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Consistent Multirobot Localization using Heuristically Tuned Extended Kalman Filter

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***Abstract*—Probabilistic algorithms have widely been used with significant success in single-robot localization as well as mapping. However, when it comes to distributed, multirobot systems, probabilistic algorithms have a tendency to quickly converge to inconsistent, often overly optimistic estimates, whenever interdependencies in such systems are ignored. This paper presents a solution to consistent, decentralized, multi-robot localization using a heuristically tuned Extended Kalman Filter. Extensive simulations show that the proposed solution is able to significantly improve the consistency of pose estimates for each robot in a system while maintaining the computational complexity of the classical Extended Kalman Filter.**

***Keywords*-Extended Kalman Filter; Decentralized Multi-robot Localization**

I. INTRODUCTION

Multirobot systems are increasingly being used to perform complex tasks. For example, Amazon uses thousands of autonomous Kiva robots in some of its warehouses in order to speed up the process of retrieving and shipping goods from the large warehouse to customers [1]. Without knowing where the robots are, and where they should go, the robots would collide with each other, resulting in damages to the robots themselves and the goods they carry, as well as delays. Therefore, accurate localization in each robot is extremely important in order to avoid such problems. In the last two decades, research in multirobot systems has gained a lot of attention [2]. The key reason is the increase in efficiency when multiple robots collaborate in order to accomplish a given task [2], [3]. As a result, a number of algorithms that support multirobot localization have been developed. These algorithms estimate the pose of every robot in a team by combining motion measurements from proprioceptive sensors of individual robots, with robot-to-robot measurements from their exteroceptive sensors [4].

Multi-robot localization schemes can be either centralized, multi-centralized or decentralized. In a centralized scheme [5]–[9], all computations are done in one processing center, which can be one of the robots (the leader). Although this scheme can provide optimal and consistent pose estimates for all robots in a group, it is poorly scalable, computationally expensive, and is vulnerable to single-point failure (for example, if the leader malfunctions). In a multi-centralized scheme [10], each robot maintains information about its own pose, and that of every other robot in the group. In other words, every robot maintains a copy of the entire group's state. This scheme is robust to single-point failure but results in even higher communication and computation costs. In a decentralized scheme [4], [11]–[13], each robot maintains only the information about its own pose, and only updates this information locally when relative robot-to-robot measurements become available. This scheme can provide approximate solution to a multirobot localization problem at reduced communication and computation costs but often leads to inconsistent (overly optimistic or pessimistic) estimates if the interdependencies between robots are neglected [14]. This paper presents an empirical methodology for tuning the classical Extended Kalman Filter (EKF) in order to achieve consistent pose estimates for every robot in decentralized multirobot systems while maintaining the computation and communication costs of the classical EKF.

II. RELATED WORK

In the early 90's, Kurazume *et. al.* developed the initial concept of multirobot localization [7]. The main idea in multirobot localization is the use of robots themselves as *portable landmarks* to others in order to improve the localization accuracy, especially in GPS-denied and unchartered areas with unstructured or insufficient landmarks [15]. Subsequent studies in multirobot localization adopted and improved upon Kurazume's initial concept. The biggest challenge in multirobot localization is calculating *consistent* pose estimates of every robot in a team through tractable algorithms. Inconsistencies in decentralized systems arise when the interdependencies within such systems are ignored. The Extended Kalman Filter is one of the most popular, tractable, probabilistic algorithm that has seen widespread application in localization as well as mapping due to its low computational cost and ease of implementation. Despite its popularity, EKF has some significant shortcomings when used to estimate robot poses, most notably its tendency to quickly converge to inconsistent values and its inability to

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handle non-linear systems where the degree of non-linearity is too high [16], [17].

One of the strategies for dealing with inconsistencies in multirobot localization involves joint estimation of every robot's pose in a team every time new information arrives [8]. This strategy however, is only tractable for small groups of robots due to quadratic growth in computation and communication costs with the number of robots. Some of the most recent strategies such as [13] and [4] provide consistent estimates through the use of a technique called Covariance Intersection, which fuses information under unknown interdependencies through convex combination of mean and covariances. The main drawback of this technique is that it needs additional computational time to iteratively determine the most optimal way of combining mean and covariances in order to achieve consistent estimated results [18].

