ELG3311: Assignment 2

Problem 5-22:

A 100-MVA, 12.5-kV, 0.85-PF-lagging, 50-Hz, two-pole, Y-connected synchronous generator has a per-unit synchronous reactance of 1.1 and a per-unit armature resistance of 0.012.
(a) What are its synchronous reactance and armature resistance in ohms?
(b) What is the magnitude of the internal generated voltage E_A at the rated conditions? What is its torque angle δ at these conditions?
(c) Ignoring losses in this generator, what torque must be applied to its shaft by the prime mover at full load?

Solution:

(a) The base phase voltage of this generator is

$$V_{\phi,base} = \frac{V_T}{\sqrt{3}} = \frac{12.5k}{\sqrt{3}} = 7217V$$

The base impedance of the generator is

$$Z_{base} = \frac{3V_{\phi,base}{}^2}{S_{base}} = \frac{3(7217)^2}{100M} = 1.56\Omega$$

The generator impedance in ohms are

$$R_A = R_{pu} \times Z_{base} = (0.012)(1.56) = 0.0187\Omega$$
$$X_S = X_{pu} \times Z_{base} = (1.1)(1.56) = 1.716\Omega$$

(b) The rated armature current is

$$I_A = I_L = \frac{S}{\sqrt{3}V_T} = \frac{100M}{\sqrt{3}(12.5k)} = 4619A \quad \Rightarrow \quad I_A = 4619\angle -31.79° A$$
$$\theta = -\cos^{-1} PF = -\cos^{-1}(0.85) = -31.79°$$

The internal generated voltage is

$$E_A = V_\phi + R_A I_A + jX_S I_A$$
$$E_A = 7217\angle 0° + (0.0187)(4619\angle -31.79°) + j(1.716)(4619\angle -31.79°) = 13275\angle 30.3° V$$

Therefore, the magnitude of the internal generated voltage E_A = 13275 V, and the torque angle is δ = 30.3°.
(c) Ignoring losses, the input power would be equal to the output power.

$$P_{in} = P_{out} = PF \times S = (0.85)(100M) = 85MW$$

And

$$n_m = \frac{120 f_e}{P} = \frac{120(50)}{2} = 3000 r/\min$$

The applied torque would be

$$\tau_{app} = \frac{P_{in}}{\omega_m} = \frac{85M}{(3000 r/\min)(2\pi rad/r)(1\min/60s)} = 270563 N \cdot m$$

Problem 5-23:

A three-phase Y-connected synchronous generator is rated 120-MVA, 13.2-kV, 0.8-PF-lagging, and 60-Hz. Its synchronous reactance is 0.9-Ω, and its resistance may be ignored.
(a) What is its voltage regulation?
(b) What would the voltage and apparent power rating of this generator be if it were operated at 50 Hz with the same armature and filed losses as it had at 60 Hz?
(c) What would the voltage regulation of the generator be at 50 Hz?

Solution:

(a) The phase voltage is

$$V_{\phi,base} = \frac{V_T}{\sqrt{3}} = \frac{13.2k}{\sqrt{3}} = 7621V$$

The rated armature current is

$$I_A = I_L = \frac{S}{\sqrt{3}V_T} = \frac{120M}{\sqrt{3}(13.2k)} = 5249A \quad \Rightarrow \quad I_A = 5249\angle -36.87° A$$

$$\theta = -\cos^{-1} PF = -\cos^{-1}(0.8) = -36.87°$$

The internal generated voltage is

$$E_A = V_\phi + R_A I_A + jX_S I_A$$
$$E_A = 7621\angle 0° + j(0.9)(5249\angle -36.87°) = 11120\angle 19.9°\text{V}$$

The resulting voltage regulation is

$$VR = \frac{E_A - V_T}{V_T} \times 100\% = \frac{11120 - 7621}{7621} \times 100\% = 45.9\%$$

(b) The voltage of the generator is directly proportional to the speed of the generator, the voltage rating and the apparent power rating of the generator will be reduced by a factor of 50/60.

$$V_{T,50} = \frac{50}{60} V_{T,60} = \frac{50}{60}(13.2k) = 11.0kV$$
$$S_{50} = \frac{50}{60} S_{60} = \frac{50}{60}(120M) = 100MVA$$

Also, the synchronous reactance will be reduced by a factor of 50/60.

