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Experimental Study of Data Merging Techniques for Workspace Modeling with Uncertainty

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Abstract - *The vast majority of sensors used in autonomous robotic systems are submitted to uncertainty sources that often generate contradictory data that must be interpreted in order to optimize the reliability of the information extracted from the models that are built from those measurements. When certainty occupancy maps are used to represent the workspace of a robot, the estimation of the uncertainty level becomes a critical issue as it must become an active part of the model. Numerous techniques such as the Bayesian theory, the Dempster-Shafer theory of evidence and fuzzy logic inference schemes have been proposed to achieve data fusion of uncertain measurements. However, the performance of these approaches has not been extensively investigated and compared in the specific context of certainty occupancy maps construction. This paper presents the results of an experimental investigation that has been conducted to adapt, implement and evaluate these three data merging techniques to achieve smooth progressive refinement in the construction of occupancy grids based on cumulative uncertain range measurements. The context of the application considered is that of collision-free path planning for mobile robots.*

I. INTRODUCTION

Merging data from various sensors and from numerous viewpoints became mandatory with the evolution of sensing technologies and autonomous robotic systems. Intrinsic limitations of specific technologies are now stimulating the development of multi-modal sensing devices [1, 2]. However, the combination of measurements representing the same physical reality but collected with different approaches for which error models differ brings important concerns on the handling of associated uncertainties.

Various strategies have been proposed in the literature. The classical Bayesian theorem [3] has been widely used in many applications mainly due to its relative simplicity and computational efficiency. A generalization of the Bayesian method known as the Dempster-Shafer theory of evidence [4, 5] provides inference mechanisms that are closer to human reasoning while its statistical performance mostly remains similar to that of the Bayesian theory [6]. Leung *et al*. [7] have investigated the two statistic-based approaches of Bayes and Dempster-Shafer for the recognition of targets in noisy radar images and observed their respective strengths and limitations, namely a higher robustness in Dempster-Shafer at the expense of a higher computational cost. Wu *et al*. [8] proposed an extended weighted Dempster-Shafer theory of evidence which is able to take into account the reliability, or error level, of respective sensors as a means to improve sensor fusion accuracy in progressive refinement modeling applications. This aspect cannot be easily handled with the Bayesian inference method. Yang [9] conducted an analytical comparison of the Bayesian and Dempster-Shafer theories in the context of mapping and localization of mobile robots and put in evidence another advantage of the Dempster-Shafer approach, that is the capability to encode and track a distinction between parts of the environment that are unknown because they have not been scanned and regions that are uncertain because of numerous contradictory measurements.

Another class of data fusion methods consists of fuzzy logic inference engines. Fuzzy logic appears as a more intuitive approach to merge uncertain information. Fuzzy memberships and rules definition are mainly inspired by designer's experience, perception and knowledge or the average performance of sensors. Zhu et al. [10] investigated the Dempster-Shafer evidence theory and proposed a parallel with fuzzy sets representation, demonstrating that the approaches can be made consistent according to the way the probabilistic evidence is represented. In our previous work, an elementary fuzzy logic inference system has been developed to merge range measurements in a progressive refinement modeling strategy with constructive and destructive contributions [11]. This initial experimentation demonstrated the relevance of the method. A refinement is proposed in the present work to allow direct comparison with the other theoretical approaches.

On the other hand, if uncertainty is to be encoded in the virtual representation resulting from data fusion, the encoding scheme must provide a sufficient flexibility. The pioneer work of Moravec on certainty grids [12] thoroughly explored the concept of probabilistic modeling. Certainty models offer an opportunity to propagate uncertainty from the sensors through the registration estimation and to encode the resulting confidence on the information directly in the model. This option is not possible with classical discrete representations where information is encoded in an absolute binary way (true/false, empty/occupied). Certainty maps allow the progressive refinement of the representation which is critical in exploration of unknown environments. New measurements, even if provided from uncertain sensors, can contribute to increase the level of confidence in the mapping. Rendas *et al.* [13] observed the importance of taking into account the past knowledge about the environment to make future decisions in safely operating an autonomous vehicle in unknown space.

However, a mathematically rigorous estimation of the probability of the state of a variable, such as space occupancy, is hardly achievable as this probability heavily relies on an accurate knowledge of the sensor's error characteristics, which are difficult to estimate. Often, a relative knowledge of the risk to perform an operation is sufficient to make the safest decision possible. The fusion method must then ensure the coherence of the model through its constructive and destructive refinements, the relative level of certainty being based on the best available estimate of the sensor's accuracy.