In the past, a common approach used to overcome inconsistencies involved heuristic tuning of the Extended Kalman Filter by deliberately adding artificial noise into the system which offsets the effects of interdependencies and linearization, and consequently results in consistent estimations. This process is referred to as *injection of stabilization noise* and was introduced by Maybeck [19]. Since then, it has been shown that under certain conditions, such as when the degree of nonlinearity of a system is not too high, the consistency of EKF estimates can be adequately maintained through deliberate addition of stabilization noise [16], [17]. In the context of robot localization, consistency in EKF poses can be achieved through deliberate inflation of landmark covariances after every update [16], or through the inflation of sensor covariances [20]. However, formal ways for determining the adequate inflation of the covariances have not yet been established. Thus, in this paper, an empirical methodology for improving the consistency of EKF estimates through artificial inflation of landmark covariances in decentralized multirobot systems is proposed and evaluated through extensive simulations performed in MATLAB.

III. PROBLEM STATEMENT

At any given time, t, the EKF represents the robot's position and its associated uncertainty with the mean, μ_t, and covariance matrix, $\sum_t$, respectively. Collectively, the robot's pose and its associated uncertainty is also known as the robot's belief, bel_t, about its pose. That is:

$$bel_t = (\mu_t, \textstyle\sum_t)$$

As a robot moves around, the EKF recursively propagates and updates its latest belief with new information coming from various sensors. This process is usually broken down into two steps, namely *prediction* and *correction*.

In the *prediction* step, the EKF makes a less accurate estimation of the robot's new belief, $\overline{bel}_t = (\overline{\mu}_t, \overline{\sum}_t)$, from its previous belief, $bel_{t-1} = (\mu_{t-1}, \sum_{t-1})$, using its motion model, $f(\mu_{t-1}, u_t)$, as shown in Equations 1 and 2:

$$\overline{\mu}_t = f(\mu_{t-1}, u_t) \tag{1}$$

$$\overline{\textstyle\sum}_t = \left(\frac{\partial \overline{\mu}_t}{\partial \mu_{t-1}}\right) \textstyle\sum_{t-1} \left(\frac{\partial \overline{\mu}_t}{\partial \mu_{t-1}}\right)^T + \left(\frac{\partial \overline{\mu}_t}{\partial u_t}\right) Q_t \left(\frac{\partial \overline{\mu}_t}{\partial u_t}\right)^T \tag{2}$$

where Q_t is the covariance matrix which specifies the uncertainties associated with the robot's control input, u_t.

In the *correction* step, the EKF uses the robot's observation model, $g(\overline{\mu}_t, \mu_L)$, to estimate relative distance and bearing, $\overline{Z}_t$, of a landmark with known location, μ_L, and uncertainty, $\sum_L$, with respect to the robot's pose estimated in the *prediction* step (Equation 3). The known uncertainty of the landmark is propagated, and compounded with the robot's uncertainty estimated in the *prediction* step, $\overline{\sum}_t$, using the same observation model as shown in Equation 4

$$\overline{Z}_t = g(\overline{\mu}_t, \mu_L) \tag{3}$$

$$S = \left(\frac{\partial \overline{Z}_t}{\partial \overline{\mu}_t}\right) \overline{\textstyle\sum}_t \left(\frac{\partial \overline{Z}_t}{\partial \overline{\mu}_t}\right)^T + \left(\frac{\partial \overline{Z}_t}{\partial \mu_L}\right) \textstyle\sum_L \left(\frac{\partial \overline{Z}_t}{\partial \mu_L}\right)^T + R_t \tag{4}$$

where R_t is the covariance matrix of the relative observations, which specifies the uncertainties associated with relative distance and angle measurements.

