited generator has the following data:

$R_a = 0.60\ \Omega$ $R_f = 240\ \Omega$

$L_a = 0.012$ H $L_f = 120$ H K = 1.8 H

The generator is operated at 900 rpm with $V_f = 360$ V.

Calculate:

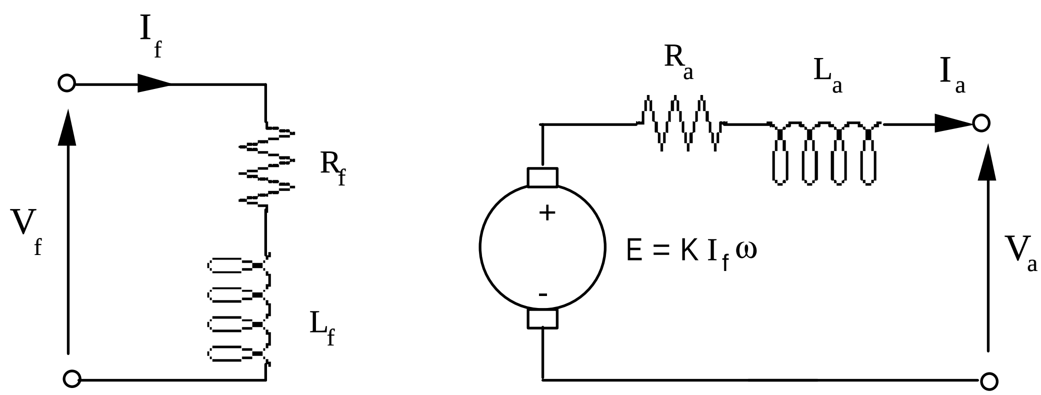
a) The no load output voltage

b) The output voltage at full load of 15A

c) The regulation.

Solution:

The equivalent circuit for a separately excited generator is shown below:



a) No load output voltage:

$$V_a = KI_f\omega - I_aR_a = KI_f\,\omega \text{ for no load where } I_a = 0$$

Also:

$$I_f = \frac{V_f}{R_f} = \frac{360}{240} = 1.5 \text{ A}$$

Substitute for I_f to obtain:

$$V_a = KI_f\omega = 1.8 \times 1.5 \times \frac{2\pi \times 900}{60} = 254 \text{ V}$$

b) Full load output voltage:

$$V_a = KI_f\omega - I_aR_a$$

Substitute for:

$I_f = 1.5$ A and $I_a = 15$ A

$$V_a = 1.8 \times 1.5 \times \frac{2\pi \times 900}{60} - 15 \times 0.6 = 245 \text{ V}$$

c) Regulation:

$$VR = \frac{V_{oc} - V_L}{V_L} = \frac{254 - 245}{245} = 3.67\%$$

3.2.2 Torque and Power in a Separately Excited DC Machine

The power converted within the machine, P_e, will be the power produced or absorbed by the voltage source E;

$$P_e = EI_a = KI_fI_a\omega = \omega T_e$$

The torque, T_e, produced by the machine will be;

$$T_e = P_e/\omega = KI_fI_a$$

It is important to note that the electrical input to the field winding, V_f and I_f, do not contribute to the mechanical power of the motor/generator. The only "power" winding is the armature or rotor winding. Thus the armature winding usually carries much more current and dissipates much more heat losses than the field winding.

The machine will be motoring if power is flowing into the voltage source E, whereas the machine will be generating if power is flowing out of the voltage source E.

Also the net mechanical power and torque available from the machine in the motoring mode are:

$$P_m = P_e - P_f = KI_fI_a\,\omega - P_f$$

$$T_m = T_e - T_f = KI_fI_a - T_f = KI_fI_a - P_f/\omega$$

Similarly the total mechanical power and torque required to operate the machine in the generating mode are:

$$P_m = P_e + P_f = KI_fI_a\,\omega + P_f$$

$$T_m = T_e + T_f = KI_fI_a + T_f = KI_fI_a + P_f/\omega$$

3.2.3 Torque and Speed for a Separately Excited DC Motor

The equivalent circuit for a separately excited DC motor is shown in **Figure 3.7**

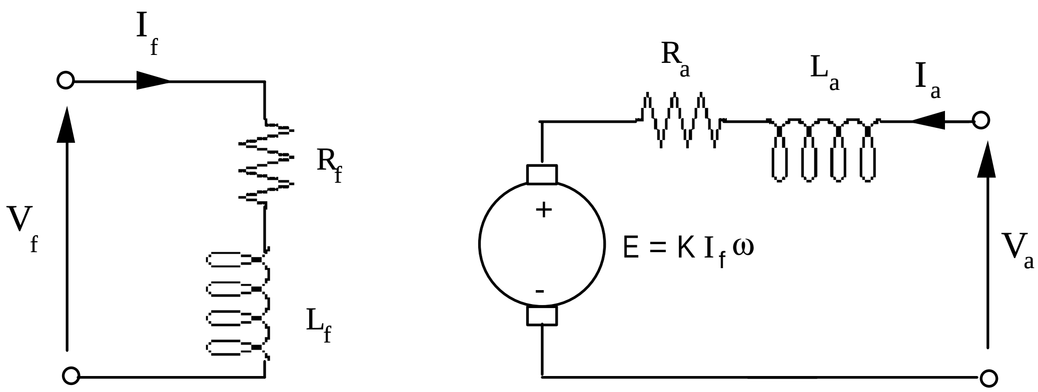


Figure 3.7 Equivalent Circuit for a Separately Excited DC Motor

The relevant equation is:

$$V_a = KI_f\omega + I_aR_a$$

This equation can be solved for ω;

$$\omega = \frac{V_a - I_aR_a}{KI_f}$$

Thus the motor speed can be controlled by varying V_a and/or I_f. It is important to note that the speed varies inversely with I_f and that;

$$\omega \rightarrow \infty \quad \text{as} \quad I_f \rightarrow 0$$

Also the equation for V_a can be used to solve for the armature current, I_a ;

$$I_a = \frac{V_a - KI_f\omega}{R_a}$$

Substitute for I_a into the equation for torque to obtain;

$$T_e = KI_fI_a = KI_f \frac{V_a - KI_f\omega}{R_a}$$

Which can be simplified to;

$$T_e = \frac{KI_f\left[V_a - KI_f\omega\right]}{R_a}$$

The above equation is shown graphically in **Figure 3.8**.

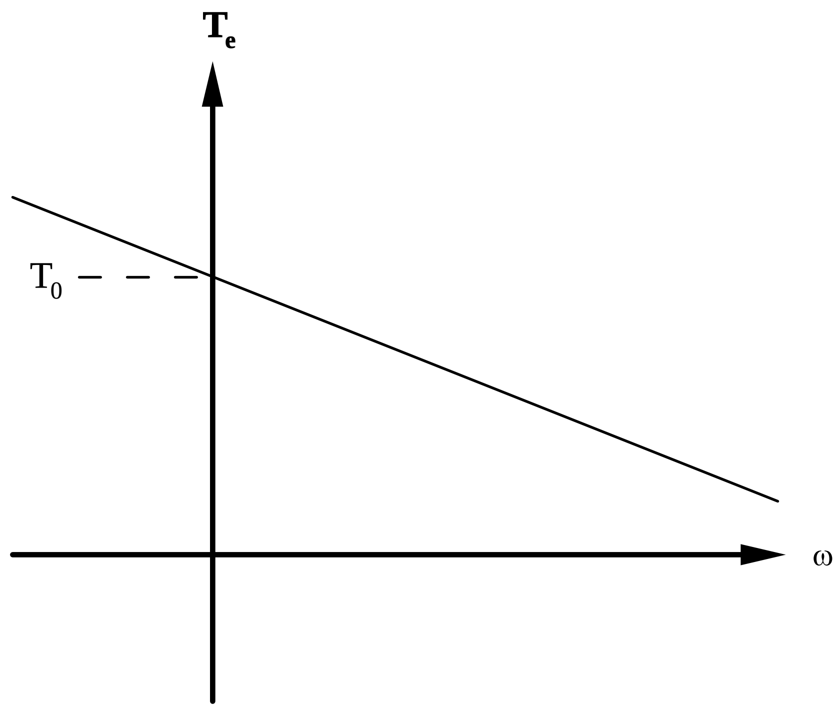


Figure 3.8 Torque vs. Speed Curve for a Spearately Excited Machine

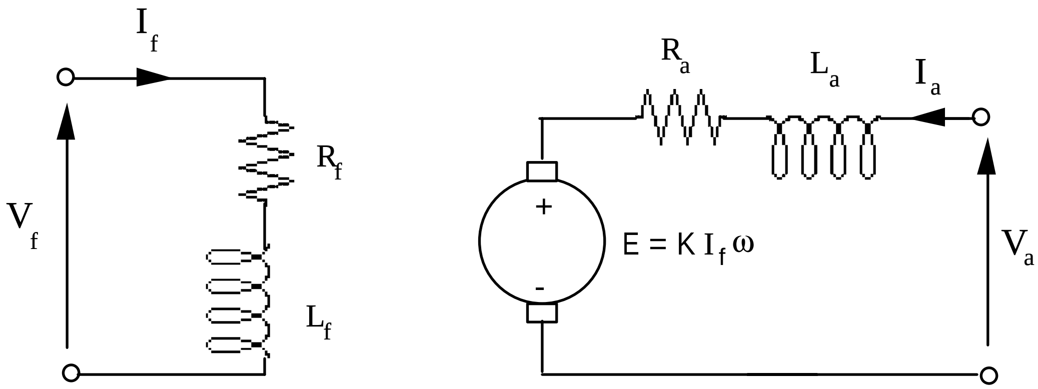
An important characteristic of a machine is the starting torque, T_0, which is defined as the torque at $\omega = 0$. For a separately excited motor;

$$T_0 = \frac{KI_fV_a}{R_a}$$

The starting torque is an indication of how fast a machine will accelerate from a standstill.

Example 3.2
When a separately excited motor is operating at full load and rated speed of 125 rpm it draws 38 A at 225 V. With the shaft stalled, (locked rotor), the machine draws 40 A at 16 V. Windage and friction losses are determined to be 550 W at rated speed.
a) What is the net available torque at full load and rated speed
b) Determine the motor efficiency at full load and rated speed, assuming zero losses in the field winding.

