MAT 3100: Alexandria after Archimedes

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Greek Mathematics after Archimedes

Alexandria had many brilliant mathematicians:

Aristarchus	$\sim$	310-250 B.C.	
Archimedes	$\sim$	287-212 B.C.	
Eratosthenes	$\sim$	276-194 B.C.	
Apollonius of Perga	$\sim$	250-176 B.C.	
Hipparchus of Bithynia	$\sim$	190-120 B.C.	
Heron	$\sim$	1st century	
Diophantus	$\sim$	3rd century	
Pappus	$\sim$	4th century	
Hypatia	$\sim$	355-415	(Primarily a scholar)
Proclus	$\sim$	410-485	(Primarily a scholar)
Plato's Academy destroyed by Emperor Justinian	$\sim$	529	
Library of Alexandria destroyed	$\sim$	642	

Aristarchus (Elder to Archimedes)

- Aristarchus $\sim$ 310-250 B.C.
- From Samos (like Pythagorus, 250 yrs earlier)
- Applied math to astronomy. First to propose sun-centred solar system. Believed apparent daily rotation of earth on its axis. Wrote several works, but only surviving one is "On the sizes and distances of the Sun and Moon". Interesting geometric proofs of distance(earth,sun), distance(earth,moon), relative sizes earth/moon, etc. Amazingly accurate theory, given primitive apparatus.
- He believed the reason the stars appeared unmoving was enormous distances of earth to stars. Even Archimedes considered him a crackpot, but Aristarchus was right! Also his theories were considered too radical with "impiety" against established religions.

Eratosthenes of Cyrene $\sim$ 276-194 B.C.

- Chief librarian of Alexandria. Interested in many things: philosophy, poetry, history, philology, geography, astronomy, and mathematics. Heath says: called by students β: why? Also *Pentathlos*. In fact, great scholar.
- Invented sieve method for finding primes, extraordinarily accurate calculation of circumference of the earth using geometrical reasoning, invented the Julian calendar.
- Wrote *Platonicus* (now lost): history of math taught at Plato's academy. Many other books, now lost. Developed mechanical devices for solving the classical problems. Made major contributions to geography (Nile, Yemen, floods).
- Received letter from Archimedes: what was it about?

Appolonius of Perga $\sim$ 262-190 B.C.

- Known as "The Great Geometer". His famous book *Conics* (eight books; seven surviving in Arabic, but only four original Greek volumes survive). Introduced the theory of conics, with terminology: "parabola, ellipse, hyperbola". Studied with Euclid and taught at Alexandria.
- Leibniz "He who understands Archimedes and Appolonius will admire less the achievements of the foremost of men of later times".
- His works are hard to read: extant books contain 387 separate propositions. Studied asymptotes of hyperbolae, tangents of conics, theory of parabolic mirrors, circles tangent to 3 objects.

cont'd.

Appolonius of Perga ∼ 262-190 B.C.

- Despite his brilliance, both Anglin-Lambek and Katz claim analytic geometry of Descartes (1596-1650) and Fermat (1601-1665) essentially made Appolonius's methods obsolete and his work forgotten. Too bad: there are probably still interesting things there!
- According to MacTutor, there are other books that he's known to have written, including one on irrational numbers (discussed by Proclus) and *Quick Delivery* in which he gave a better approximation to π than Archimedes.
- Although Eudoxus's complicated spherical models of the universe were influential, Appolonius developed epicyclic and eccentric circle models of the cosmos.

Hipparchus of Bithynia $\sim$ 190-120 B.C.

- Many contributions to astronomy. Calculated length of year to within 6 minutes, lunar parallax, eccentricity of solar orbit, higher-order epicycles for modelling motions of planets.
- Founded *trigonometry* and developed trig tables.
- Seems to have known various trig identities, e.g. $sin(x \pm y) = sin(x)cos(y) \pm cos(x)sin(y)$, and some half-angle formulas.

Ptolemy ∼ 100-178

- Rough proportion: Ptolemy: Astronomy :: Euclid: Geometry.
- Greatest astronomer: his book (13 volumes) in Arabic was called *Almagest* or "greatest". Gave a complete theory of all celestial motion of known bodies, theory of eclipses, tables of observational data, etc.
- He was a prolific writer, and his work far extended Appolonius' ideas, using epicycles; deep and interesting models of the astronomical bodies which roughly fit the data of the time, and lasted over 1400 years.
- The books developed elaborate chord tables, trignometric charts, theory of epicycles, etc.
- *Almagest* was the standard textbook on advanced Astronomy until the end of the Renaissance.
- He also wrote the standard textbooks on Geography and Cartography, which lasted for almost 1500 years. His maps were used by early European sailors (e.g. Columbus, et. al.).