Since in multirobot systems the robots carry portable landmarks with them, μ_L and $\sum_L$ in Equation 3 can be considered as μ_t and $\sum_t$ of the robots that carry the portable landmarks.

In Equation 4, S is usually referred to as the innovation or residual covariance, and is used by the EKF to compute the gain, which specifies the extent by which the less accurate, estimated belief, $\overline{bel}_t = (\overline{\mu}_t, \overline{\sum}_t)$, is refined to more accurate belief, $bel_t = (\mu_t, \sum_t)$. In practice, EKF has a tendency of providing overconfident estimates. This means that estimated covariance matrix, $\sum_t$, of the belief, bel_t, converges to incorrect, often smaller values than the actual covariance matrix. The convergence can be due to incorrect modeling of the system, ignored interdependencies or high degree of non-linearity in the system beyond the EKF's ability.

IV. TUNING OF CLASSIC EKF FOR DECENTRALIZED MULTIROBOT LOCALIZATION

This section describes the proposed empirical methodology for improving the consistency of EKF estimates by means of artificial inflation of landmark covariances. The proposed approach is validated through extensive simulations of multirobot localization using the Extended Kalman Filter.

A. Multirobot Localization using Classic Extended Kalman Filter

In order to study the behaviour of the classical EKF algorithm, a number of simulations involving different number of robots were performed in MATLAB. The following assumptions were made in the simulations:

- The belief of a portable landmark, $bel_L = (\mu_L, \sum_L)$ is the same as that of the carrier robot, $bel_t = (\mu_t, \sum_t)$.
- All robots in a team are identical with the same motion and observation models.
- The initial belief (EKF pose and associated uncertainty) of each robot is randomly initialized around the actual ground truth of the robot using the errors specified by the initial covariance matrix. This is done in order to reflect the discrepancies between ground truth and EKF estimates, which may arise if the robots were to use their own sensors to estimate their initial poses.
- Robots move within sensing and communication range from each other.
- Zero mean, white Gaussian random noise is added into each of the robot's modeled sensors in order to reflect the uncertainties associated with measurements.

In addition to the above assumptions, simulation parameters for each robot were initialized as follows:

- Encoder error (each wheel) = 10% of distance traveled.
- Initial actual pose (m, m, rad) = 5[randn, randn, randn] where randn represents a random scalar drawn from the standard normal distribution.
- Initial pose error (m, m, rad) = [0.001, 0.001, 0.001].
- Range sensor error (m, rad) = [0.1, 0.1].
- Robot size = 0.3m x 0.3m.

Under the above assumptions, 20 simulation runs were repeated for 5 robots using the classic EKF. In all simulation runs, each robot moves along its predefined ground truth path in an open area in such way that at any given moment at least one of the five robots remains stationary while the other robots (one or more) move to their next locations. The order in which the robots move is completely random. Furthermore, each moving robot refines its estimates through relative observation of stationary robot(s) after having moved a random distance between 0.1m and 1m from its last estimated location. For each robot, the Average Normalized Estimation Error Squared (ANEES), averaged over 20 simulation runs, is used to evaluate the consistency of the EKF estimates at 95% confidence interval. ANEES can be interpreted as the ratio of the actual error to the estimated error [21]. On the other hand, Mean Absolute Error (MAE) in position, averaged over the distance moved by the robot, is used to evaluate the accuracy of the EKF estimates. For 20 simulations and at 95% confidence interval, an estimate is consistent if its ANEES value falls within the interval $[0.6746, 1.3884]$. Figure 1 shows the ground truths and average trajectories of the classic EKF for the five robots (top) and the corresponding ANEES curves for each robot (bottom).