Solution:
The equivalent circuit for a separately excited motor is shown below:



Basic equation is:

$$V_a = KI_f\omega + I_aR_a$$

Solve for R_a to obtain:

$$R_a = \frac{(V_a - KI_f\omega)}{I_a}$$

During blocked rotor, $\omega = 0$, $V_a = 16$, $I_a = 40$, therefore:

$$R_a = \frac{(16-0)}{40} = 0.4\Omega$$

The basic equation can also be used to solve for:

$$KI_f = \frac{V_a - I_aR_a}{\omega} = \frac{225 - 38\times 0.4}{125\times\frac{2\pi}{60}} = 16.0\ \Omega$$

Also:

$$T_m = T_e - T_f = KI_fI_a - \frac{P_f}{\omega}$$

$$= 16.0\times 38 - \frac{550}{125\times\frac{2\pi}{60}} = 566 \text{ n-m}$$

Efficiency:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{(P_{in} - P_{losses})}{P_{in}}$$

Where:

$$P_{in} = V_aI_a = 38\times 225 = 8550 \text{ W}$$

$$P_{losses} = I_a^2 R_a + P_f = 38^2\times 0.4 + 550 = 1127.6 \text{ W}$$

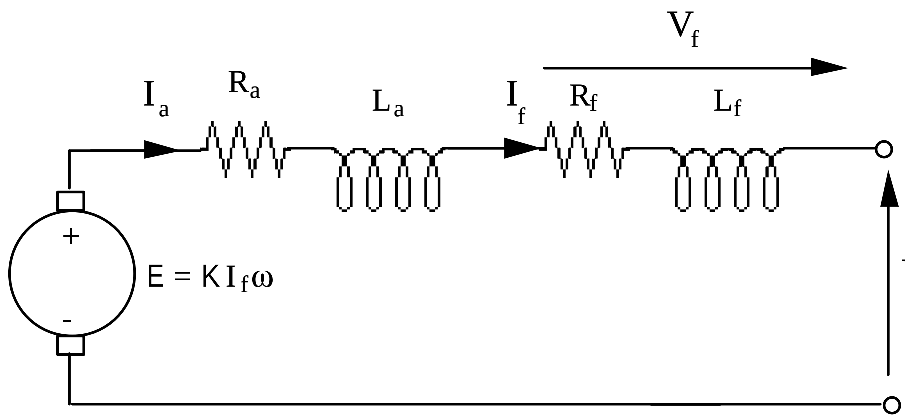
Therefore:

$$\eta = \frac{(P_{in} - P_{losses})}{P_{in}} = \frac{(8550 - 1127.6)}{8550} = 86.8\%$$

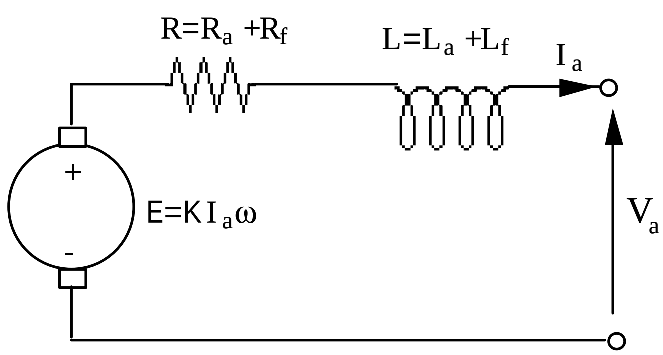
3.3 Series Machine

3.3.1 Equivalent Circuit for a Series Machine

A separately excited machine configuration requires a separately controllable field excitation circuit which usually means additional expense. Thus the more common low cost DC machines have their field windings connected in series or in parallel to the armature so as to avoid the need for a separate field excitation circuitry and to produce unique characteristics.



a) Field winding connected in series with the armature



a) Simplified equivalent circuit

Figure 3.9 Equivalent Circuit for a Series Machine

The equivalent circuit for a series machine is shown in **Figure 3.9a** and it consists of the field inductance and resistance connected in series with the armature inductance and resistance.
Thus:

$$I_f = I_a$$

The equivalent circuit can be simplified by combining the field and armature inductances into one inductance, L, and combining the armature and series resistances into one resistance, R, as shown in **Figure 3.9b**.

3.3.2 Series Motor

The equivalent circuit for a series motor is shown in **Figure 3.10.** It is basically the same circuit as **Figure 3.9** except that for a motor the armature current is shown going into the machine.

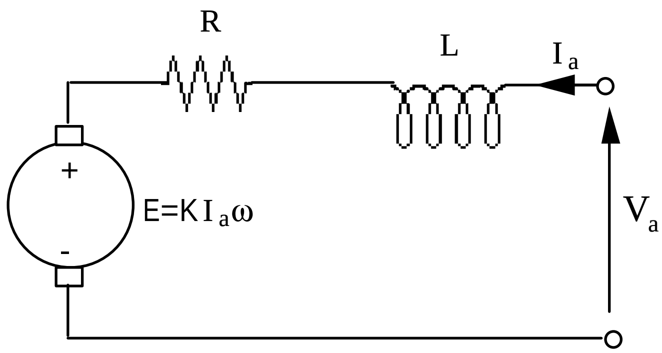


Figure 3.10 Equivalent Circuit for a Series Motor

The basic equations for a series motor can be determined by substituting for;

$$I_f = I_a$$

Into;

$$V_a = KI_f\omega + I_aR$$

And;

$$T_e = KI_f I_a$$

To obtain;

$$V_a = KI_a \omega + I_aR = I_a(K\omega + R)$$

$$T_e = KI_a^2$$

It is important to note that the torque produced by the series machine is always positive regardless of the polarity of the armature current. Thus, if;

$$I_a(\omega t) = \hat{I} \sin(\omega t)$$

Then;

$$T_e = K\hat{I}^2 \sin^2(\omega t)$$

Thus even though the armature current is alternating the motor torque will not alternate, it will always be in the same direction and the motor speed will be in the one direction. The end result is that a series motor will work equally well with an AC or DC power source. It is therefore often referred to as a *universal* motor.

The equations for V_a and T_e can solved to obtain;

$$I_a = \frac{V_a}{K\omega + R}$$

And;

$$T_e = K\frac{V_a^2}{(K\omega + R)^2}$$

And also:

$$T_0 = K\frac{V_a^2}{R^2}$$

It is important to note that for most series motors R is deliberately made small so as to minimize I^2R losses. Thus the $V_a^2 \;/R^2$ term is usually very large and thus a series motor can produce very high starting torque. This makes it very useful for electric vehicle applications and similar traction drives. The torque vs speed characteristic for a series motor is shown graphically in **Figure 3.11**.

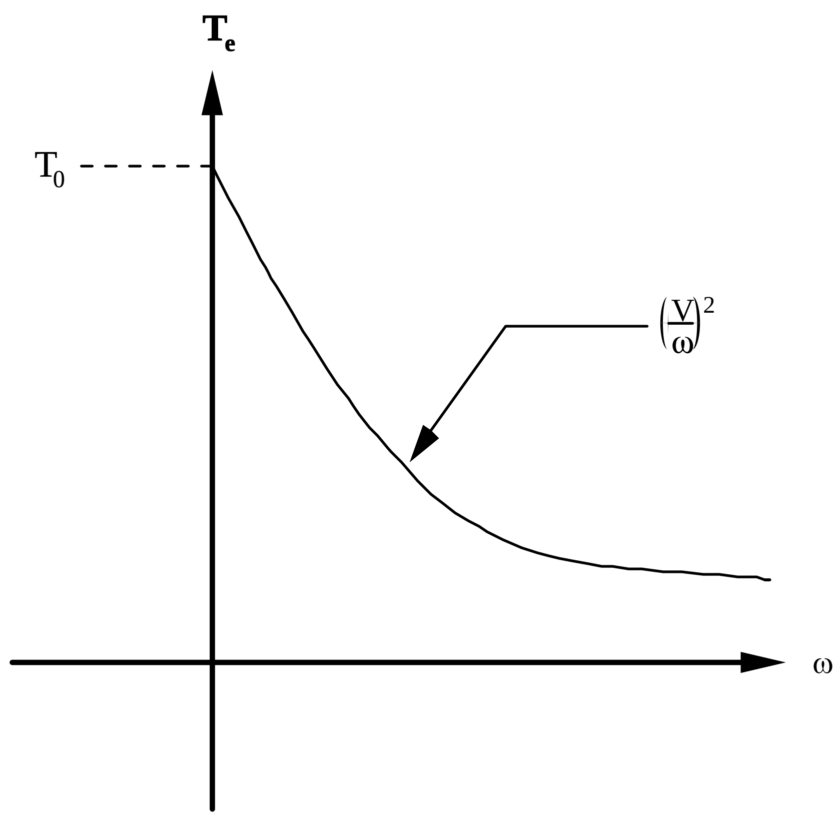


Figure 3.11 Torque vs. Speed Curve for a Series Motor

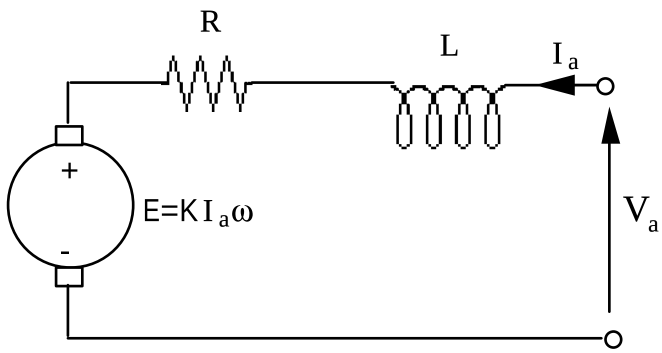
The preceeding equations can also be solved for ω;

$$\omega = \frac{V_a}{I_a K} - \frac{R}{K}$$

This equation illustrates that the most convenient method of speed control is by controlling the motor voltage, V_a. However, in many application where the motor voltage is fixed, it is very difficult to control the speed of a series motor because I_a can not be independently controlled. The resistance R, is also difficult to control because it is a very low value and carries the full machine current. Thus series motors are not used where speed control is required unless the motor voltage can be easily controlled.

Example 3.3
A series motor has ratings of: 4 A. 125 V, 3000 rpm.
At blocked rotor and rated current, the voltage is 60V.
The friction and windage losses are proportional to speed and equal 225W at 3000 rpm.
Calculate the no load current and speed, in rpm, at 125 V.