$$X_{S,50} = \frac{50}{60} X_{S,60} = \frac{50}{60}(0.9) = 0.75\Omega$$

(c) At 50 Hz rated conditions, the phase voltage is

$$V_{\phi,base} = \frac{V_T}{\sqrt{3}} = \frac{11.0k}{\sqrt{3}} = 6351V$$

And the armature current would be

$$I_A = I_L = \frac{S}{\sqrt{3}V_T} = \frac{100M}{\sqrt{3}(11.0k)} = 5249A \quad \Rightarrow \quad I_A = 5249\angle -36.87° A$$
$$\theta = -\cos^{-1} PF = -\cos^{-1}(0.8) = -36.87°$$

The internal generated voltage is

$$E_A = V_\phi + R_A I_A + jX_S I_A$$
$$E_A = 6351\angle 0° + j(0.75)(5249\angle -36.87°) = 9264\angle 19.9°\text{V}$$

The resulting voltage regulation is

$$VR = \frac{E_A - V_T}{V_T} \times 100\% = \frac{9264 - 6351}{6351} \times 100\% = 45.9\%$$

Because voltage, apparent power, and synchronous reactance all scale linearly with frequency, the voltage regulation at 50 Hz is the same as that at 60 Hz.

Problem 5-24:

Two identical 600-kVA, 480-V synchronous generators are connected in parallel to supply a load. The prime movers of the two generators happen to have different speed droop characteristics. When the field currents of the two generators are equal, one delivers 400-A at 0.9-PF-lagging, while the other delivers 300-A at 0.72-PF-lagging.
(a) What are the real power and reactive power supplied by each generator to the load?
(b) What is the overall power factor of the load?
(c) In what direction must the field current on each generator be adjusted in order for them to operate at the same power factor?

Solution:

(a) The real and reactive powers are

$$P_1 = \sqrt{3} V_T I_L \cos\theta = \sqrt{3}(480)(400)(0.9) = 299kW$$
$$Q_1 = \sqrt{3} V_T I_L \sin\theta = \sqrt{3}(480)(400)(\sin(\cos^{-1} 0.9)) = 145kVAR$$
$$P_2 = \sqrt{3} V_T I_L \cos\theta = \sqrt{3}(480)(300)(0.72) = 180kW$$
$$Q_2 = \sqrt{3} V_T I_L \sin\theta = \sqrt{3}(480)(300)(\sin(\cos^{-1} 0.72)) = 173kVAR$$

(b) The overall power factor can be found from the total real power and reactive power supplied to the load.

$$P_{total} = P_1 + P_2 = 299k + 180k = 479kW$$
$$Q_{total} = Q_1 + Q_2 = 145k + 173k = 318kVAR$$

The overall power factor is

$$PF = \cos\left(\tan^{-1}\frac{Q_{total}}{P_{total}}\right) = \cos\left(\tan^{-1}\frac{318k}{479k}\right) = 0.83 - lagging$$

(c) The field current of generator 1 should be increased, and the field current of generator 2 should be simultaneously decreased.

Problem 5-25:

A generating station for a power system consists of four 120-MVA, 15-kV, 0.85-PF-lagging synchronous generators with identical speed droop characteristics operating in parallel. The governors on the generators' prime movers are adjusted to produce a 3-Hz drop from no load to full load. Three of these generators are each supplying a steady 75-MW at a frequency of 60-Hz, while the fourth generator (called the swing generator) handles all incremental load changes on the system while maintaining the system's frequency at 60-Hz.

(a) At a given instant, the total system loads are 260 MW at a frequency of 60 Hz. What are the no-load frequencies of each of the system's generators?

(b) If the system load rises to 290 MW and the generator's governor set points do not change, what will the new system frequency be?

(c) To what frequency must the no-load frequency of the swing generator be adjusted in order to restore the system frequency to 60 Hz?

(d) If the system is operating at the conditions described in part c, what would happen if the swing generator were tripped off the line (disconnected from the power line)?

Solution:

(a) The full-load power of these generators is (120 MVA)(0.85) = 102 MVA and the droop from no-load to full-load is 3 Hz. Thus the slope of the power-frequency curve for these four generators is

$$s_P = \frac{Power}{Frequency} = \frac{102M}{3} = 34MW/Hz$$

The power supplied of generator 4 is

$$P_4 = P_{load} - 3P_{1,2,3} = 260M - 3(75M) = 35MW$$

The no-load frequency of the first three generators is

$$\begin{aligned} P_{1,2,3} &= s_P(f_{nl1,2,3} - f_{sys}) \\ 75M &= (34M)(f_{nl1,2,3} - 60) \end{aligned} \quad \Rightarrow \quad f_{nl1,2,3} = 62.21Hz$$