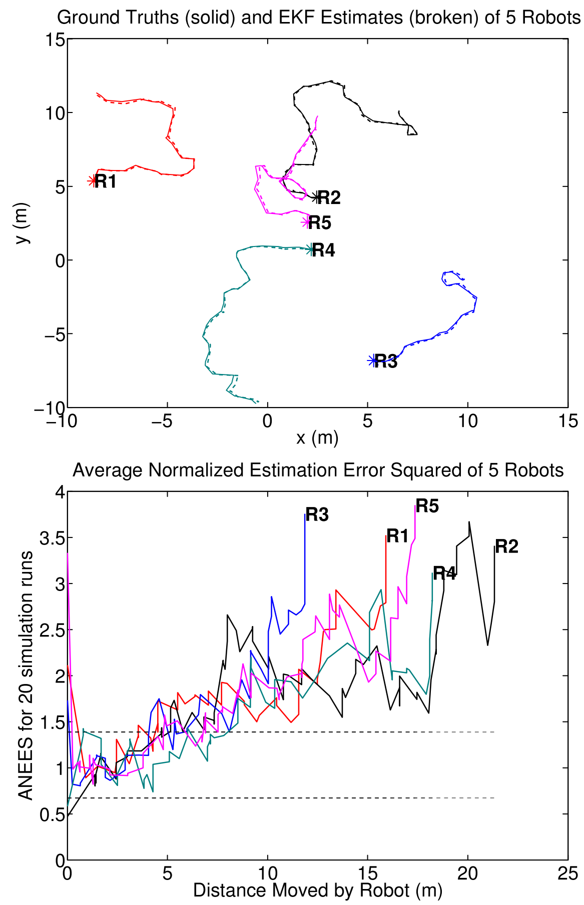

Figure 1: Top: Trajectories of the classic EKF averaged over 20 runs. Bottom: ANEES of 5 robots using classic EKF averaged over 20 runs.

From Figure 1, it can be seen that the ANEES for each robot rapidly grows out of the consistent interval, $[0.6746, 1.3884]$ (black broken lines) despite having their EKF pose estimates very close to their respective ground truths. This happens because the covariances estimated by the classical EKF for each robot converge to wrong, small values, while the actual covariances grow unbounded.

B. Heuristic Tuning of the Classic EKF

Suppose Equation 5 is the shorthand representation of Equation 4 introduced in Section III.

$$S = S_R + S_L + R_t \tag{5}$$

where

$$S_R = \left(\frac{\partial \overline{Z}_t}{\partial \overline{\mu}_t}\right) \overline{\sum}_t \left(\frac{\partial \overline{Z}_t}{\partial \overline{\mu}_t}\right)^T ; \; S_L = \left(\frac{\partial \overline{Z}_t}{\partial \mu_L}\right) \sum_L \left(\frac{\partial \overline{Z}_t}{\partial \mu_L}\right)^T$$

R_t holds its original meaning.

In order to overcome the convergence of the estimated covariance matrices to wrong values, the covariance matrices of the stationary robots (and therefore of the portable landmarks) are artificially inflated during their propagation such that S_L becomes:

$$S_L = \left(\frac{\partial \overline{Z}_t}{\partial \mu_L}\right)(C\textstyle\sum_L)\left(\frac{\partial \overline{Z}_t}{\partial \mu_L}\right)^T \tag{6}$$

In this paper, C will be referred to as the *Covariance Inflation Index*. At first glance, it appears that the Covariance Inflation Index amplifies the entire covariance matrix of the stationary robot, thereby inflating both linear and angular components of the uncertainty associated with the pose of the stationary robot. However, a closer examination of the term S_L in the classical Extended Kalman Filter reveals that only the linear components of the landmark covariances are propagated through the observation model during the *correction* step. Thus, the landmark covariances provide the ideal medium through which classical EKF can be tuned by manipulating linear components of the system (*i.e* position), which are less prone to errors, while bypassing the angular components (*i.e* orientation), which is more susceptible to linearization errors.

C. Multirobot Localization using Heuristically Tuned Extended Kalman Filter

The simulations described in Section IV-A were repeated using 5 robots with artificially inflated covariance matrices during the *correction* step. For the given system, the value of C (Covariance Inflation Index), was empirically determined and experimentally verified to be proportional to the distance traveled by the robot up to the point where it acts as a landmark to others [22]. Thus:

$$C = A\Delta D \tag{7}$$

where A is an experimentally determined constant, and ΔD is the distance covered by the robot up to the point where it acts as a landmark to others. Thus, Equation 6 becomes:

$$S_L = \left(\frac{\partial \overline{Z}_t}{\partial \mu_L}\right)(A\Delta D\textstyle\sum_L)\left(\frac{\partial \overline{Z}_t}{\partial \mu_L}\right)^T \tag{8}$$

For each set of 20 simulations involving 5 robots as described in IV-A, a different value of A in Equation 8 was used until the best ANEES results were obtained as shown in Figure 2. For the given multirobot system, the best ANEES results were obtained when the value of A was 15 because most of the ANEES values fall within the valid interval (black, broken horizontal lines).

V. EXPERIMENTAL VALIDATION

Using this value of A, the performance of the tuned EKF was evaluated against its classic counterpart in four different test scenarios, each of which consists of 20 repeated simulation runs. In each scenario, the initial starting locations, the trajectories of the robots, and the order of their movements are different. Simulation results for the four scenarios are depicted graphically in Figures 3 through 6 and summarized in Tables I through IV.

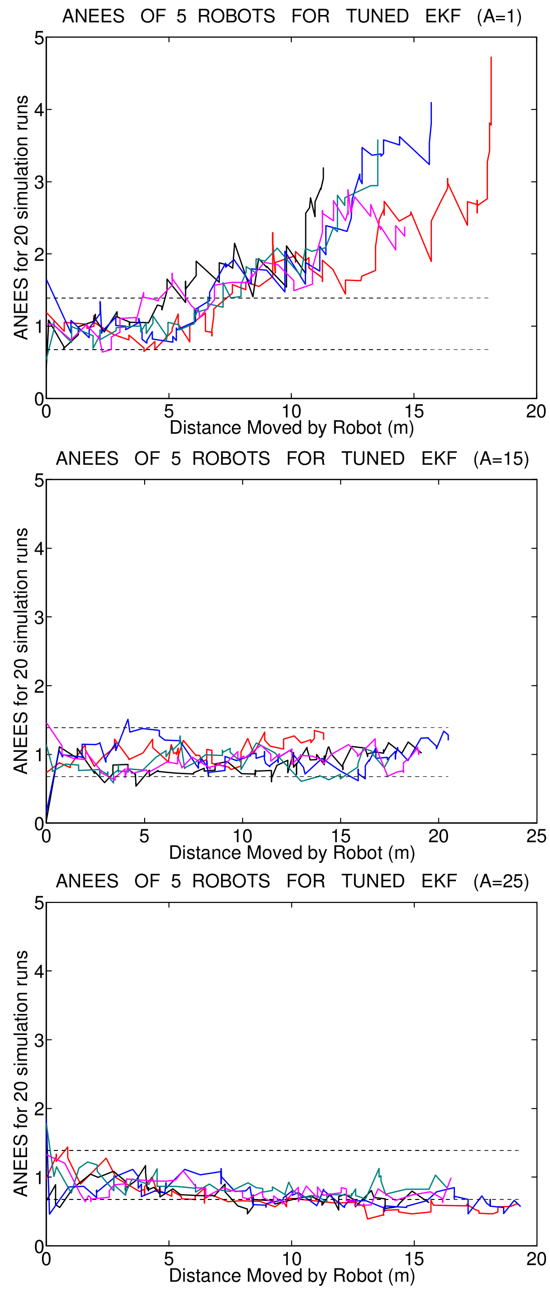

Figure 2: ANEES results for EKF tuning with 5 robots over 20 runs with A=1, 15, 25.