Solution:
The equivalent circuit for a series motor is shown below:



R can be determined from the blocked rotor test, (where $\omega = 0$):

$$V = I_a(K\omega + R)$$

Solve for:

$$R = \frac{V}{I_a} - K\omega = \frac{60}{4} - 0 = 15\ \Omega$$

And, at rated speed and voltage, solve for:

$$K = \frac{(V - RI_a)}{I_a\omega} = \frac{(125 - 15\times 4)}{4\times 3000} = 5.42\times 10^{-3} \text{ with respect to rpm}$$

At "no load" $P_m = 0$, however there is still P_f and:

$$P_f \propto \omega \text{ or } P_f = K_f\omega$$

Solve for:

$$K_f = \frac{P_f}{\omega} = \frac{225}{3000} = 0.075 \text{ with respect to rpm}$$

Also:

$$P_e = P_f = EI_a = K_f\omega$$

Substitute for:

$$E = K\omega I_a$$

To obtain:

$$K\omega I_a I_a = K_f\omega$$

Solve for:

$$I_a = \sqrt{\frac{K_f}{K}} = \sqrt{\frac{0.075}{5.42 \times 10^{-3}}} = 3.72 \text{ A}$$

Also:

$$V = I_a K\omega + I_a R$$

Solve for:

$$\omega = \frac{V - I_a R}{K I_a} = \frac{125 - 3.72 \times 15}{5.42 \times 10^{-3} \times 3.72} = 3{,}430 \text{ rpm}$$

3.3.3 Series Generator

The basic equivalent circuit for a series generator with a resistive load is shown in **Figure 3.12**.

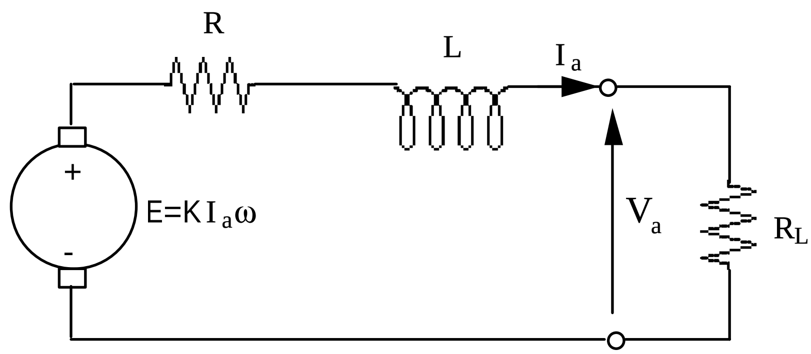


Figure 3.12 Equivalent Circuit for a Series Generator with Resistive Load

This situation is represented graphically in **Figure 3.13**. The load equation is represented by a straight line going through the origin:

$$E = I_aR + I_aR_L$$

The back EMF for a series generator voltage is represented by another equation going through the origin:

$$E = KI_a\omega$$

The operating point for a series generator is represented by the intersection of the above two equations which will only intersect at the origin, $E = 0$, $I_a = 0$, which means no output voltage or current.

However, in a real series generator the effects of hysteresis and saturation distort the generator voltage equation to produce the curve shown in **Figure 3.13**.

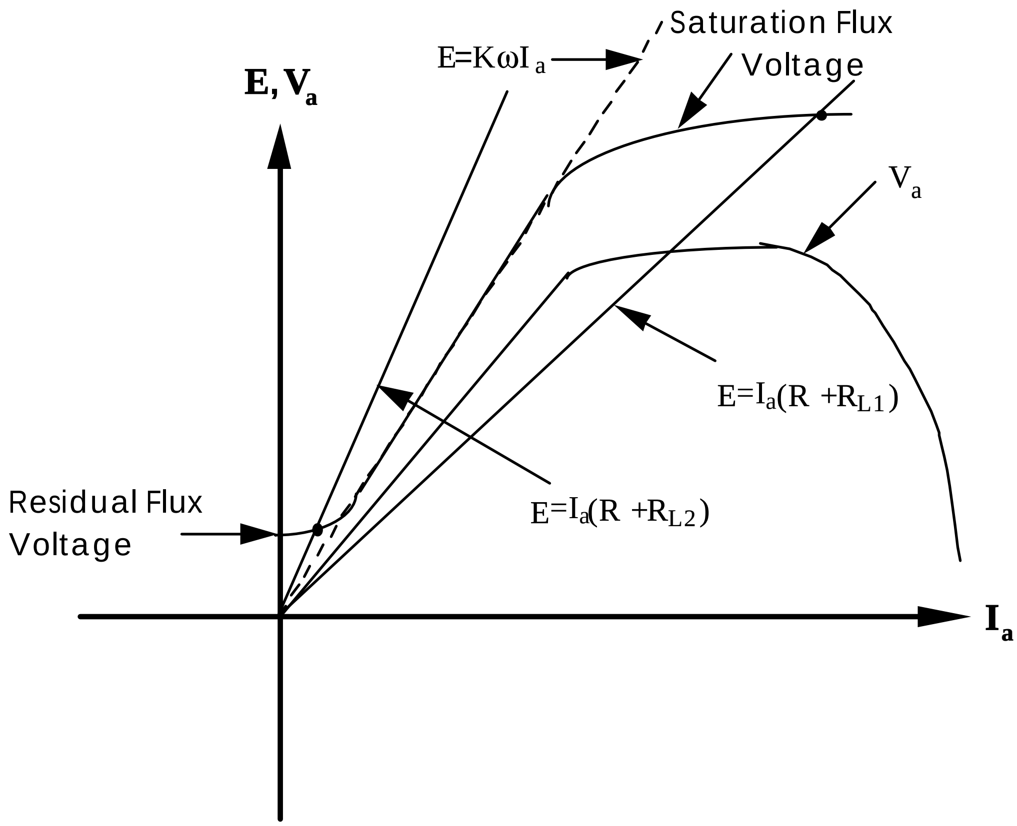


Figure 3.13 Voltage Current Resistance Relationships for a Series Generator

Thus two operating points are possible. One intersection point is at the residual flux level which will result in very low output voltage and and current. A more suitable operating point is represented by the intersection on the saturation part of the curve. This will be reached only if the load line is "shallow" enough to intersect the generator line;

$$R + R_L = K\omega$$

This implies a maximum starting value for the load, R_L , and a minimum starting value for shaft speed, ω. Also the steady state operation will always be in the non-linear region requiring graphical solutions. That makes for a rather unreliable generator and series machines are thus not used as a source of electricity. However, series motors can readily be used in a generator mode when feeding power back to a voltage source, as in braking. A typical output voltage, V_a, vs. current, I_a, curve for a series generator is shown in **Figure 3.13.**

3.4 Shunt Machine

3.4.1 Equivalent Circuit for a Shunt Machine

A DC machine in which the field winding is connected in parallel to the armature is called a shunt machine. The equivalent circuit for such a machine is shown in **Figure 3.14**.

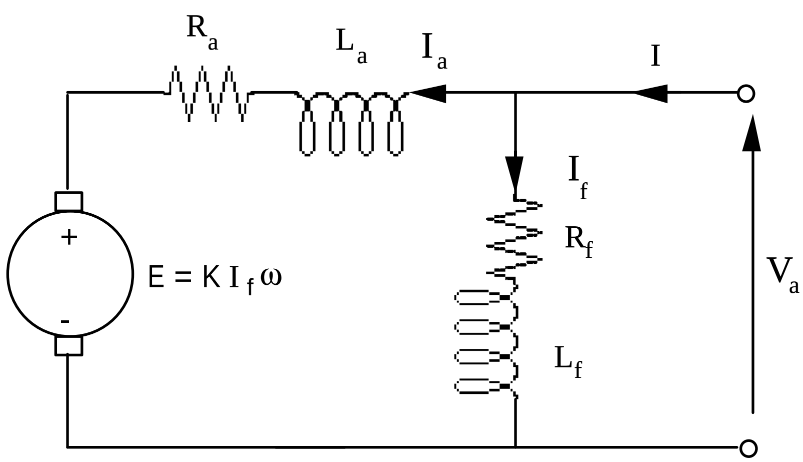


Figure 3.14 Equivalent Circuit for a Shunt Machine (Motor Configuration)

The relevant equations are;

$$I = I_a + I_f$$

Where I is the total current into the machine. Also, in the steady state;

$$V_a = K\omega I_f + R_a I_a$$

And;

$$I_f = V_a/R_f$$

The equation for torque is;

$$T_e = KI_f I_a$$

Note that for a shunt machine the field resistance, R_f, includes both the reistance of the field winding itself and any external (variable) resistor that is often used to control the field current.

3.4.2 Shunt Motor

The equations for a shunt machine can be rearranged to obtain a torque versus speed equation;

$$T_e = \frac{KV_a^2}{R_f R_a}\left[1 - \frac{K\omega}{R_f}\right]$$

This equation is shown graphically in **Figure 3.15**.

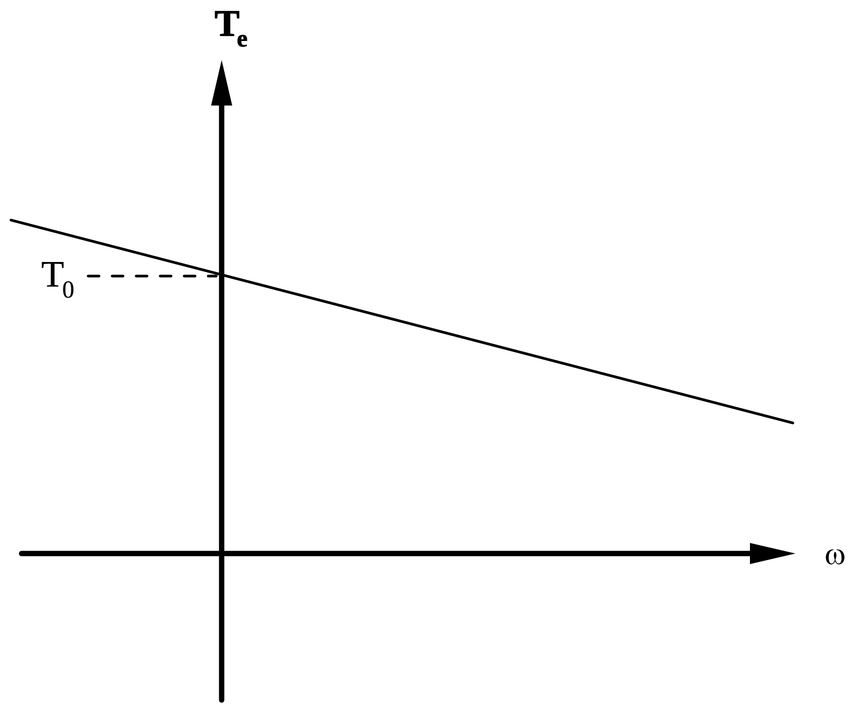


Figure 3.15 Torque vs. Speed Curve for a Shunt Motor

The starting torque, T_0, can be determined as:

$$T_0 = \frac{KV_a^2}{R_f R_a}$$

The above expression for a shunt motor will generally result in a much lower value of starting torque than for a series motor because,

$$R_f >> R_a$$

The preceeding equations can also be solved for ω;

$$\omega = \frac{V_a - R_a I_a}{K I_f}$$

Thus the speed of a shunt motor can be readily controlled by controlling the field current, I_f.