Facing this need to evaluate various merging schemes and to adapt them for operation in workspace modeling with progressive refinement, an experimental study was conducted with a simulated mobile platform equipped with a range finder to map cluttered environments. This paper presents an experimental implementation and an adaptation of three data merging approaches, the Bayesian theory, the Dempster-Shafer theory of evidence and a fuzzy logic inference engine, to identify strategies that are the most appropriate in this specific context. The aspects of richness and accuracy of the environment map, ease of tune, capability of refinement and overall computational workload are mainly considered.

The following sections define the sensor model used for experimentation and summarize the merging schemes. Details of the adaptation made to merging techniques to achieve certainty occupancy maps with progressive refinement are discussed. The experimental testbed is presented to illustrate how uncertain measurements were collected and manipulated. An extensive comparative analysis is proposed to identify the strengths and limitations of each of the merging schemes for this application.

II. SENSOR UNCERTAINTY MODEL

In order to obtain an certainty occupancy map through the merge of uncertain range measurements a typical model of the performance of a range sensor was used. This model maps the distance, ρ, between the sensor and the surface of an object as a percentage representing the risk that space is occupied, $P(\rho)$, given that the sensor's accuracy follows a Gaussian distribution of variance, σ_ρ^2.

$$P(\rho) = \frac{7}{20} + \frac{3}{20}\left(e^{-\left(\frac{2((\rho-\bar{\rho})+2\sigma_\rho)}{\sigma_\rho^2}\right)^{-1}}\right) + \frac{3}{20}\left(e^{-\left(\frac{(\rho-\bar{\rho})^2}{\sigma_\rho^2}\right)}\right) \tag{1}$$

where $\bar{\rho}$ corresponds to the numerical measurements provided by the sensor.

The parameters of this model were empirically estimated to match those of a standard laser range finder used in our previous work. This sensor model is designed to match the evolution of the probabilistic state of occupancy along the scan line on a physical scene. That is, the region between the sensor and the object is empty (low $P(\rho)$), the area located close to the object's surface is most probably occupied (high $P(\rho)$) and the region behind the surface of the object is occluded, that is in an unknown state ($P(\rho) \approx 0.5$). This occupancy mapping is presented in figure 1.

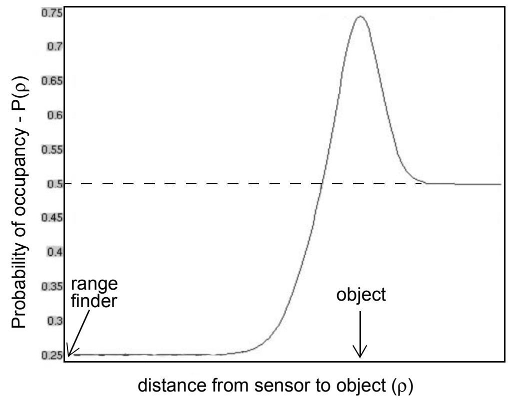


Fig. 1. Range sensor model with Gaussian uncertainty.

III. MERGING SCHEMES

A. Bayesian theorem

The classical Bayesian theorem provides a straightforward mathematical means for estimating the probability of a given parameter's state, P, resulting from the direct merge of two uncertain measurements having their own level of confidence, P_1 and P_2 respectively.

$$P = \frac{P_1 \cdot P_2}{P_1 \cdot P_2 + (1-P_1)\cdot(1-P_2)} \tag{2}$$

This rule can be interpreted in two ways: first, if P_1 and P_2 represent two separate measurements, eq. (2) will combine them in an estimate of the state that depends on both contributions; second, if P_1 represents the current state of the model and P_2 corresponds to a new measurement, the result of the merge will be an update of the model state that takes into account the memory of the past measurements and the information provided by the new data.

Starting from a fully unknown model, that is P(ρ)=0.5 everywhere, and applying eq. (2) recursively leads to the desired progressive refinement of the state estimate as information is accumulated in the model following the availability of new measurements. According to the nature of the information perceived in the past, that is empty or occupied space for our application, the memory encoded in the model might evolve in various ways, preserving the past knowledge on a short-term or long-term horizon.

This characteristic is illustrated in figure 2 where the Bayesian method is initially applied iteratively to merge a sequence of 10 consistent measurements (fig. 2a) that are followed by a sequence of 20 perturbations (fig. 2b) before another set of 10 consistent measurements is provided by the sensor (fig. 2c). The sensor is assumed to be located at 0 cm.

