ELG2331: Tutorial for Chapter 4

P: 4.35:

From the problem, we have the angular frequency as 3000 rad/s. First step is to find the reactive impedance of the inductor L = 190 mH:

$$X_L = \omega L = 3000 \times 190 \times 10^{-3} = 0.57\ \text{k}\Omega$$

Then find Z_L which will be equal to $j0.57$ kΩ. Second step is to find the capacitive impedance of the capacitor C = 55 nF:

$$X_C = \frac{1}{\omega C} = \frac{1}{3000 \times 55 \times 10^{-9}} = 6.061\,\text{k}\Omega$$

Then find $Z_C = -j6.061$ kΩ.

Third step is to combine R_1 with Z_L to be called Z_{eq1}. The following combination is in rectangular form

$$Z_{eq1} = R_1 + Z_L = 2.3 + j0.57$$

Convert the rectangular form into polar form. In the polar form we need the magnitude and the angle: $A\angle\theta$. In order to find the magnitude A we should do the following:

$$\sqrt{2.3^2 + 0.57^2} = 2.37$$

The angle θ is found as:

$$\tan^{-1}\left(\frac{0.57}{2.3}\right) = 13.92$$

Now combine the magnitude and angle in the polar form to have Z_{eq1} as: $2.37\ \angle 13.92^o$ kΩ.

Follow the same procedure to find Z_{eq2} in rectangular form first and polar form second:

$$Z_{eq2} = R_2 - jX_C = 1.1 - j6{,}061 = 6.16\angle -79.71^o\ \text{k}\Omega$$

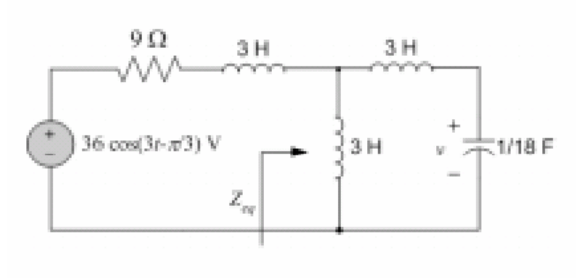
Since Z_{eq1} and Z_{eq2} are in parallel, then their total equivalent impedance is

$$Zeq = \frac{Z_{eq1} \times Z_{eq2}}{Z_{eq1} + Z_{eq2}} = \frac{(2.37\angle 13.92^o)(6.16\angle -79.71^o)}{(2.3 + j0.57) + (1.1 - j6.061)}$$

$$= \frac{14.60\angle -65.79^o}{3.4 - j5.491} = \frac{14.60\angle -65.79}{6.458\angle -58.23^o} = 2.261\angle -7.56^o\ \text{k}\Omega$$

You may convert the value of Z_{eq} from polar form into rectangular form again. We will get: $Z_{eq} = 2.241 - j0.297$. This is obtained by calculating ($2.261 \times \cos -7.56^o = 2.241$ as the real part) and ($2.262 \times \sin -7.56^o = -0.297$ as the imaginary part). This is a "**capacitive load**". A capacitive load means a load (Z) that has a resistance and a capacitance.

P4.52:



$$w = 3 \text{ rad/s} \; (\;)$$
$$V_s = 36\angle\text{-}60^o \text{ (polar form)}$$
$$Z_{L1} = jwL_1 = j3\times 3 = j9\Omega$$
$$Z_{L2} = jwL_2 = j3\times 3 = j9\Omega$$
$$Z_{L3} = jwL_3 = j3\times 3 = j9\Omega$$
$$Z_C = \frac{1}{jwC} = \frac{1}{j3\times\left(\frac{1}{18}\right)} = -j6\Omega$$
$$Z_{eq} = Z_{L2} // Z_{L3} + Z_C = j9 // \left(j9 - j6\right) = 2.25\angle 90^o\,\Omega$$
$$Z_T = Z_R + Z_{L1} + Zeq = 9 + j9 + j2.25 = 9 + j11.25 = 14.4\angle 51.34^o\,\Omega$$

Now find the total current from the source I

$$I = \frac{V_S}{Z_T} = \frac{36\angle\text{-}60^o}{14.4\angle 51.34^o} = 2.499\angle -111.34^o \text{ A}$$