Example 3.4

A 240 V shunt motor has the following parameters:

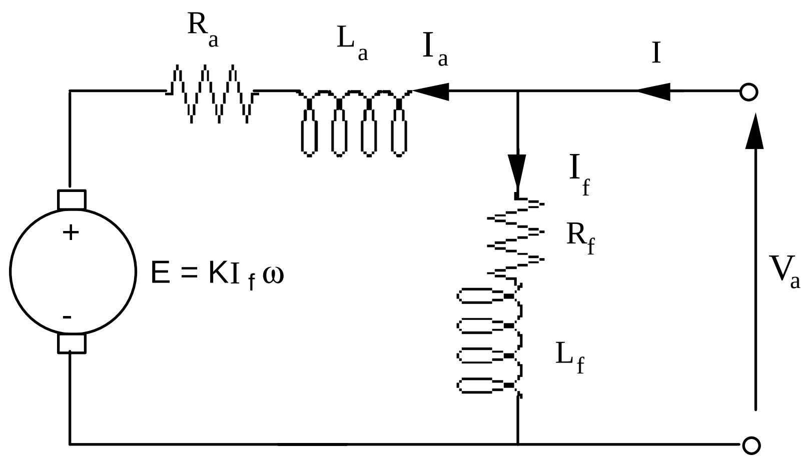
$R_a = 0.11\ \Omega$, $R_f = 120\ \Omega$

When driving rated load torque the machine draws 82 A at 240 V and its speed is 900 rpm.

Determine current and speed if the torque is increased by 50% but voltage is unchanged.

Solution:

The equivalent circuit for a shunt motor is shown below:



$$I_f = \frac{V}{R_f} = \frac{240}{120} = 2\text{ A}$$

$$I_a = I_L - I_f = 82 - 2 = 80\text{ A}$$

The machine voltage is given by:

$$V_a = K\omega I_f + R_a I_a$$

Solve for:

$$K = \frac{V_a - R_a I_a}{\omega I_f} = \frac{240 - 0.11 \times 80}{900 \times \frac{2\pi}{60} \times 2.0} = 1.23\text{ H}$$

Assume rated torque is T_r and therefore:

$$T_r = K I_f I_a$$

Thus, if

$$T_2 = 1.5 T_r$$

Then,

$$I_{a2} = 1.5 I_a = 1.5 \times 80 = 120\text{ A}$$

Solve the basic equation for:

$$\omega_2 = \frac{V_a - R_a I_{a2}}{K I_f} = \frac{240 - 0.11 \times 120}{1.23 \times 2.0} = 92.2 \text{ r/s} = 880 \text{ rpm}$$

3.4.3 Shunt Generator

The basic equivalent circuit for a shunt generator is shown in **Figure 3.16**.

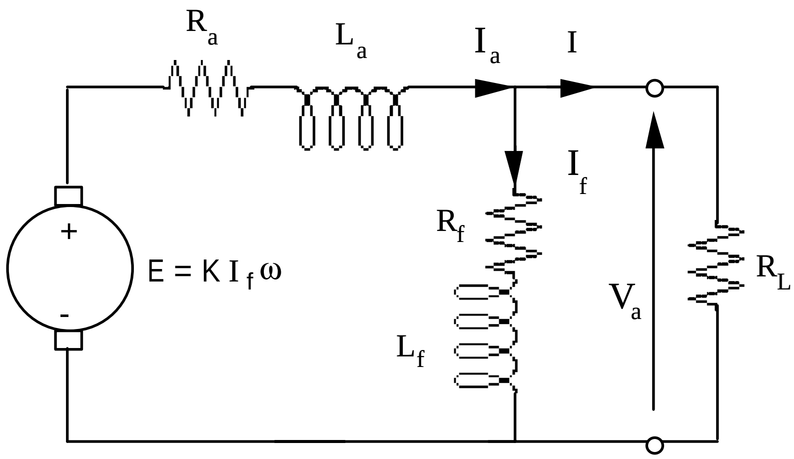


Figure 3.16 Equivalent Circuit for a Shunt Generator with a Resistive Load

The load equation is represented by a straight line going through the origin:

$$E = I_a R_a + I_a (R_f || R_L)$$

Substitute for;

$$I_a = I \frac{R_L}{R_f || R_L}$$

To obtain:

$$E = I \frac{R_L R_a}{R_f || R_L} + IR_L = I \frac{R_f R_L + R_L R_f + R_a R_f}{R_f}$$

The back EMF for a shunt generator is represented by another equation going through the origin:

$$E = KI_f \omega = K \omega I \frac{R_L}{R_f}$$

The operating point for a shunt generator is represented by the intersection of the above two equations which will only intersect at the origin, $E = 0$, $I = 0$, which means no output voltage or current.

However, in a real shunt generator the effects of hysteresis and saturation distort the generator voltage equation to produce the curve shown in **Figure 3.17**.

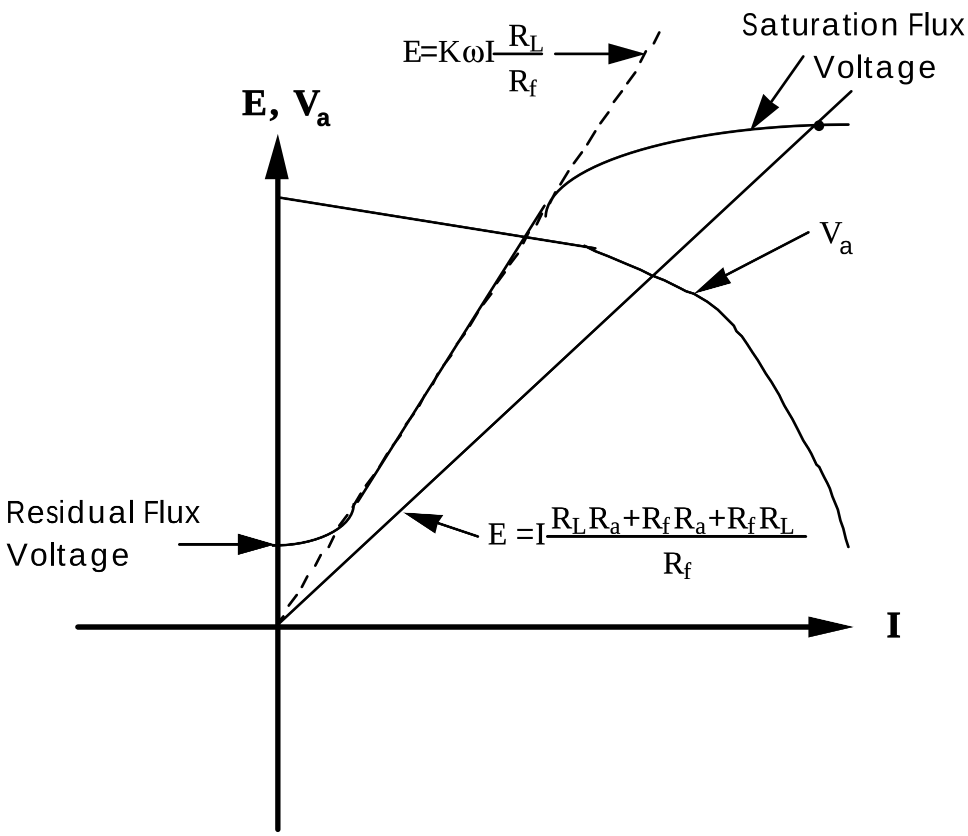


Figure 3.17 Voltage, Current, Resistance Relationships for a Shunt Generator

Thus an operating point will be reached only if the load line, (including armature and field resistance), is "shallow" enough to intersect the generator line;

$$R_a + (R_f \,||\, R_L) = K\omega$$

This implies a maximum starting value for the load, R_L, and field resistance, R_f, as well as a minimum starting value for shaft speed, ω. Also the steady state operation will always be in the non-linear region requiring graphical solutions. That makes for a rather unreliable generator and shunt machines are thus not used as a source of electricity. However, shunt motors can readily be used in a generator mode when feeding power back to a voltage source, as in braking. A typical output voltage, V_a, vs current, I, curve is also shown in **Figure 3.17**.

3.5 Compound Machine

3.5.1 Equivalent Circuit

Many DC machines have two field windings, one of which is connected in series with the armature and the other is connected in parallel to the armature. Such a machine is called a compound machine. The equivalent circuit for such a machine is shown in **Figure 3.18**.

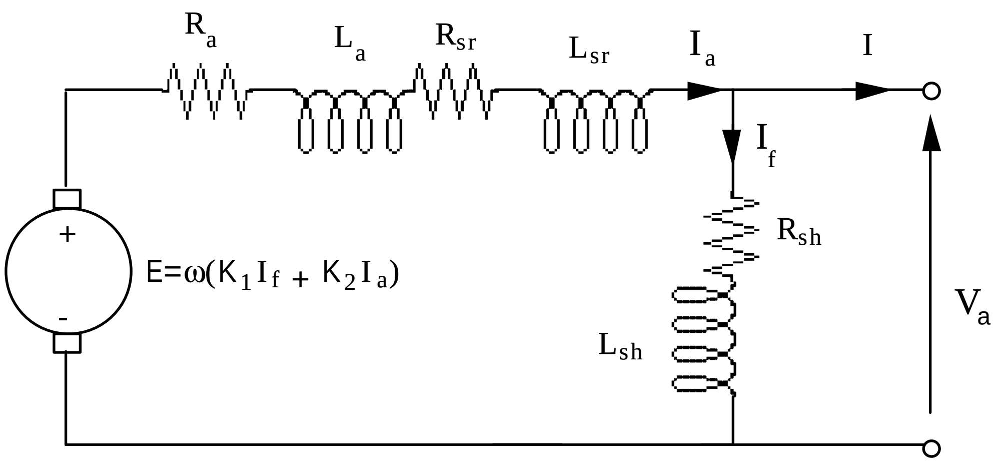


Figure 3.18 Equivalent Circuit for a Compound Machine

The series and shunt fields can be connected either aiding or opposing each other. If the fields are aiding each other the machine is said to be "cummulatively compounded." If the fields are opposing each other the machine is said to be "differentially compounded."

3.5.2 Compound Generator

The equation for the output voltage of a compound generator is ;

$$V_a = \omega(K_f I_f + K_a I_a) - I_a(R_a + R_{sr})$$

For a cummulatively compound machine K_a is positive, whereas for a differentially compound machine K_a is negative. The different types of voltage versus current curves for a compound generator are shown in **Figure 3.19**.

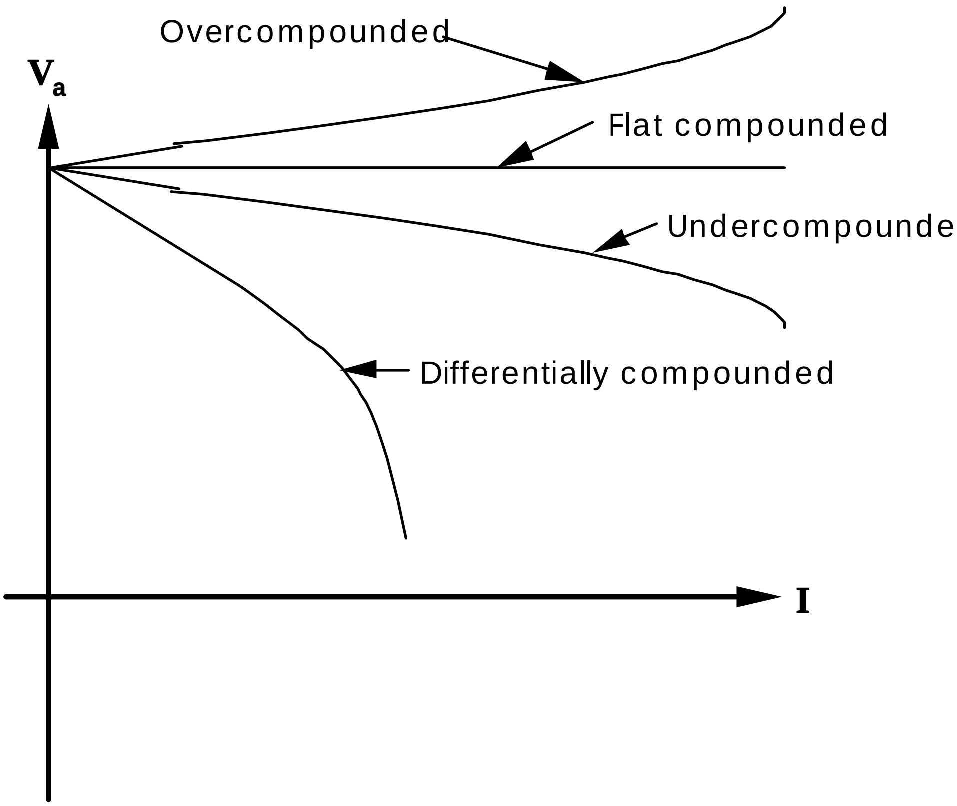


Figure 3.19 Output voltage and current curves for compound generators with various compounding factors.

The most common type of compound generator is a cummulatively compound configuration.   There are several types of curves for cummulatively compound generators depending on the relative magnitude of ωK_a versus R_a and R_{sr}. For a flat compound curve ωK_a should be equal to the sum of R_a and R_{sr}. This is only possible at one speed. Thus compound generators are relatively good at a fixed speed but do not offer much advantage at a varying speed.

3.5.2 Compound Motor

The equation for the torque generated by a compound motor is;

$$T_e = K_f I_f I_a + K_a I_a I_a$$

$$= K_f I_f I_a + K_a I_a^2$$

The above equation can not be simplified to the same extent as the corresponding equations for series or shunt motors. Nevertheless the result is a torque vs speed curve that is a hybrid between that of a series motor and that of a shunt motor, as shown in **Figure 3.20**.

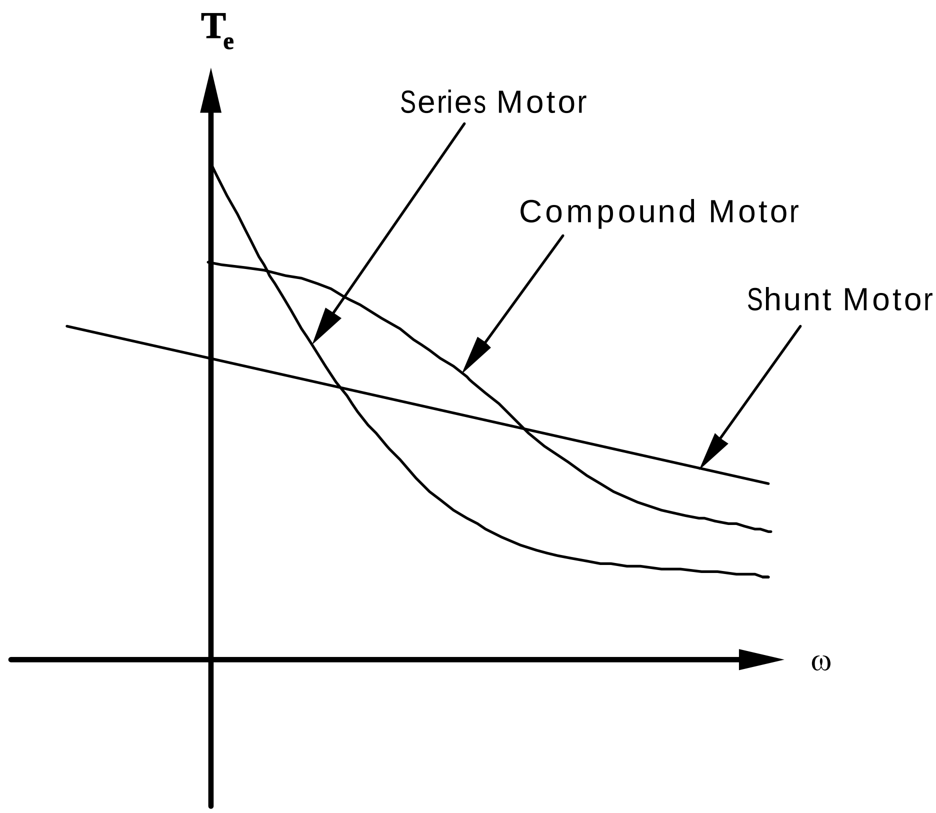


Figure 3.20 Torque vs. Speed for a Compound Motor Compared to Series and Shunt Motors

Also the expression for starting torque can be shown to be;

$$T_0 = K_f I_f I_a + K_a I_a I_a$$

$$= \frac{K_f V_a^2}{R_f(R_a + R_{sr})} + \frac{K_a V_a^2}{(R_a + R_{sr})^2}$$

The resultant value for T_0 is between that of a series motor and a shunt motor. Compound motors are thus used where moderately high starting torque is required but keeping starting current low is also an important consideration. A common such application is in locomotives, where rapid acceleration is not important but it is important to keep starting current moderately low. The 'starting mode' for a locomotive can be several hours and thus it is important to keep the current within limits to prevent motor burnout.

3.6 Limitations of DC Machines

The main limitation for a DC machine is the commutator. For high current machines the commutator can be the largest part of the machine. It requires maintenance to replace worn brushes. The brushes rubbing against the commutator produce a ’brush’ resistance which dissipates power and produces arcing. This tends to make high voltage DC machines impractical.

The rotor windings of a DC machine are the power carrying windings and thus dissipate most of the power. Because they are rotating, it is virtually impossible to water cool them, and thus high power DC machines are impractical because of cooling limitations.

DC generators have been virtually eliminated by solid state power supplies. Even the DC generator in automobiles has been replaced by a synchronous AC generator and diodes that rectify the AC to produce DC.

DC motors are still in widespread us for variable, controllable speed systems such as traction drives, locomotives, rolling mills, etc., because the control system for a DC system is simpler than for an AC system. However, the continuing decrease in the cost and complexity of AC power controllers, due to power electronics, has reduced even these applications.

3.7 Traction Drives

3.7.1 Traction Drive Requirements

A typical torque vs speed requirement for a traction (vehicle) drive system is shown in **Figure 3.21**. In general, a high starting torque is required in the accelerating mode followed by a constant power mode for cruising. In decelerating it is important to provide maximum braking torque over the entire speed range.

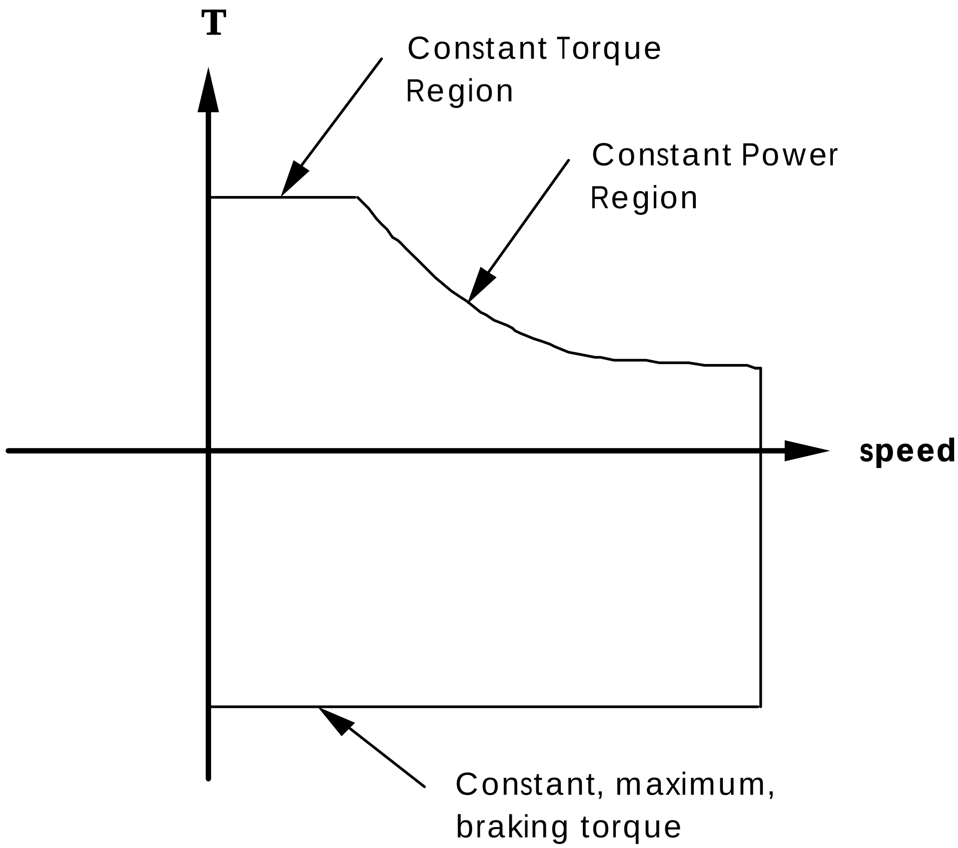


Figure 3.21 Torque vs Speed Requirements for Traction Drives (Vehicle Drives)

DC machine characteristics are particularly well suited for such applications because it is relatively easy to vary their speed over a wide range and it is relatively easy to provide braking torque by changing them over to a generator mode. This is called electrical braking and saves wear and tear on brake pads, brake linings etc. that would be required for mechanical type friction braking. There are two types of braking possible with electrical drive systems; dynamic braking and regenerative braking. In both cases the machine acts as a generator and converts the kinetic energy of the moving vehicle into electrical energy.