As consistent measurements indicating the surface of an object at 50 cm are successively provided, the probability of occupancy rises toward 1.0 around the object's surface while the zone located between the sensor and the object progressively drops to 0.0, making the confidence in the model stronger. When erroneous measurements start to report an object farther away from the sensor (located at 60 cm), a new region starts to see its probability of occupancy grow until it reaches the maximum. However, as the region where the object was supposed to reside (around 50 cm) is now seen as empty by the sensor, its probability progressively decreases to reach 0.0. This illustrates the effect of short-term memory when new measurements are collected over a given area.

Finally, an extra set of measurements indicate that the object is really located at 50 cm. When this information is merged with the perturbed model, the probability of occupancy around 50 cm is brought back progressively. However, the zone around 60 cm is now occluded by the object and cannot be measured by the sensor. Therefore, subsequent merges combine the current state of the model for this region with 0.5, according to the sensor model. Given the Bayesian theorem, a merge with 0.5 is neutral. As a result, the model keeps a long-term memory that there were measurements indicating the presence of an object around 60 cm. These results are used as a reference for the experimental study.

B. Dempster-Shafer theory of evidence

Even though the Bayesian method is a valuable approach to merge uncertain measurements, this approach does not provide a strong mechanism to differentiate between various types of unknown states. For example, following the example of section A, no difference is made between zones that have never been explored and regions where contradictory measurements led to unknown occupancy state (0.5).

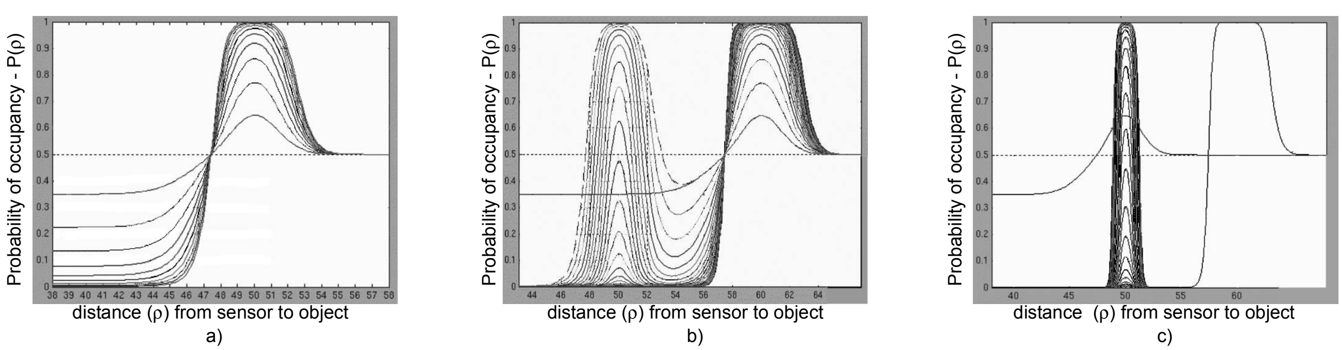


Fig. 2. Progressive model refinement with Bayesian theorem.

The generalized theory of Dempster-Shafer implies an enumeration of all mutually exclusive alternatives, as defined by the *frame of discernment*, θ. This frame of discernment includes all possible combinations of events. For example, in the case of occupancy representation, a valid frame of discernment is given by:

$$\theta = \{\varnothing, O, E, \{O, E\}\} \quad (3)$$

where O represents occupied space, E represents empty space, $\{O, E\}$ represents unknown space state, that is a lack of information that preempts any conclusion, and $\varnothing$ represents any other state, excluding "occupied", "empty" and "unknown".

A *basic probability density function*, m, determines the confidence, or belief, in the information provided by the sensor. When applied on a measurement from a sensor, the estimate of this belief comes from a sensor uncertainty model.

From there, two important concepts dictate the behavior of the Dempster-Shafer method, and the uncertainty estimation on the occupancy model in our application. These are the *belief* and the *plausibility*. The belief is the level of confidence that one can have into a given event to happen based on the information reported by the sensors that directly supports this event. The plausibility measures the degree of confidence that one can have into that event based on the information provided by the sensor that does not directly contradict this event. In other words, evidences that say something else than what we are looking for.

Given an evidence, S_i, by a sensor that a region of space is occupied. The belief and the plausibility are computed as:

$$Belief(O) = \sum_{S_i \subseteq O} m(S_i) \quad (4)$$

$$Plausibility(O) = 1 - \sum_{S_i \cap O = \varnothing} m(S_i) \quad (5)$$

Adapting these relationships to the frame of discernment of interest in our workspace occupancy modeling, we can rewrite eqs. (4) and (5) as:

$$Belief(O) = m(O) \quad (6)$$

$$Plausibility(O) = 1 - Belief(E) \quad (7)$$

And similarly for empty and unknown space:

$$Belief(E) = m(E) \quad (8)$$

$$Plausibility(E) = 1 - Belief(O) \quad (9)$$

$$Belief(\{O, E\}) = m(\{O, E\}) + m(O) + m(E) \quad (10)$$

$$Plausibility(\{O, E\}) = 1 - Belief(\varnothing) \quad (11)$$

Given that we want to use the same sensor uncertainty model as with the Bayesian theorem to provide a valuable comparison basis, a mapping function from the Gaussian occupancy probability distribution, $P(\rho)$, to the basic probability density function, m, must be defined. To achieve this, a sequence of linear functions have been used as presented in figure 3. This diagram puts in evidence the capability to explicitly encode the lack of information on the status of a region of space resulting from uncertainty or errors in the measurements when a Dempster-Shafer approach is used. For example, if a region has 37% probability of being occupied. With the Bayesian model, this would automatically correspond to 63% probability of being empty. On the other hand, the Dempster-Shafer theorem does not conclude on the evidence that this region is empty. It rather estimates a plausibility of 63% (1-Belief(O)) that this space is empty. This opens a whole new set of opportunities to refine the encoding of the occupancy model by offering the capability to represent unknown state by assigning belief to generic events such as {O,E}.

This makes the strength of Dempster-Shafer theory as everything is not simply true or false, empty or occupied. There might be sensors clearly reporting that a region is occupied, thus increasing the belief, while other sensors do not confirm that it is occupied nor confirm that it is empty, thus not reducing the plausibility. The confidence interval for a given measurement is bound by the belief and the plausibility for a given state.

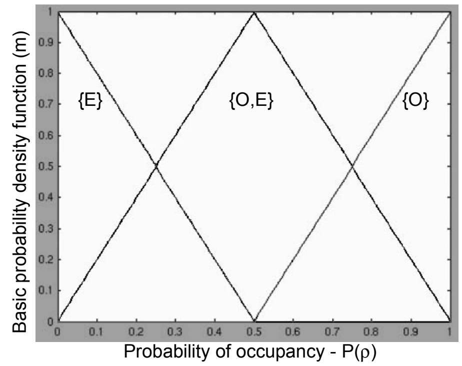


Fig. 3. Conversion between the Gaussian sensor uncertainty model and the basic probability density function of Dempster-Shafer.

To merge successive information provided by the sensors, m_1 and m_2, in a consistent way, Dempster-Shafer define a merging rule as follows:

$$[m_1 \oplus m_2](A) = \frac{\sum_{S_i \cap S_j = A} m_1(S_i) \cdot m_2(S_j)}{1 - \sum_{S_i \cap S_j = \varnothing} m_1(S_i) \cdot m_2(S_j)} \quad (12)$$

where $A \in \{\varnothing, O, E, \{O, E\}\}$.

This rule can be applied recursively between two measurements or between the current state of the model and a new measurement as described in section III.A for the Bayesian merge equation.

Another particularity that has to be taken into account here is that the fully unknown state of the original model is represented by a belief and a plausibility of the unknown state that is maximum, which corresponds to belief({O,E})=plausibility({O,E})=1.0 since there is no initial evidence about the current state of occupancy of space. The initial certainty map must then be encoded in a slightly different way.

In order to test the behavior of the Dempster-Shafer theorem in a recursive merging application with consistent and contradictory measurements, a series of measurements were presented to the merging procedure when starting from a fully unknown state, as shown in table 1. The first set of 5 measurements place the region of interest in empty space, making the belief in empty state rise progressively while the belief in unknown state and the plausibility of occupancy state decrease. Next a set of 10 other measurements contradict the initial information and place the region of interest in occupied space. A transition is observed where the belief in occupied space increases while the belief in empty space is reduced. However, the belief in unknown state continues to drop as extra data are now provided about the state of the space even though they are contradictory. The plausibility of emptiness is reduced while that of occupancy increases.

	Meas.	Belief(O)	Belief(E)	Belief({O,E})	Plausibility(O)	Plausibility(E)
0	---	0.0	0.0	1.0	1.0	1.0
1	E	0.0	0.5	0.5	0.5	1.0
2	E	0.0	0.75	0.25	0.25	1.0
3	E	0.0	0.875	0.125	0.125	1.0
4	E	0.0	0.938	0.063	0.063	1.0
5	E	0.0	0.969	0.031	0.031	1.0
6	O	0.030	0.939	0.030	0.061	0.939
7	O	0.086	0.886	0.029	0.114	0.886
8	O	0.179	0.795	0.026	0.205	0.795
9	O	0.319	0.660	0.021	0.304	0.660
10	O	0.492	0.492	0.016	0.508	0.492
11	O	0.663	0.326	0.011	0.674	0.326
12	O	0.792	0.195	0.006	0.805	0.195
13	O	0.881	0.108	0.003	0.892	0.108
14	O	0.934	0.057	0.002	0.943	0.057
15	O	0.962	0.029	0.001	0.971	0.029

Table 1. Evolution of belief and plausibility values over the merge of contradictory measurements.

One can observe that after the merge with the 10th measurement, the belief of occupancy and the belief of emptiness are both around 0.5 as an equal number of contradictory measurements have been merged for each group. At the same time, the belief in unknown space is already very low as the uncertainty does not result from the lack of measurements. Would have it been the case, the belief({O,E}) would still be large. This specific encoding of the belief in unknown state of space provides an essential element for the interpretation of raw data contained in occupancy models that can reveal to be critical for efficient robot guidance. Such information is not permitted with the Bayesian scheme.

Finally to evaluate the progressive refinement process that can be achieved with the Dempster-Shafer theory of evidence in comparison with the results obtained with the Bayesian model, the resulting mapping based on belief and plausibility needed to be converted back to probability of occupancy on a 0-100% scale. A cascaded Gaussian transfer function depicted in Figure 4 has been introduced to achieve this conversion at the expense of losing the specific information about unexplored regions versus contradictory measurements. However, this conversion is not required normally and is performed here only for comparison between techniques.

Experimentation was conducted with the same range dataset as used with the Bayesian approach. That is a sequence of 10 consistent measurements were successively merged (fig. 5a), followed by 10 perturbed data that estimate the object's surface to be 10 cm farther away from the sensor (fig. 5b). Finally, a last set of 10 measurements consistent with the original set were again provided (fig. 5c). The results are very similar to those obtained with a Bayesian merge except that minimum and maximum convergence levels are slightly lower due to the boundaries imposed on the Gaussian distribution for the final conversion.

This demonstrates that the Dempster-Shafer method can be adapted to provide a similar progressive refinement modeling strategy as that obtained with a Bayesian approach. However, if the model takes full advantage of the extra information encoded in belief and plausibility values, especially about unknown state of space, unexplored areas and contradictory measurements can be distinguished. The progressive refinement provides meaningful information about the confidence that we have on what is known but also on what is unknown. These observations suggests that a model based only on a percentage scale of occupancy probability is not the best suited for modeling with uncertainty management.

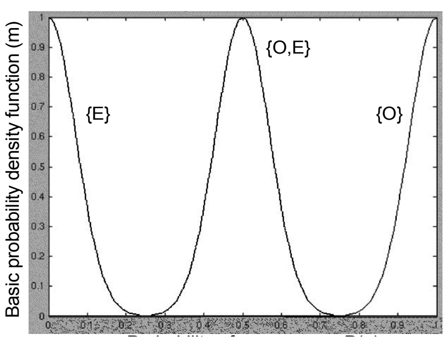


Fig. 4. Conversion from the Dempster-Shafer mapping to a probabilistic occupancy representation.

C. Fuzzy logic inference

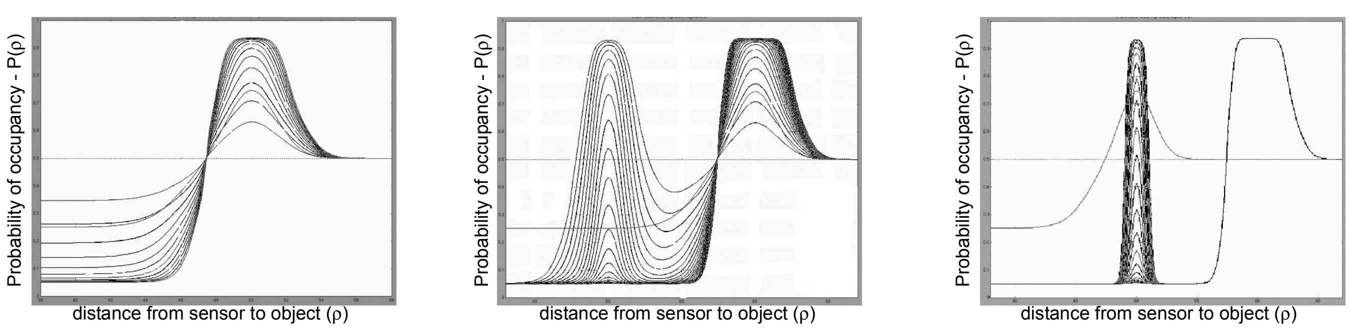
Unlike the Bayesian and Dempster-Shafer approaches, fuzzy logic inference appears as an intuitive way to merge redundant information. In previous work, it has been demonstrated that it is possible to reproduce the desired progressive refinement behavior with a fuzzy logic set of membership functions and rules [11]. A refined fuzzy logic system is proposed here that smoothens the progressive refinement in the evaluation of the space occupancy certainty level. The proposed fuzzy logic inference engine preserves the desired long-term and short-term memory effects.