Find the voltage across Z_{eq}

$$V_{eq} = IZ_{eq} = \left(2.499\angle -111.34^o\right)\left(2.25\angle 90^o\right) = 5.623\angle -21.34^o \text{ V}$$

We will perform voltage divider between Z_{L3} and Z_C to find V

$$V = \left(\frac{Z_C}{Z_C + Z_{L3}}\right)V_{eq} = \left(\frac{-j6}{j9 - j6}\right)\left(5.623\angle -21.34^o\right) = 11.25\angle 158.66^o \text{ V}$$

Now convert this value into time domain

$$v(t) = 11.25\cos(3t - 158.66^o)\text{V}$$

P4.53:

This is a current divider problem

$$w = 2 \text{ rad/s}$$
$$Z_{L2} = j\omega L_2 = j2\times 10 = j20\Omega$$
$$Z_{L3} = j2\times 1 = j2\Omega$$
$$Z_C = \frac{1}{j\omega C} = -j\Omega$$
$$I = \left(\frac{Z_{L2} + Z_C}{\left(Z_{L2} + Z_C\right) + (R + Z_{L3})}\right) I_S = \left(\frac{j20 - j}{\left(j20 - j\right) + \left(5 + j2\right)}\right)\times 6\angle 0^o = 5.28\angle 13.4^o \text{ A}$$
$$i(t) = 5.28 \cos(2t + 13.4^o) \text{ A}$$

P4.57:

In this circuit we have two meshes.

First find Z_C and Z_L

$$Z_C = \frac{1}{j1500 \times 10^{-6}} = -j666.7\Omega$$
$$Z_L = j(1500)(0.5) = j750\Omega$$

Apply KVL in the first loop

$$-V_S + R_1 I_1 + Z_C(I_1 - I_2) = 0$$

Apply KVL in the second loop

$$Z_L I_2 + I_2 R_2 + Z_C(I_2 - I_1) = 0$$

Substitute values and find I_1 and I_2

Answer in phasor form:

$$I_1 = 3.8 \times 10^{-3} \angle 46.6^o \text{ A}$$
$$I_2 = 19.6 \times 10^{-3} \angle -83.2^o \text{ A}$$

Write the answer in time domain

P4.58

We have a circuit with a current source. Also, the requirement is to find v_1 and v_2. In such case, it is better and easier to apply KCL.

First, find the capacitive impedance Z_C and the inductive impedance Z_L

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{100 \times 500 \times 10^{-6}} = -j20\Omega$$
$$Z_L = j\omega L = j100 \times 0.2 = j20\Omega$$

Apply KCL at node 1, we have three currents joining at the node, namely: I_S; current through R_1 (assume its direction from v_1 toward the reference); and current through the capacitor (assume its direction from v_1 to v_2)

$$I_S = \frac{V_1}{R_1} + \frac{V_1 - V_2}{Z_C} = \left(\frac{1}{R_1} + \frac{1}{Z_C}\right)V_1 - \frac{1}{Z_C}V_2$$
$$40\angle 0^o = \left(\frac{1}{R_1} + \frac{j}{20}\right)V_1 - \frac{j}{20}V_2$$

Now apply KCL at node 2, we have three currents joining at the node, namely: current through the capacitor (assume its direction from v_1 to v_2); current through R_2 (assume its direction from v_2 toward the reference); and current through the inductor (assume its direction from v_2 toward the reference)

$$\frac{V_1 - V_2}{Z_C} = \frac{V_2}{R_2} + \frac{V_2}{Z_L}$$
$$\frac{V_1}{Z_C} = \left(\frac{1}{R_2} + \frac{1}{Z_L} + \frac{1}{Z_C}\right)V_2$$
$$j\frac{V_1}{20} = \left(\frac{1}{10} - j\frac{1}{20} + j\frac{1}{20}\right)V_2$$
$$j\frac{V_1}{20} = \left(\frac{1}{10}\right)V_2$$
$$V_1 = -j2V_2$$

Now, substitute the values and find V_1 and V_2.

Answers in phasor form:

$$V_1 = 565.7\angle -45^o \text{ V}$$
$$V_2 = 282.85\angle 45^o \text{ V}$$

Write them in time domain: