Example 1

Determine a state space representation for a system with the transfer function

$$\frac{Y(s)}{R(s)} = T(s) = \frac{s+10}{s^4 + 12s^3 + 23s^2 + 34s + 40}$$

The state space representation is

$$\dot{x} = Ax + Br$$
$$y = Cx + Du$$

$$T(s) = \frac{s^{-3} + 10s^{-4}}{1 + 12s^{-1} + 23s^{-2} + 34s^{-3} + 40s^{-4}}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -40 & -34 & -23 & -12 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 10 & 1 & 0 & 0 \end{bmatrix}$$

Example 2

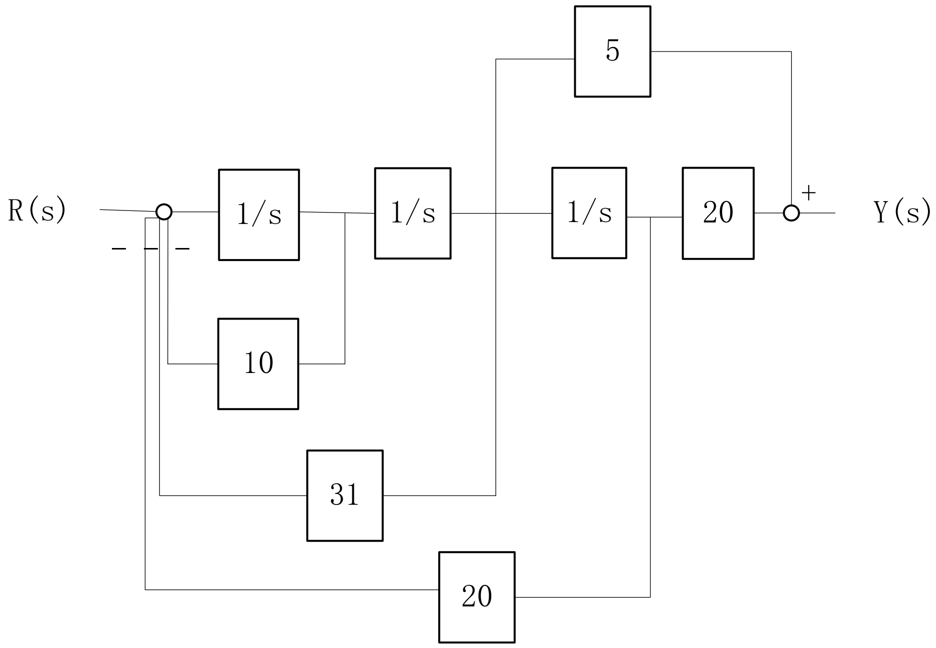
Obtain a block diagram and a state variable representation of this system

$$\frac{Y(s)}{R(s)} = T(s) = \frac{5(s+4)}{s^3 + 10s^2 + 31s + 20}$$

$$T(s) = \frac{5s^{-2} + 20s^{-3}}{1 + 10s^{-1} + 31s^{-2} + 20s^{-3}}$$

$$\dot{x} = Ax + Br$$
$$y = Cx + Du$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -20 & -31 & -10 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 20 & 5 & 0 \end{bmatrix}$$



Example 3

Have state space representation

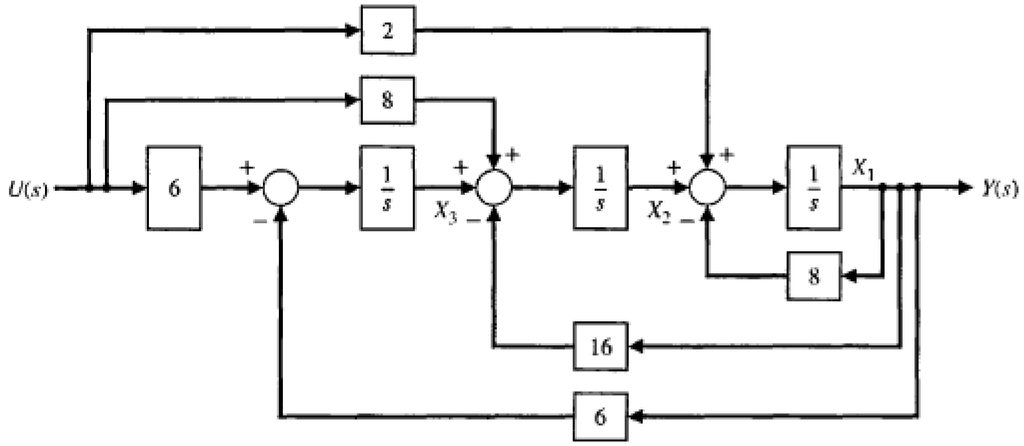
$$\dot{x} = \begin{bmatrix} 1 & 1 & -1 \\ 4 & 3 & 0 \\ -2 & 1 & 10 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} u,$$

$$y = \begin{bmatrix} -1 & 2 & 0 \end{bmatrix} x$$

Determine the transfer function

$$\dot{x} = Ax + Br$$
$$y = Cx$$

$$G(s) = C(sI - A)^{-1}B = \frac{10s^2 - 60s - 70}{s^3 - 14s^2 + 37s + 20}$$



$$\dot{\mathbf{x}} = \begin{bmatrix} -8 & 1 & 0 \\ -16 & 0 & 1 \\ -6 & 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 2 \\ 8 \\ 6 \end{bmatrix}u(t),$$

and the output is $y(t) = x_1(t)$. ■