The system input correspond to a mapping of the occupancy state of space along the line of scan of the sensor. This area is discretized and the fuzzyfication process operates successively on each of the resulting cells. The crisp input is defined as a fuzzy state of occupancy at a given distance with respect to the sensor. Input membership functions are classified in an order that matches the typical occupancy distribution. As shown in figure 6, starting from empty space just in front of the sensor located on the far left-hand side, it evolves up to unknown space behind the surface of the object, with a region of occupied space around the surface of the object located at 0. The rate of transition between these zones is adjusted according to the sensor uncertainty model as before.

The merge between measurements corresponding to a same region is achieved through a set of fuzzy inference rules that are defined in such a way that the refinement process occurs in both directions (toward empty or occupied) as the certainty on the state of space increases with the availability of new measurements. Table 2 presents the expanded set of rules that has been defined to refine the resolution of the fuzzy data merging method. As the number of membership functions has been increased from the original version, the number of rules has been enlarged. Rules have also been revised to improve performances. These rules can be applied between two measurements or between the current state of the model and a new measurement as described in section III.A.

Input 2 (model current state) \ Input 1 (new measurement)	E	M.P. E	P. E	M.A. E	A. E	A. O	M.A. O	P. O	M.P. O	O	U
E	E	M.P. E	M.P. E	M.P. E	P. E	P. E	M.A. E	A. E	A. E	U	E
M.P. E	E	M.P. E	P. E	P. E	P. E	M.A. E	M.A. E	A. E	U	A. O	M.P. E
P. E	M.P. E	M.P. E	P. E	P. E	M.A. E	M.A. E	A. E	U	A. O	A. O	P. E
M.A. E	P. E	P. E	P. E	M.A. E	A. E	A. E	U	A. O	A. O	M.A. O	P.A. E
A. E	P. E	M.A. E	M.A. E	M.A. E	A. E	U	A. O	A. O	M.A. O	M.A. O	A. E
A. O	M.A. E	M.A. E	A. E	A. E	U	A. O	M.A. O	M.A. O	M.A. O	P. O	A. O
M.A. O	M.A. E	A. E	A. E.	U	A. O	A. O	M.A. O	P. O	P. O	P. O	M.A. O
P. O	A. E	A. E	U	A. O	M.A. O	M.A. O	P. O	P. O	M.P. O	M.P. O	P. O
M.P. O	A. E	U	A. O	M.A. O	M.A. O	P. O	P. O	M.P. O	M.P. O	O	M.P. O
O	U	A. O	M.A. O	M.A. O	P. O	P. O	M.P. O	M.P. O	M.P. O	O	O
U	M.A. E	M.A. E	A. E	A. E	A. E	A. O	A. O	A. O	M.A. O	M.A. O	U

Table 2. Data fusion fuzzy inference rules.



a) b) c)

Fig. 5. Progressive model refinement with Dempster-Shafer theory of evidence.

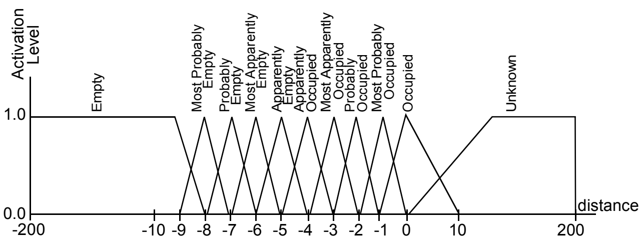


Fig. 6. Fuzzy input membership functions.

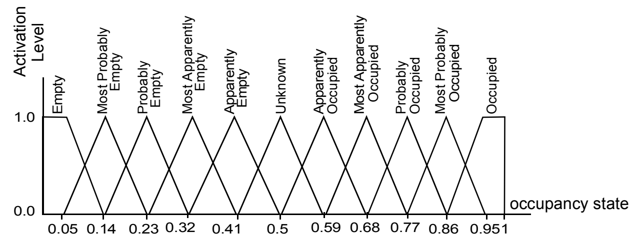


Fig. 7. Fuzzy output membership functions.