The no-load frequency of the fourth generator is

$$\begin{aligned} P_4 &= s_P(f_{nl4} - f_{sys}) \\ 35M &= (34M)(f_{nl4} - 60) \end{aligned} \quad \Rightarrow \quad f_{nl4} = 61.03Hz$$

(b) The set points of generators 1, 2, 3 and 4 do not change, so the system frequency will be for 290 MW system load

$$P_{load} = s_P(f_{nl1} - f_{sys}) + s_P(f_{nl2} - f_{sys}) + s_P(f_{nl3} - f_{sys}) + s_P(f_{nl4} - f_{sys})$$

$$290M = (34M)(62.21 - f_{sys}) + (34M)(62.21 - f_{sys}) + (34M)(62.21 - f_{sys}) + (34M)(61.03 - f_{sys})$$

$$\Rightarrow f_{sys} = 59.78Hz$$

(c) The governor set points of the swing generator must be increased until the system frequency rises back to 60 Hz. At 60 Hz, as the first three generators supply 75 MW each, so the swing generator must supply

$$P_4 = P_{load} - 3P_{1,2,3} = 290M - 3(75M) = 65MW$$

Therefore, the swing generator's set points must be set to

$$P_4 = s_P(f_{nl4} - f_{sys})$$
$$65M = (34M)(f_{nl4} - 60) \quad \Rightarrow \quad f_{nl4} = 61.91Hz$$

(d) If the swing generator trips off the line, the other three generators would have to supply all 290 MW of the load. The system frequency will become

$$P_{load} = s_P(f_{nl1} - f_{sys}) + s_P(f_{nl2} - f_{sys}) + s_P(f_{nl3} - f_{sys})$$
$$290M = (34M)(62.21 - f_{sys}) + (34M)(62.21 - f_{sys}) + (34M)(62.21 - f_{sys})$$
$$\Rightarrow f_{sys} = 59.37Hz$$

And each generator will equally supply to the loads

$$P_{1,2,3} = \frac{P_{load}}{3} = \frac{290M}{3} = 96.67MW$$

Problem 5-26:

Suppose that you were an engineer planning a new electric cogeneration facility for a plant with excess process steam. You have a choice of either two 10-MW turbine-generators or a single 20-MW turbine-generator. What would be the advantage and disadvantages of each choice?

Solution:

A single 20 MW generator will probably be cheaper and more efficient than two 10 MW generators, but if the 20 MW generator goes down all 20 MW of generation would be lost at once. If two 10 MW generators are chosen, one of them could go down for maintenance and some power could still be generated.

Problem 5-28:

A 20-MVA, 12.2-kV, 0.8-PF-lagging, Y-connected synchronous generator has a negligible armature resistance and a synchronous reactance of 1.1 per unit. The generator is connected in parallel with a 60-Hz, 16-kV infinite bus that is capable of supplying or consuming any amount of real or reactive power with no change in frequency or terminal voltage.

(a) What is the synchronous reactance of the generator in ohms?
(b) What is the internal generated voltage E_A of this generator under rated conditions?
(c) What is the armature current I_A in this machine at rated conditions?
(d) Suppose that the generator is initially operating at rated conditions. If the internal generated voltage E_A is decreased by 5 percent, what will the new armature current I_A be?
(e) Repeat part d for 10, 15, 20, and 25 percent reductions in E_A.
(f) Plot the magnitude of the armature current I_A as a function of E_A. (You may wish to use MATLAB to create this plot.)

Solution:

(a) The base phase voltage of this generator is

$$V_{\phi,base} = \frac{V_T}{\sqrt{3}} = \frac{12.2k}{\sqrt{3}} = 7044V$$

The base impedance of the generator is

$$Z_{base} = \frac{3V_{\phi,base}^{\;\;2}}{S_{base}} = \frac{3(7044)^2}{20M} = 7.44\Omega$$

The generator impedance in ohms are

$$R_A \approx 0\Omega(negligible)$$
$$X_S = X_{pu} \times Z_{base} = (1.1)(7.44) = 8.18\Omega$$

(b) The rated armature current is

$$I_A = I_L = \frac{S}{\sqrt{3}V_T} = \frac{20M}{\sqrt{3}(12.2k)} = 946A \quad \Rightarrow \quad I_A = 946\angle -36.87^\circ A$$
$$\theta = -\cos^{-1} PF = -\cos^{-1}(0.8) = -36.87^\circ$$