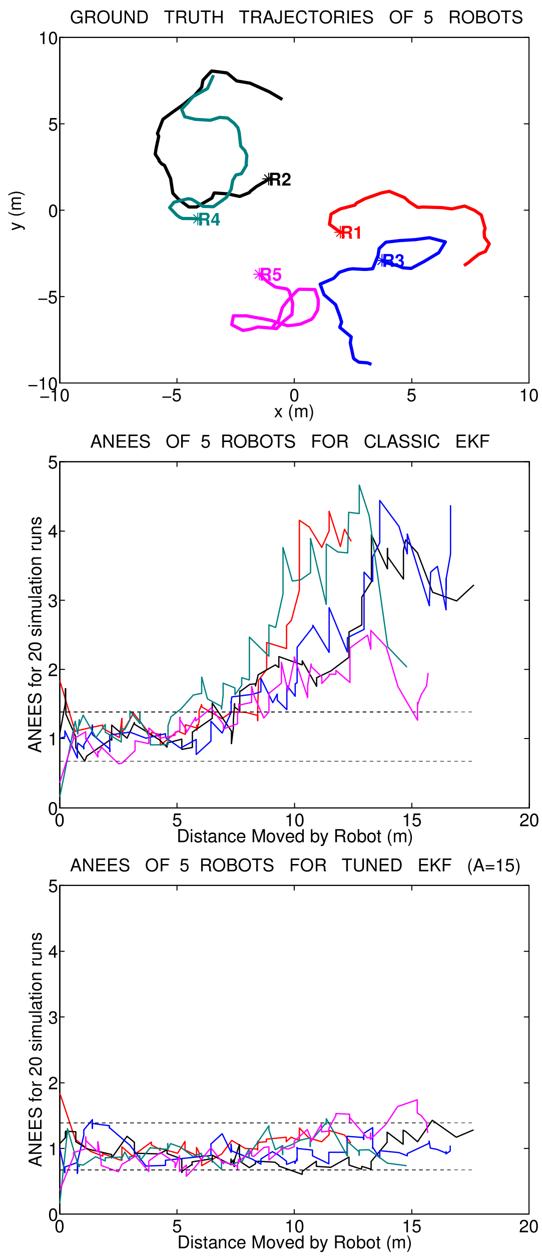

Figure 3: Scenario #1.
Ground truth trajectories of 5 robots (top). ANEES for Classic (center) and Tuned (bottom) EKF averaged over 20 runs.

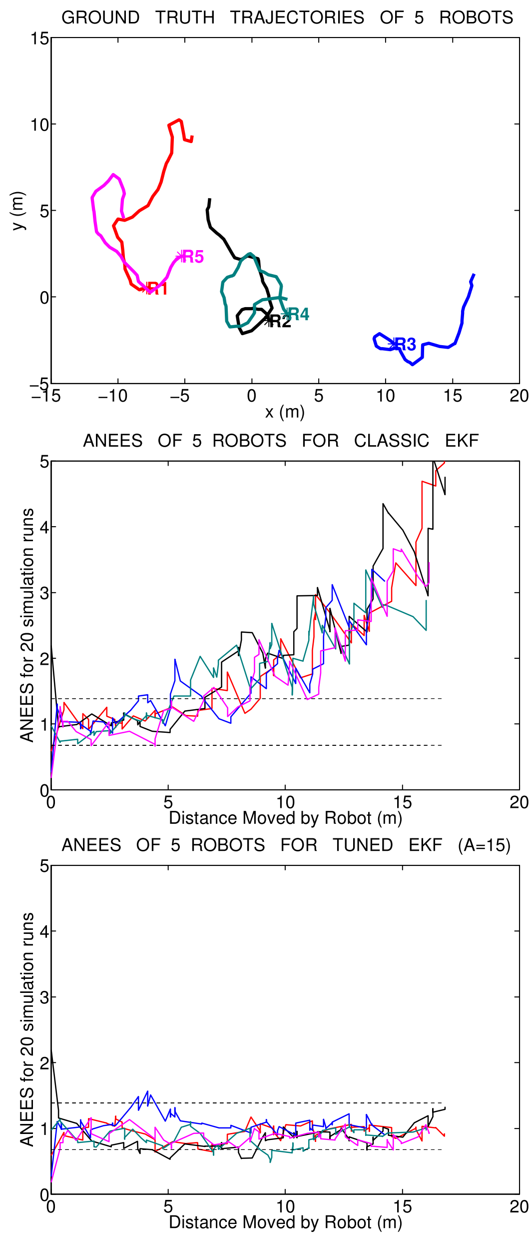

Figure 4: Scenario #2.
Ground truth trajectories of 5 robots (top). ANEES for Classic (center) and Tuned (bottom) EKF averaged over 20 runs.

Table I: Summary for Scenario #1.

	Classic EKF		Tuned EKF	
Robot	Consistent ANEES (%)	MAE (m)	Consistent ANEES (%)	MAE (m)
1	58.4906	0.0728	98.1132	0.0396
2	35.0000	0.1007	91.2500	0.0728
3	34.4086	0.1026	88.1720	0.0955
4	31.8841	0.1107	95.6522	0.0513
5	44.8718	0.1087	78.2051	0.0613

Table II: Summary for Scenario #2.

	Classic EKF		Tuned EKF	
Robot	Consistent ANEES (%)	MAE (m)	Consistent ANEES (%)	MAE (m)
1	34.1772	0.1275	94.9367	0.1204
2	23.7500	0.0646	82.5000	0.0600
3	39.1892	0.1523	93.2432	0.1454
4	24.1379	0.1010	78.1609	0.0965
5	37.3134	0.0898	97.0149	0.0880

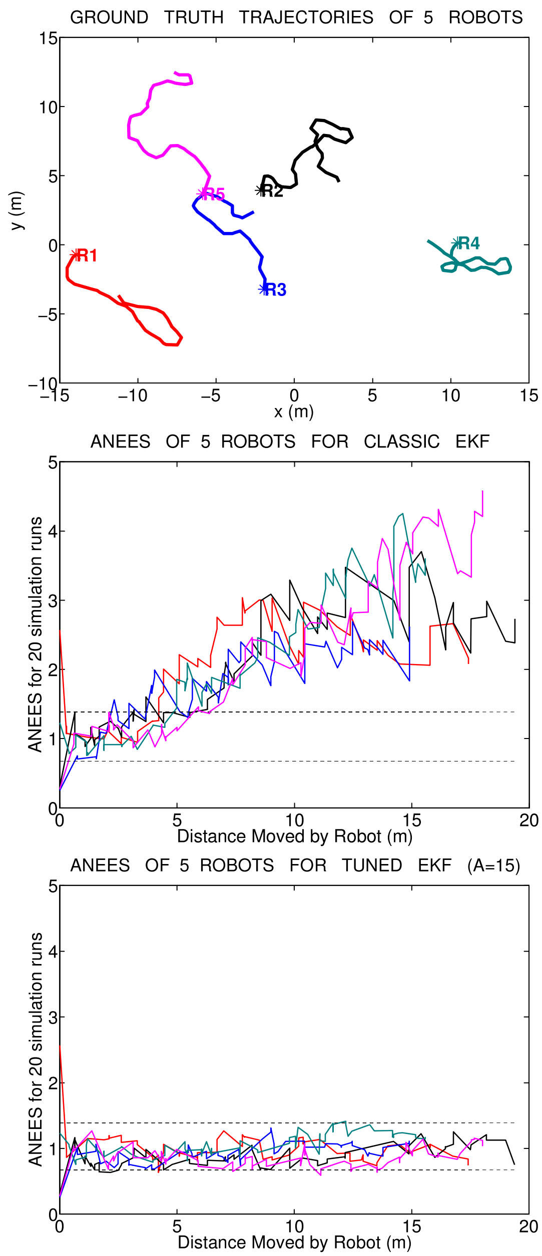

Figure 5: Scenario #3.
Ground truth trajectories of 5 robots (top). ANEES for Classic (center) and Tuned (bottom) EKF averaged over 20 runs.