When measurements are merged using this fuzzy approach, the resulting occupancy model is encoded in the fuzzy space. That is the level of certainty on the state of occupancy of space is represented by fuzzy labels. In order to allow a comparison with the Bayesian and the Dempster-Shafer schemes, as defuzzyfication stage has been defined that converts the representation from the fuzzy space to the probabilistic occupancy space (on a 0-100% scale). The defuzzyfication procedure provides a mapping between the fuzzy tags and their respective activation level and the probability of occupancy. Figure 7 presents the output membership functions that were used for this extended fuzzy logic inference engine.

In order to compare the behavior of this data fusion method with the previous ones, a similar test was made using the proposed fuzzy logic inference engine. The results are presented in figure 8. A first sequence of 7 consistent measurements were merged (fig. 8a), followed by 7 perturbed data that estimated the object's surface to be 10 cm farther away from the sensor (fig. 8b). Finally, a final set of 7 consistent measurements were again provided with an estimated distance equal to that of the very first set of measurements. (fig. 8c).

From these curves, we observe that the progressive refinement is preserved but the evolution of the certainty on the occupancy state does not evolve as smoothly as with the Bayesian theorem or the Dempster-Shafer theory of evidence. Actually the behavior extensively depends on the number of members in the input and output functions, the shape of those membership functions (triangular, Gaussian) and the setting of the fuzzy rules. Moreover, the defuzzyfication step appears to be computationally expensive and significantly slows down operation, taking up to 45 minutes for this model during our experimentation.

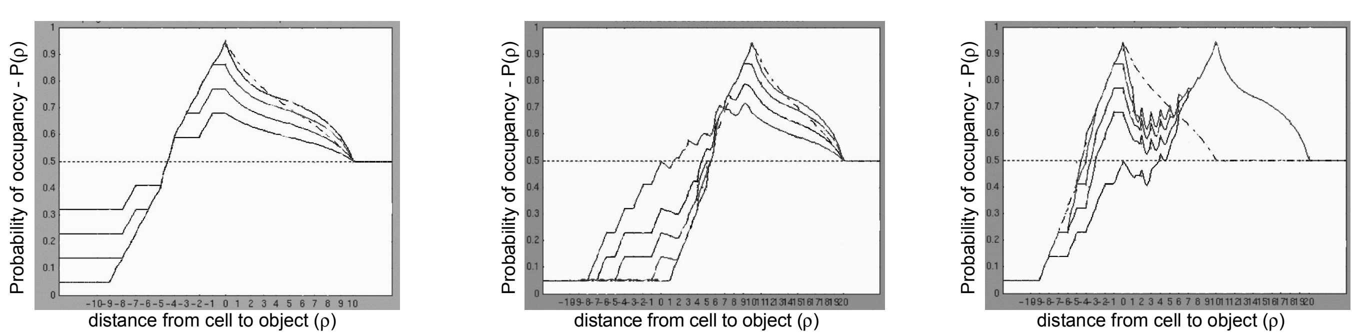
If a fuzzy merging scheme would be selected, this suggests that the certainty occupancy model would preferably be saved and manipulated in the fuzzy space where the occupancy status of each cell is represented by fuzzy tags along with their respective degree or confidence. Tuning of the fuzzy rules also revealed to be difficult in spite of its intuitive nature. Further refinement to the merging process, especially to smoothen the evolution of the progressive refinement, would imply extra membership functions and fuzzy rules to be added. This appears as major limitation of this approach.

IV. EXPERIMENTAL TESTBED

The comparison between these three data merging methods has been performed in the context of the exploration of unknown space with mobile robots. A simulation of a mobile platform equipped with a virtual laser range finder was designed on Matlab running on a Sparc 10 Sun workstation [14]. All merging schemes have been implemented using the same sensor uncertainty model and run in similar conditions with the same dataset of range measurements. Apart from generating the one-dimensional curves presented above, the simulator provides a visual interface to render 2D certainty maps resulting from successive merge of range measurements collected from various viewpoints over a limited field of view.

Figure 9 presents the resulting 2D certainty maps obtained with this simulator when the measurements collected from 8 different viewpoints are merged using the three data fusion techniques that are analyzed. White cells corresponds to regions where the certainty of occupancy is high while black cells represented a high certainty of emptiness. We observe that all techniques are able to provide similar realistic occupancy mappings of a cluttered space with progressive refinement of the certainty level.

The computational workload implied by the use of the Bayesian theory is the lowest among all methods. However, the use of Dempster-Shafer theory of evidence only slightly raises the computational workload as more equations need to be process. But this slight increase provides supplementary information, especially a specific encoding of unknown space, which is not available with the Bayesian approach. The main constraint imposed by the Dempster-Shafer method comes from the need to define a more complex encoding scheme for the certainty occupancy model which cannot map occupancy state only with a percentage value but rather requires a specific encoding of belief and plausibility values for each region of space. The resulting model is therefore slightly larger. Finally, the fuzzy logic encoding scheme has a higher computational workload that becomes intractable for real-time systems if the defuzzyfication stage is used. Otherwise, when the certainty occupancy model is to be encoded in the fuzzy space, the computational workload remains tractable but the tuning of the membership functions and fuzzy inference rules is challenging in spite of its intuitive nature. On the other hand, a fuzzy logic



a) b) c)

Fig. 8. Progressive model refinement with fuzzy logic inference.

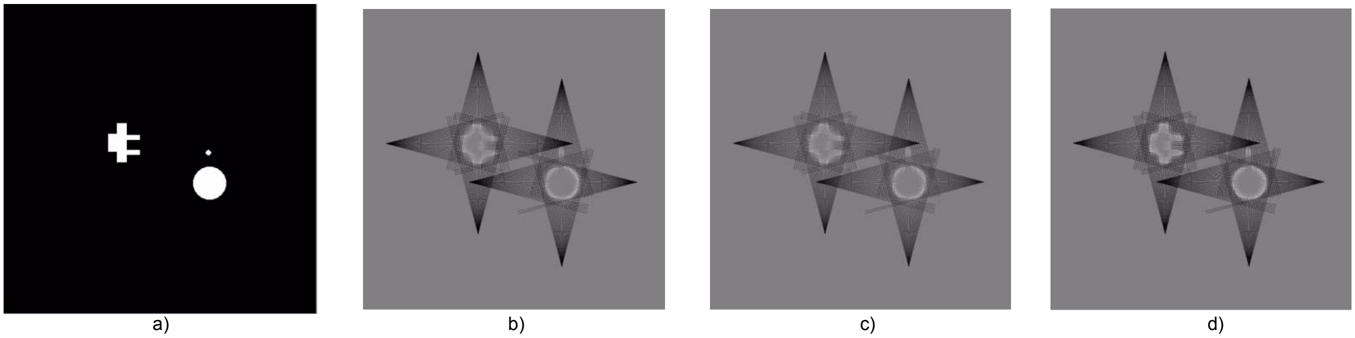


Fig. 9. 2D certainty maps resulting for the merge of range measurements from 8 viewpoints on a) a predefined environment, using b) the Bayesian theorem, c) the Dempster-Shafer theory of evidence, and d) the fuzzy logic inference engine.

inference approach for data fusion offers extra flexibility on the rate of transition of the uncertainty estimation when consistent and erroneous measurements are combined. Not being limited to predefined merging rules as with the Bayesian (eq. (2)) and the Dempster-Shafer (eq. (12)) methods gives a maximum flexibility to tune the behavior of the data fusion. This might reveal advantageous when safe operation is a major concern.

V. CONCLUSION

This research work proposes an adaptation and a comparison of three data merging approaches for the construction of occupancy maps where the level of uncertainty of range sensors and the confidence resulting from the merge of overlapping measurements is directly estimated. Experimentation demonstrated that the Bayesian theorem, the Dempster-Shafer theory of evidence and a fuzzy logic inference engine can all be used to create occupancy maps that are representative of the physical reality with a progressive refinement on the certainty level of occupancy or emptiness for a given region of space.

The Bayesian approach appears as a simple and efficient technique that requires a minimum of analysis to obtain a meaningful representation. It provides both an extreme simplicity of implementation and a high computational efficiency.

Dempster-Shafer theory of evidence is a generalization of the Bayesian theory which is able to handle aspects that are neglected with the Bayesian scheme such as a clear differentiation between unknown occupancy state resulting from contradictory measurements or from the lack of exploration of a given region. This characteristic reveals strategically advantageous to facilitate and optimize collision-free path planning in synchronization with optimal sensor positioning. It also provides an excellent method of selection of the input when multi-modal sensing technologies are used as unknown areas resulting from many contradictory measurements can be identified to switch the sensor to another mode. On the other hand, a Dempster-Shafer approach is slightly more difficult to implement as sensor's error level must be mapped to the basic probability density function.

Finally, the fuzzy logic inference fusion method appears to be the most difficult to tune and the less efficient in terms of computational workload. However, this approach opens the door to a different modeling scheme of uncertain representations where occupancy is no longer represented by percentage or degree of belief but rather by a set of fuzzy tags with their respective degree of confidence. When combined with a fuzzy logic inference engine for navigation and control as found on many mobile robot platforms, this scheme might be advantageous as it directly provides fuzzy input data.

Future investigation of data merging techniques for the construction of certainty occupancy models will extend the comparison to Kalman filters which provide means to explicitly estimate both the occupancy state of space and the associated uncertainty level in parallel.

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