In dynamic braking the electrical energy is dissipated in resistors that are connected to the output of the machine. These are sometimes used to provide heat in passenger vehicles.

In regenerative braking the electrical energy is recovered by feeding it back to the power source. The power source has to be able to receive this regenerated energy. This means that the power source has to be able to tolerate both positive and negative currents. Such a power source is referred to as being able to "source and sink current". A suitable power source may be a battery in an electric car or a DC grid that feeds more than one vehicle such as in a subway system.

3.7.2 Traction Drive with a Series Motor

a) Motoring
Series DC motors have an inherent torque-speed characteristic that closely matches the requirements of **Figure 3.21**. Their initial torque is high and is usually more than sufficient for starting a vehicle and in fact often has to be limited to keep the current from becoming excessive. Once the machine speed increases to a sufficient level the torque inherently drops off in an approximately constant power curve.

b) Dynamic Braking
A typical series motor configuration for traction drives with dynamic braking is shown in **Figure 3.22** A series motor inherently produces torque in the same polarity even if the armature current reverses. Thus simply reversing the polarity of the voltage applied to the machine will not cause braking action. In order to generate a negative, braking, torque a series motor has to have its field winding reversed with respect to the armature winding, as shown by the reversing switches in **Figure 3.22**. During dynamic braking the series motor behaves in a nonlinear mode as described in **section 3.3.3**. This requires that the braking resistor, R_B, must start off at a low value and then gradually increase until braking action "kicks in" and then continually reduced as the speed decreases. This makes the control system for braking rather complex and often requires manual operation.

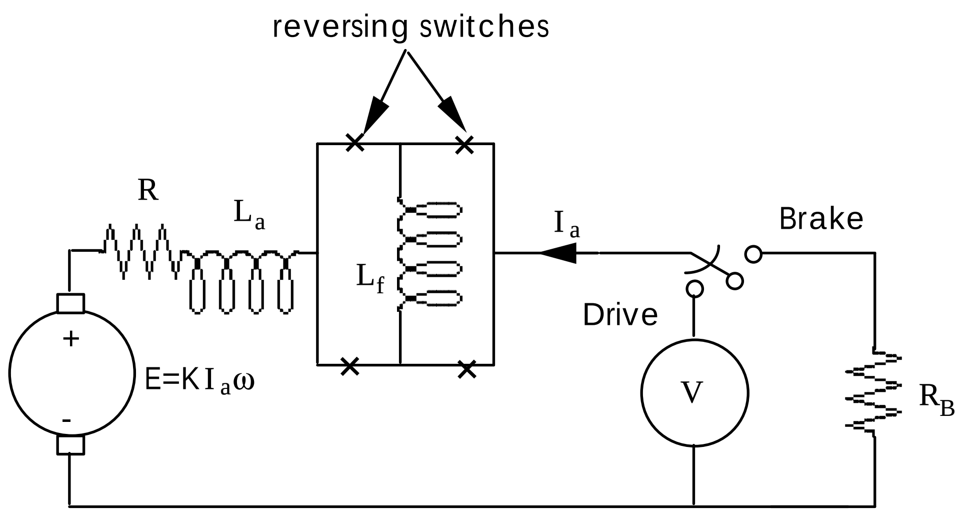


Figure 3.22 Series Motor in Traction Application with Dynamic Braking

c) Regenerative Braking

Regenerative braking with a series motor requires that the power source driving the motor be capable of accepting power as well as providing power. The configuration shown in **Figure 3.22** can still be used for regenerative braking, the field winding still has to be reversed from the motoring mode to the braking mode but instead of switching in the braking resistor, R_B, the machine is connected back to the voltage source V. The equivalent circuit during the regenerating mode is shown in **Figure 3.23**.

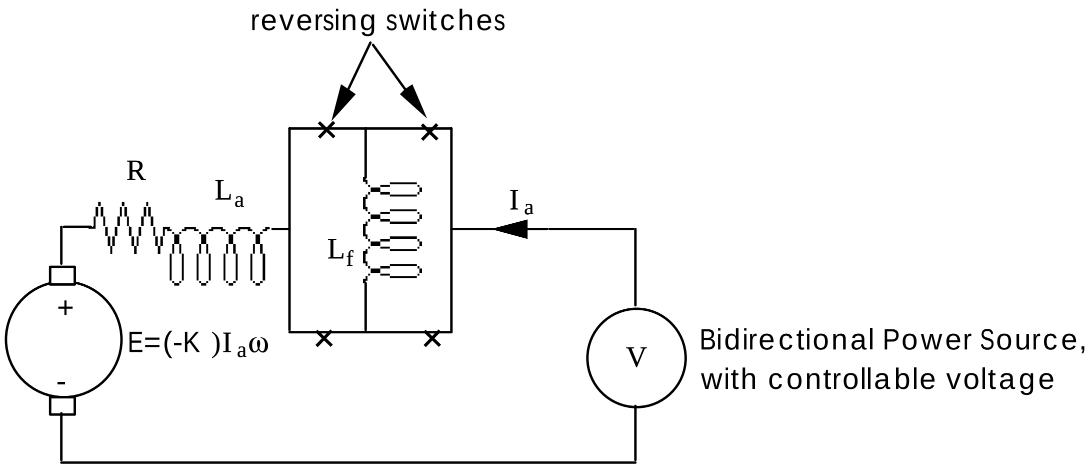


Figure 3.23 Equivalent Circuit for Regenerative Braking with a Series Motor

It is convenient in this sort of analysis to assume that the machine is still a motor and therefore the motor current, I_a, is flowing into the machine. However the polarity reversal of the field winding is captured in the equivalent circuit by changing the polarity of K. The circuit equation now becomes:

$$V_a = (-K)\omega I_a + RI_a$$

Solve for;

$$I_a = \frac{V_a}{R - K\omega}$$

Assuming that the switchover from motoring to braking occurred quickly enough then the machine speed, ω, will not change during the switchover and it can be shown that for any appreciable machine speed;

$$K\omega > R$$

And therefore;

$$I_a < 0$$

And the machine back EMF will be;

$$E = (-)K\omega I_a \quad > 0$$

Thus power will be flowing out of the machine and back into the power source.

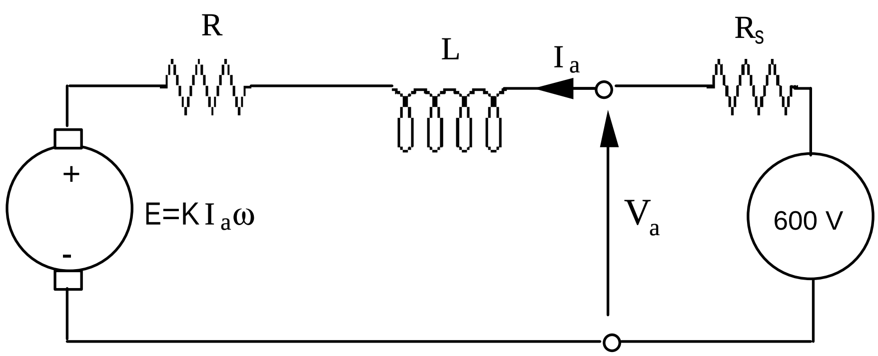
Also the machine torque will be;

$$T_e = (-K)I_a^2 = -K\frac{V_a^2}{(R - K\omega)^2}$$

It is important to note that in regenerative braking there is no means of controlling the braking torque in a series motor except by controlling the power source voltage, V_a. Thus regenerative braking requires a controllable DC voltage that can source and sink current. It has to be able to supply positive current during motoring and accept negative current during regenerative braking.

Example 3.5
A subway car weighing 10 tonnes is driven by a **series motor** fed from a 600 V rail with a source impedance of 0.1 ohms. Upon starting, the initial current is 1000A. The machine developes a torque of 500 n-m at a top speed of 80 kph, and draws 100A.
Determine:
i) Starting torque
ii) Motor rpm at 80 kph
iii) The braking torque if regenerative braking is attempted by disconnecting the supply, reversing the field and reconnecting the supply.

Solution:
The equivalent circuit for a series motor and 600 V source is shown below:



First determine R and K so that all other characteristics of the machine can be determined.
The basic equation is:

$$V = I_a(K\omega + R + R_s)$$

Solve for R and substitute for starting conditions:, $I_a = 1000$ A, $\omega = 0$, to obtain:

$$R = \frac{V}{I_a} - K\omega - R_s = \frac{600}{1000} - 0 - 0.1 = 0.5\ \Omega$$

Also:

$$T = KI_a^2$$

Solve for:

$$K = \frac{T}{I_a^2} = \frac{500}{100^2} = 0.05\ \mathrm{H}$$

i) Starting torque:

$$T_0 = KI_a^2 = 0.05 \times 1000^2 = 50{,}000\ \text{n-m}$$

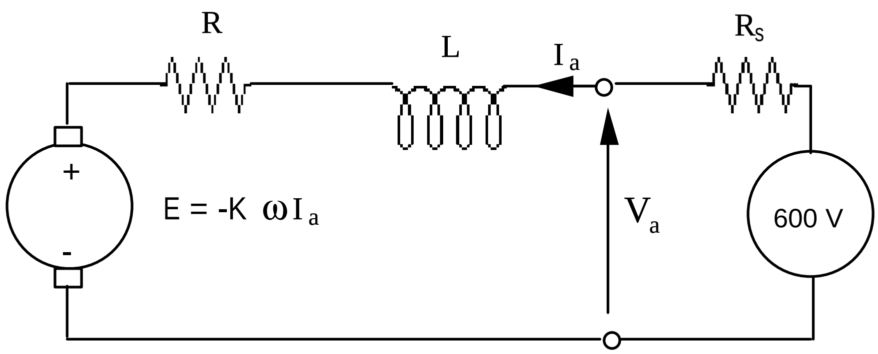
ii) RPM at 80 kph:

$$V = I_a(K\omega + R + R_s)$$

Solve for:

$$\omega = \frac{V - I_a(R + R_s)}{KI_a} = \frac{600 - 100(0.5 + 0.1)}{0.05 \times 100} = 108 \text{ r/s} = 1031 \text{ rpm}$$

iii) The braking torque with field reversed:
Equivalent circuit with field reversed is:



From part i) and ii): K= 0.05 H and ω = 108 r/s

The basic equation now is:

$$V = I_a(-K\omega + R + R_s)$$

Solve for:

$$I_a = \frac{V}{-K\omega + R + R_s} = \frac{600}{-0.05 \times 108 + 0.5 + 0.1} = -125 \text{ A}$$

Therefore the assumed polarity for I_a is negative and thus I_a is flowing towards and into the 600V source and thus energy is flowing back to the source.

$$T = KI_a^2 = -0.05 \times (-125)^2 = -781 \text{ n-m}$$

This is a braking torque.

Power Balance Check:
Power from the source:

$$P_{source} = V_a I_a = 600 \times (-125) = -75 \text{ kW}$$

The negative value means that power is going back into the source, ie, power is being regenerated.
Power going into the machine:

$$P_{machine} = T\omega = (-781) \times 108 = -84.375 \text{ kW}$$

The negative value means that power is coming out of the machine, ie, the machine is braking.
Power dissipated in resistors R and R_s is:

$$P_{dissipated} = (R+R_s)I_a^2 = (0.5+0.1) \times 125^2 = 9.375 \text{ kW}$$

Thus:

$$|T\omega| = |V_a I_a| + (R+R_s)I_a^2$$

Or, in other words, the power coming out of the machine is equal to the power dissipated in the resistors plus the power fed back to the source.

3.7.3 Traction Drive with a Separately Excited Motor

a) Motoring
Separately excited DC motors can be controlled so as to provide the torque-speed characteristic required for traction drive.

For the constant torque region, the field current can be controlled such that:

$$T_e = KI_fI_a = \text{constant}$$

Or;

$$I_f = \frac{T_e}{KI_a}$$

For the constant power region, the field current can be controlled such that;

$$P_e = \omega KI_fI_a = \text{constant}$$

Or;

$$I_f = \frac{P_e}{\omega KI_a}$$

b) Dynamic Braking
A separately excited machine offers significant advantages in ease of braking control. The configuration shown in **Figure 3.24** can be used for either dynamic or regenerative.

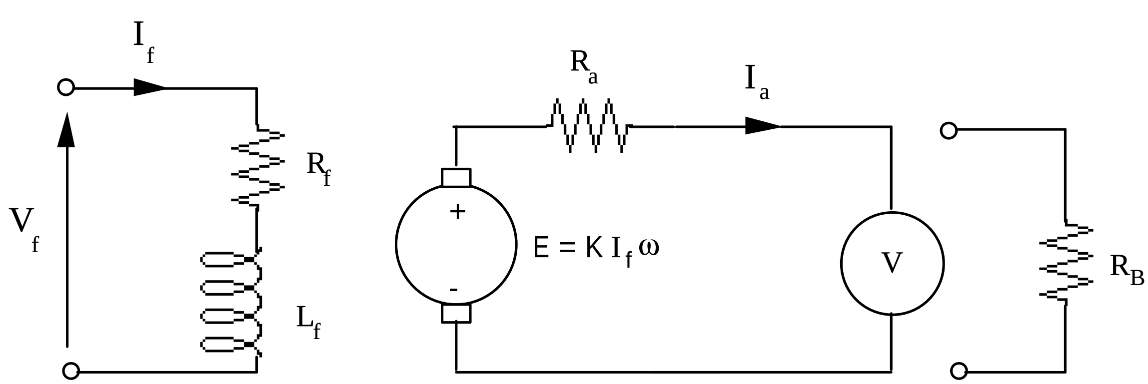


Figure 3.24 Eguivalent Circuit for Dynamic and Regenerative Braking with a Separately Excited Machine

For dynamic braking the armature is disconnected from the power source and the braking resistor, R_B , is connected to the armature. There is no need to reverse the field because the armature current will automatically reverse and that will reverse the torque. The braking torque can be controlled by controlling the field current such that;

$$I_f = \frac{P_e}{\omega K I_a}$$

And;

$$I_a = \frac{-V_a}{R_a + R_B} = \frac{-\omega K I_f}{R_a + R_B}$$

Substitute for I_a into the equation for I_f to obtain;

$$I_f = \frac{1}{\omega K}\sqrt{P_e(R_a + R_B)}$$

Note that the field current does not have to be reversed to produce dynamic braking.

c) Regenerative Braking
For regenerative braking the armature is left connected to the power source and the the field current is simply increased such that:

$$\omega K I_f > V_a$$

That will cause the armature current, I_a , to reverse polarity;

$$I_a = \frac{V_a - \omega K I_f}{R_a}$$

Thus power will flow out of the machine, causing braking action, and into the power source, enabling energy recovery. This is particularly useful in battery operated vehicles such as electric cars and in vehicles that start and stop a lot such as subway cars.

The braking torque can be kept constant by controlling the amplitude of the field current so that the $I_f I_a$ product is constant.

d) Plugging

A more severe form of braking is called "plugging" in which the polarity of the field current is reversed. The expression for armature current, I_a , becomes;

$$I_a = \frac{V_a - \omega K(-I_f)}{R_a} = \frac{V_a + \omega K I_f}{R_a}$$

The armature current is now several times what it was during motoring, but it is still positive. Thus power will still be coming out of the voltage source, P_s , and it will be several times as high as during the motoring mode;

$$P_s = I_a V_a = \frac{V_a^2 + \omega K I_f V_a}{R_a}$$

The back EMF will be negative and the machine torque will be negative, thus the machine will be braking. Since the armature current is now several times the motoring current then the braking torque will be several times the motoring torque. The power converted by the machine will be;

$$P_e = \omega K I_f I_a = \frac{\omega K I_f V_a + (\omega K I_f)^2}{R_a}$$

Both of the above expression represent electrical power *produced* by power sources. There is no recovery of power. The sum of the above power expressions, P_t , can be determined;

$$P_t = P_e + P_s = \frac{\omega K I_f V_a + (\omega K I_f)^2}{R_a} + \frac{V_a^2 + \omega K I_f V_a}{R_a}$$

This expression can be rearranged in the form;

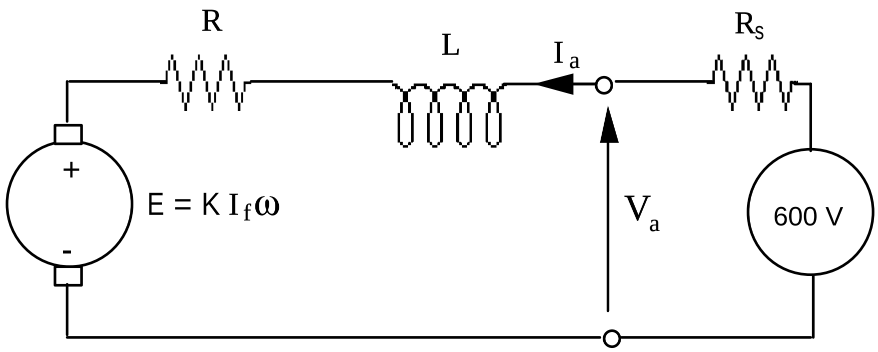
$$P_t = \frac{V_a^2 + 2\omega K I_f V_a + (\omega K I_f)^2}{R_a} = \frac{(V_a + \omega K I_f)^2}{R_a}$$

$$= I_a^2 R_a$$

Thus the combined power produced by the machine and the voltage source is dissipated in the armature resistance R_a. This power is usually an order of magnitude higher than the power drawn by the machine when motoring. This form of braking is dynamic braking and not regenerative braking, and furthermore the total power is dissipated in the motor windings. This is a very severe form of braking in that the braking torque is several times higher than the motoring torque and the thermal stresses in the machine windings are about an order of magnitude higher than normal. Therefore, during plugging one has to be careful not to burn out the armature windings due to excessive current and not to damage the motor shaft (or other torque bearing mechanical parts) due to excessive braking torque.

Example 3.6
A subway car weighing 10 tonnes is driven by a **separately excited motor** fed from a 600 V rail with a source impedance of 0.1 Ω.
Upon starting, the initial current is 1000A. The machine developes a torque of 500 n-m at a top speed of 80 kph, and draws 100A.
Assume constant field current and determine:
i) Starting torque
ii) Motor rpm at 80 kph
iii) The braking torque if regenerative braking is attempted by disconnecting the supply, reversing the field and reconnecting the supply.
iv) The braking torque if dynamic braking is attempted by disconnecting the supply and connecting a 5 Ω resistor across the armature, (field current is not reversed, but is the same polarity as in parts i) and ii).
v) The braking torque if regenerative braking is attempted doubling the field current, (field current is not reversed, but is the same polarity as in parts i) and ii).

Using a separately excited machine the equivalent circuit becomes:



The basic equation is:

$$V_a = KI_f\omega + I_a(R + R_s)$$

Solve for R and substitute for starting conditions:, I_a = 1000 A, ω = 0, to obtain:

$$R = \frac{V_a - KI_f\omega}{I_a} - R_s = \frac{600 - 0}{1000} - 0.1 = 0.5\ \Omega$$

Also:

$$T = KI_fI_a$$

Solve for:

$$KI_f = \frac{T}{I_a} = \frac{500}{100} = 5.0$$

i) Starting torque:

$$T_0 = KI_fI_a = 5.0 \times 1000 = 5{,}000 \text{ n-m}$$

ii) Motor rpm:

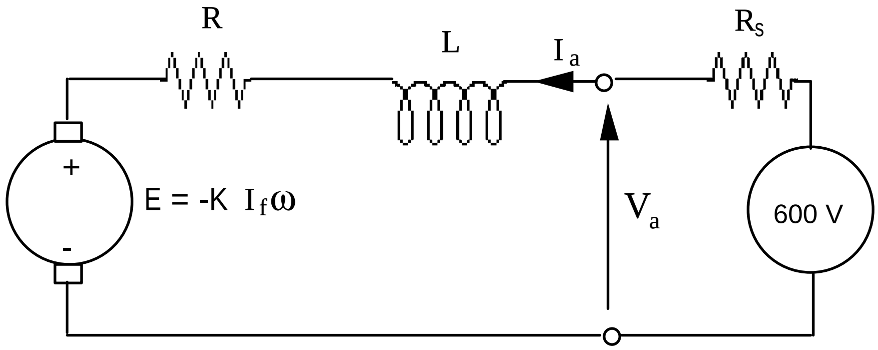
$$V = KI_f\omega + I_a(R + R_s)$$

Solve for:

$$\omega = \frac{V - I_a(R + R_s)}{KI_f} = \frac{600 - 100 \times (0.5 + 0.1)}{5.0}$$

$$= 108 \text{ r/s} = 1031 \text{ rpm}$$

iii) Braking torque with field reversed:
the equivalent circuit now becomes:



Note that $KI_f\omega$ is unchanged from part Bii), where:

$$KI_f\omega = KI_f \times \omega = 5.0 \times 108 = 540$$

Also the basic equation becomes:

$$V_a = -KI_f\omega + I_a(R + R_s)$$

Solve for:

$$I_a = \frac{V_a + KI_f\omega}{R + R_s} = \frac{600 + 540}{0.5 + 0.1} = 1900 \text{ A}$$

Note that I_a is still positive and still drawing power from the source.

$$T = -KI_fI_a = -5.0 \times 1900 = -9{,}500 \text{ n-m}$$ which is a <u>braking</u> torque.