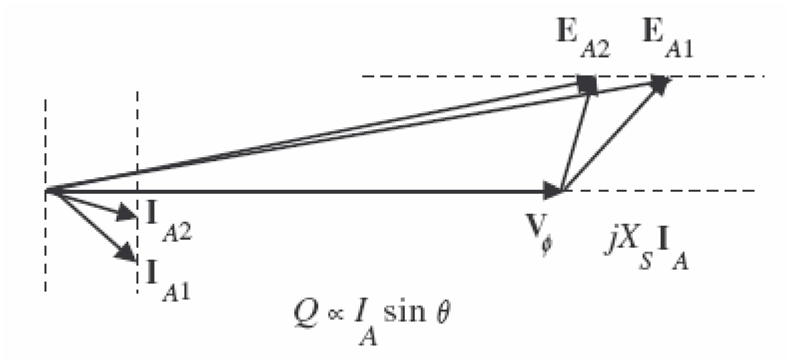
The internal generated voltage is

$$E_A = V_\phi + R_A I_A + jX_S I_A$$
$$E_A = 7044\angle 0^\circ + j(8.18)(946\angle -36.87^\circ) = 13230\angle 27.9^\circ V$$

(c) From the above calculations, the armature current is

$$I_A = 946\angle -36.87° A$$

(d) If E_A is decreased by 5%, the armature current will change as shown below. Note that the infinite bus will keep $V\phi$ and ω_m constant. Also, since the prime mover hasn't changed, the power supplied by the generator will be constant.



$$P = \frac{3V_\phi E_A}{X_S}\sin\delta = constant \text{, so } E_{A1}\sin\delta_1 = E_{A2}\sin\delta_2$$

With a 5% decrease, E_{A2} = 95% E_A = 12570 V, and

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right) = \sin^{-1}\left(\frac{13230}{12570}\sin 27.9°\right) = 29.5°$$

Therefore, the new armature current is

$$I_A = \frac{E_{A2} - V_\phi}{jX_S} = \frac{12570\angle 29.5° - 7044\angle 0°}{j8.18} = 894\angle -32.2°$$

(e) Repeating part (d):

With a 10% decrease, E_{A2} = 90% E_A = 11907 V, and

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right) = \sin^{-1}\left(\frac{13230}{11907}\sin 27.9°\right) = 31.3°$$

Therefore, the new armature current is

$$I_A = \frac{E_{A2} - V_\phi}{jX_S} = \frac{11907\angle 31.3° - 7044\angle 0°}{j8.18} = 848\angle -26.8°$$

With a 15% decrease, E_{A2} = 85% E_A = 11246 V, and

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right) = \sin^{-1}\left(\frac{13230}{11246}\sin 27.9°\right) = 33.4°$$

Therefore, the new armature current is

$$I_A = \frac{E_{A2} - V_\phi}{jX_S} = \frac{11246\angle 33.4° - 7044\angle 0°}{j8.18} = 809\angle -20.7°$$

With a 20% decrease, E_{A2} = 80% E_A = 10584 V, and

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right) = \sin^{-1}\left(\frac{13230}{10584}\sin 27.9°\right) = 35.8°$$

Therefore, the new armature current is

$$I_A = \frac{E_{A2} - V_\phi}{jX_S} = \frac{10584\angle 35.8° - 7044\angle 0°}{j8.18} = 780\angle -14.0°$$

With a 25% decrease, E_{A2} = 75% E_A = 9923 V, and

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right) = \sin^{-1}\left(\frac{13230}{9923}\sin 27.9°\right) = 38.6°$$

Therefore, the new armature current is

$$I_A = \frac{E_{A2} - V_\phi}{jX_S} = \frac{9923\angle 38.6° - 7044\angle 0°}{j8.18} = 762\angle -6.6°$$

(f) A MATLAB program to plot the magnitude of the armature current as a function of the internal generator voltage is shown below.

```
% M-file: prob5_28f.m
% M-file to calculate and plot the armature current
% supplied to an infinite bus as Ea is varied.

% Define values for this generator
Ea = (0.65:0.01:1.00)*13230; % Ea
Vp = 7044; % Phase voltage
```

```
d1 = 27.9*pi/180; % torque angle at full Ea
Xs = 8.18; % Xs (ohms)

% Calculate delta for each Ea
d = asin( 13230 ./ Ea .* sin(d1));

% Calculate Ia for each flux
Ea = Ea .* ( cos(d) + j.*sin(d) );
Ia = ( Ea - Vp ) ./ (j*Xs);

% Plot the armature current versus Ea
figure(1);
plot(abs(Ea)/1000,abs(Ia),'b-','LineWidth',2.0);
title ('\bfArmature current versus \itE_{A}\rm');
xlabel ('\bf\itE_{A}\rm\bf (kV)');
ylabel ('\bf\itI_{A}\rm\bf (A)');
grid on;
hold off;
```