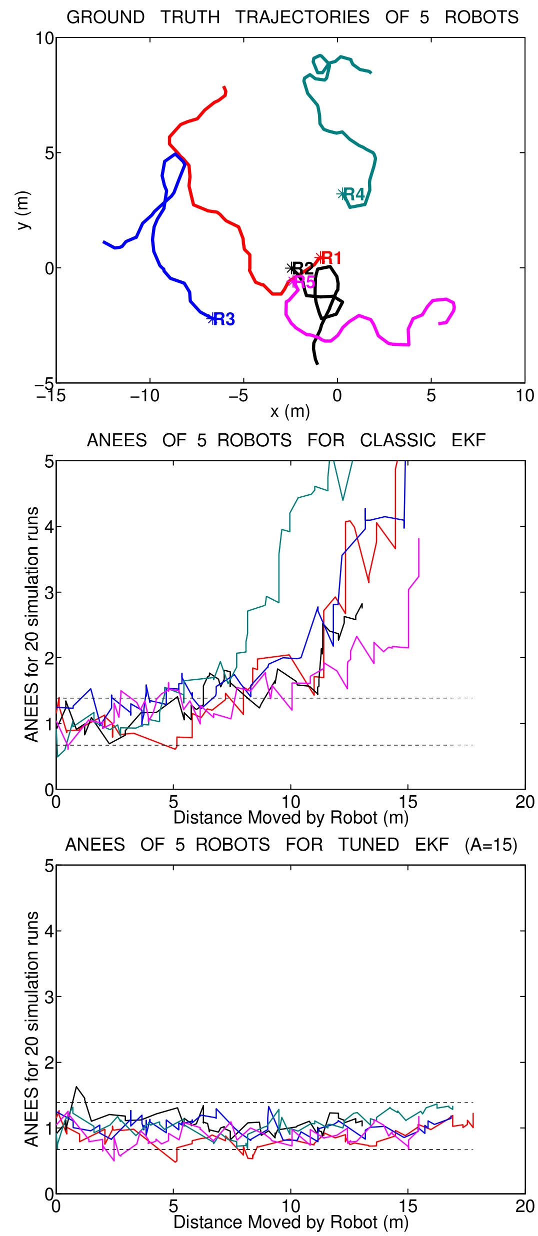

Figure 6: Scenario #4.
Ground truth trajectories of 5 robots (top). ANEES for Classic (center) and Tuned (bottom) EKF averaged over 20 runs.

Table III: Summary for Scenario #3.

	Classic EKF		Tuned EKF	
Robot	Consistent ANEES (%)	MAE (m)	Consistent ANEES (%)	MAE (m)
1	22.8571	0.1392	94.2857	0.0481
2	32.9268	0.1178	87.8049	0.0658
3	21.2121	0.0574	98.4848	0.0557
4	26.3736	0.2085	90.1099	0.1000
5	27.9070	0.1765	88.3721	0.0982

Table IV: Summary for Scenario #4.

	Classic EKF		Tuned EKF	
Robot	Consistent ANEES (%)	MAE (m)	Consistent ANEES (%)	MAE (m)
1	32.5000	0.0966	78.7500	0.0538
2	36.5079	0.0902	95.2381	0.0507
3	25.6410	0.0981	93.5897	0.0962
4	22.3529	0.1323	94.1176	0.1162
5	38.6667	0.0875	89.3333	0.0544

The results in Tables I through IV show a significant improvement in the consistencies (ANEES columns) and a slight improvement in the accuracies (MAE columns) of the estimates for all robots in all scenarios after tuning the EKF. From the simulations, it was revealed that provided the initial errors of the robots is small, the tuned EKF maintains its integrity even in scenarios different from which it was tuned in, and can handle random motions of continuously varying number of moving or stationary robots.

VI. CONCLUSION

In this paper, an empirical methodology for tuning the classic EKF for consistent multirobot localization in decentralized systems is proposed. The methodology involves controlled artificial inflation of the covariance matrices of the stationary robots which act as landmarks to the moving robots. The performance of the EKF tuned through the proposed methodology was evaluated and compared against its classic counterpart through extensive simulations involving different scenarios, four of which are presented in the paper. The results from the simulations show that the tuning methodology improves both the consistency and accuracy of the classic EKF, at no additional computation or communication cost.

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