Note that power is being drawn from the source and from the machine, this is still called dynamic braking but it is not regenerative braking.

Power Balance Check:
Power from the source:

$$P_{source} = V_aI_a = 600 \times 1900 = 1.140 \text{ MW}$$

The positive value means that power is still being drawn from the source, power is not being recovered.
Power going into the machine is:

$$P_{machine} = T\omega = (-9500) \times 108 = -1.026 \text{ MW}$$

The negative value means that electrical power is being generated by the machine, ie, it is braking.
Power dissipated in resistors R and R_s is:

$$P_{dissipated} = (R+R_s)I_a^2 = (0.5+0.1) \times 1900^2 = 2.166 \text{ MW}$$

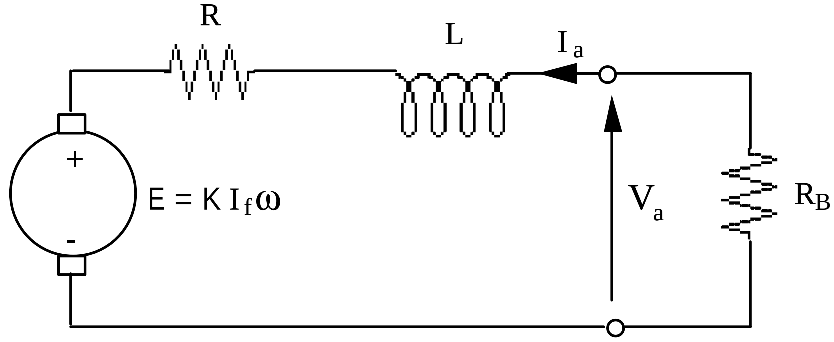
Thus all the power drawn from the source and generated by the machine is being dissipated in the resistors R and R_s .

This is called <u>plugging</u>.
Thus:

$$(R+R_s)I_a^2 = |T\omega| + V_aI_a$$

Or, in other words, the power dissipated in the resistors is equal to the power drawn from the source plus the power regenerated by the machine.

iv) Braking torque with a 5 Ω resistor:
the equivalent circuit now becomes:



From part Bii) we know that:

$\omega = 108$ r/s and $KI_f = 5.0$

The basic equation is:

$V_a = KI_a\omega + I_aR = -I_aR_B$

Solve for:

$$I_a = \frac{-KI_f\omega}{R + R_B} = \frac{-5.0 \times 108}{0.5 + 5.0} = -98.18 \text{ A}$$

This current is negative meaning that it is coming out of the machine, and thus power is being drawn out of the machine.

$T = KI_fI_a = 5.0 \times -98.18 = -491$ n-m which is a <u>braking</u> torque.

This is dynamic braking, but not regenerative braking.

Power Balance Check:
There is no power drawn from the source.
Power going into the machine is:

$P_{machine} = T\omega = (-491) \times 108 = 53.0$ kW

Power dissipated in resistor R is:

$P_R = RI_a^2 = 0.5 \times 98.18^2 = 4.8$ kW

Power dissipated in resistor R_B is:

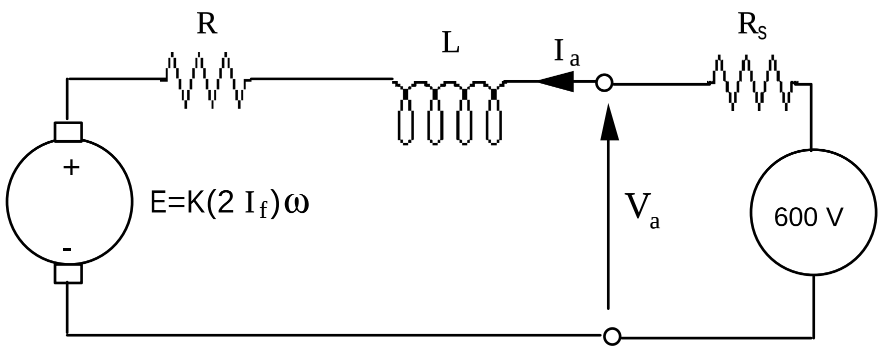
$P_{RB} = R_BI_a^2 = 5.0 \times 98.18^2 = 48.2$ kW

Thus:

$T\omega = RI_a^2 + R_BI_a^2$

Or, in other words, the power regenerated by the machine is equal to the total power dissipated in the resistors. This is called dynamic braking rather than regenerative braking because no power is recovered.

v) Braking torque by doubling the field current:
the equivalent circuit becomes:



From part ii) we know that:

$\omega = 108$ r/s and $KI_f = 5.0$

Therefore:

$K(2I_f) = 10.0$

The basic equation is:

$V_a = K(2I_f)\omega + I_a(R + R_s)$

Solve for:

$$I_a = \frac{V_a - K(2I_f)\omega}{R + R_s} = \frac{600 - 10.0 \times 108}{0.5 + 0.1} = -800 \text{ A}$$

Note I_a is flowing out of the machine and into the source thus power is flowing out of the machine and into the source, thus we have regenerative braking.

$T = K2I_fI_a = 10.0 \times -800 = -8{,}000$ n-m which is a braking torque.

Power Balance Check:

Power from the source

$P_{source} = V_aI_a = 600 \times (-800) = -480$ kW

The negative value means that power is going back into the source.

Power going into the machine

$P_{machine} = T\omega = (-8000) \times 108 = -864$ kW

The negative value means that power is coming out of the machine, ie, the machine is braking.

Power dissipated in resistors R and R_s is $= (R+R_s)I_a^2$

$= (0.5+0.1) \times 800^2 = 384$ kW

Thus:

$|T\omega| = |V_aI_a| + (R+R_s)I_a^2$

Or, in other words, the power regenerated by the machine is equal to the power dissipated in the resistors plus the power fed back to the source.

3.7 Problems

3.7.1 Separately Excited Machines

1. The armature of a separately excited motor draws 40 A at 225 V and 1000 rpm. The armature resistance is known to be 0.4 ohms. What is the mechanical power and torque developed by the motor assuming no friction. (8360W,79.8n-m)

2. The armature of a separately excited motor draws 50 A at 150 V and 1000 rpm. The armature resistance is known to be 0.5 ohms. What is the mechanical power and torque available from the motor assuming internal friction losses are 750W. (5500W,52.5 n-m)

3. A 25kW, 250 V, separately excited generator has an efficiency of 85% at full output. Friction losses are constant and, at full output, are 20% of the total losses in the motor. Determine the efficiency of the generator at 15KW, 250 V, assuming the field current power is negligible. (87.4%)

4. A separately excited motor has the following test data:

Armature			Field
Volts	Amps	RPM	Amps
25	200	Zero	1.0
600	100	100	1.0

Assume that there is no friction or other mechanical losses.

a)Determine the steady state equivalent circuit for this motor. (R_a =0.125 Ω, K=56.1H)

b)What is the field current and the armature current in order to produce 400 n-m of torque at a shaft speed of 50 radians per second with an armature voltage of 100 V. (0.0178A, 400A)

3.7.2 Series Machines

5. A 440 V series motor draws 100 A at 1000 rpm. The armature resistance is 0.11 ohms, field resistance is 0.09 ohms. Calculate the speed and torque at 440 V, 50 A. (341 r/s, 63.5 n-m)

6. A 225 V series motor draws 80 A at 1000 rpm. During a blocked rotor test the machine draws 150 A at 22.5 V. Calculate the speed and torque at 440 V, 50 A. (214.5 r/s, 100.25 n-m)

7. For the machine of problem 5, operating at 440 V, 50 A, calculate the braking torque, and regenerated power if;
a) only the field winding is reversed, (110 n-m, 23 kW)
b) only the supply voltage is reversed, (-100.25 n-m, -22 kW)
c) both supply voltage and field winding are reversed. (110 n-m, 23 kW)

8. For the machine of problem 6, operating at 440 V, 50 A, calculate the torque, and power if;
a) only the field winding is reversed, (-68.1 n-m, -22.8 kW)
b) only the supply voltage is reversed, (63.5 n-m, 22 kW)
c) both supply voltage and field winding are reversed. (-68.1 n-m, -22.8 kW)

9. A series motor has the following test data:

Volts	Amps	RPM
600	100	100
595	50	200

a) Determine the steady state equivalent circuit for this motor. (R=0.1 Ω, K=0.563 H)
b)What is the torque developed by the machine in each of the above cases. (5630 n-m, 1410 n-m)

10. A series motor has the following test data:

Volts	Amps	RPM
300	80	100
295	50	200

a) Determine the steady state equivalent circuit for this motor. (R=1.6 Ω, K=0.205 H)
b) What is the torque developed by the machine in each of the above cases. (1312 n-m, 512.5 n-m)

3.7.3 Shunt Machines

11. You are given a shunt machine with the following characteristics when operated as a motor;

Speed (rpm)	Torque (n-m)	Volts	Amps
0	6.0	27	25.2
500	12.83	54	27.1

Determine the equivalent circuit parameters, excluding inductances.
(R_a=1.08 Ω, R_f =135 Ω, K=1.2 H)

12. You are given a shunt machine with the following test data;

	Speed (rpm)	Torque (n-m)	Volts	Amps
R_f = maximum	500	30	250	26.0
R_f = max./2	500	-60	250	2.0

Determine the armature, field current and field resistance in each case as well as the armature resistance and field constant, K. (I_a=16.67A, -16.67A, I_f=4.67A, 9.33 A, R_f=26.8 Ω @ maximum, R_a=5 Ω K= 0.193 H)