Diophantus $\sim$ 200-284

- Father of algebra. Introduced abstract symbolic expressions for the first time in history.
- His book *Arithmetica* of 13 volumes was a series of solutions of a wide range of algebraic equations (and systems of equations) over $\mathbb{Q}$.
- His innovations immediately attracted many mathematicians and commentators, and Diophantine problems and methods became a fundamental part of number theory and mathematics ever since.
- Was aware of rules for multiplying "minus" (for example a minus times a minus is a plus, etc.) but for him, this was just symbolic. The Greeks did not deal with negative numbers.
- **Read my handout on Diophantus. Also read Wei Lu's notes.**

Diophantus $\sim$ 200-284

- A quote from Diophantus:
  If a problem leads to an equation in which certain terms are equal to terms of the same species but with different coefficients, it will be necessary to subtract like from like on both sides, until one term is found equal to one term. If by chance there are on either side or on both sides any negative terms, it will be necessary to add the negative terms on both sides, until the terms on both sides are positive, and then again to subtract like from like until one term only is left on one side. This should be the object . . . to reduce the equations, if possible until one term is left equal to one term; but I will show you later how, in the case also where two terms are equal to one term, such a problem is solved.

- *To divide a given square number into two squares*

Method: The following is a quotation from Diophantus, in modern notation (from Katz, p 179).

Let it be required to divide 16 into two squares. And let the first square $= x^2$*; then the other will be* $16 - x^2$*. It shall be required therefore to make* $16 - x^2$ *equal to a square. I take a square of the form* $(ax - 4)^2$*, a being any integer and* 4 *the square root of* 16*. For example, let the side be* $2x - 4$*, and the square itself* $4x^2 + 16 - 16x$*. Then* $4x^2 + 16 - 16x = 16 - x^2$*. Add to both sides the negative terms and take like from like. Then* $5x^2 = 16x$*, and* $x = 16/5$*. One number will therefore be* $256/25$ *, the other* $144/25$ *and their sum is* $400/25 = 16$*, and each is a square.*

Diophantus: More of his Problems

- Find 3 numbers such that the product of any two added to the third is a square; i.e. find x, y, z such that: for some s, t, u, $xy + z = t^2, xz + y = s^2, yz + x = u^2$.
- According to Katz, Diophantus knew the formula for $(x + y)^3$.
- In Book VI he solves problems such as finding x such that $4x + 2$ is a cube and $2x + 1$ is a square (easy). Much harder is to write 13 as a sum of two squares > 6, and to write 10 as a sum of 3 squares, each > 3. (For the solutions, see MacTutor bio on Diophantus).
- Know the connection of Diophantus's work with the unsolvability of Hilbert's 10th problem (see Wei Lu's notes.)

Diophantus ∼ 200-284

- Diophantus was forgotten for hundreds of years. Revived by Bombelli (1526-1572) in his book on algebra.
- Fermat (1600-1665) studied the 1621 Latin edition of Diophantus's Arithmetica.
- In thinking about one of Diophantus's problems, Fermat used the method of infinite descent to show: One cannot find three positive integers a, b and c such that $a^4 - b^4 = c^2$. He wrote in the margin: *One cannot split a cube into two cubes, nor a fourth power into two fourth powers, nor in general any power beyond the square in infinitum into two powers . . . .*

  He then said he had a "truly marvelous demonstration... which this margin...is too narrow to contain". This is Fermat's Last Theorem, proved finally by Andrew Wiles in 1995.

Pappus of Alexandria $\sim$ 290-350

- Last of the great Greek geometers. The school of mathematics at Alexandria had declined, and Pappus was its sole member.
- He wrote *The Mathematical Collection*, a guide (in 8 volumes) to all of the Greek mathematics, and is meant to be read as a commentary with the original works. Not particularly original, except for Volume VII, but it shows he had mastered almost all of the mathematics and astronomy of the Greek academy.
- At the time, young mathematicians studied astronomy, physics, trigonometry, number theory, and algebra, not geometry (it was an "old" subject).
- The training of professional mathematicians at the time involved studying Euclid's elements, as well as Volume VII of Pappus.

Pappus of Alexandria $\sim$ 290-350

- Book VII contains the Theorem of Pappus, to be discussed in class. This is related to Pascal's Mystical Hexagram (1614). It expresses commutativity of multiplication, and is fundamental to projective geometry, as well as a key theorem in Hilbert's axiomatization of Euclidean geometry.
- See Wei Lu's notes (2.4.26, 2.4.27), and the Wikipedia link to Pappus' (and Pascal's) theorems on course website (under Greek Mathematics)

Hypatia of Alexandria $\sim$ 370-415

- First woman mathematician known.
- Head of Platonist school in Alexandria. Taught mathematics by her father, Theon of Alexandria, and wrote papers and books with him.
- Apparently a charismatic teacher, eloquent, great scholar.
- Killed by a fanatic Christian mob, who considered her a pagan, and yet who considered her a threat because of her scholarship and scientific knowledge.
- All her work, except for titles and references and a few letters, is lost.
- She's been the heroine of many novels and at least one movie (Agora). For interesting references, go to Youtube and look up Hypatia. Lots of interesting videos and scholarly stuff, as well as fluff.

Proclus$\sim$ 412- 485

- Philosopher and great scholar.
- Studied in Alexandria, but took up philosophy and moved to Athens.
- Became the leader of Plato's Academy at this time; was prolific writer, deep thinker. Much of his huge output of writings is lost, but still a lot remains.
- Wrote critical and important commentaries on history of Euclid and Euclidean geometry, which have been fundamental in our understanding of early Greek mathematics.