MAT 3100: Homework 1

Prof. P.J. Scott, Jan. 18, 2016

Due: Tuesday, January 26, 2016 by 16:00, in the MAT3100 box in the math department.

No late assignments accepted.

Instructions: Please carefully read the course handout about plagiarism and homework (on the webpage: `http://www.site.uottawa.ca/ phil/courses/MAT3100.16/MAT3100.16.html`).

You may talk to your friends and me, but under no circumstances should you copy or submit shared homework. Homework must be your own work. U. Ottawa has very strict policies about academic fraud. The marker will pay careful attention. If in doubt of what's allowed, just ask me.

Study problems and Reading:

(a) Make sure to read on the course webpage (Egyptian and Mosopotamian Mathematics) `http://www.site.uottawa.ca/ phil/courses/MAT3100.16/MAT3100.13.mesopot.html` the documents (mostly from St. Andrews Archives) with a **Red Dot** beside them.

(b) Read Anglin-Lambek, Katz, or Eves' books on reserve in the library.

(c) Represent 375 and 4856 in Egyptian hieroglyphics and Babylonian cuneiform. Use Egyptian techniques to multiply $\overline{2}\,\overline{14}$ by $1\,\overline{2}\,\overline{4}$

(d) Give addition and multiplications tables for base $b = 4, 5, 6, 7$ (recall, this means give addition and multiplication tables over the alphabet $\{0, 1, \cdots, b-1\}$, where for entry (i, j) express the sum and product in base b. For example, base 4, $4 + 3 = 13$, since $(4 + 3)_{10} = (7)_{10} = (13)_4$.

(e) Recall, in the theory of Egyptian fractions, we mentioned the following conjecture of the famous mathematician P. Erdos (from the book of Anglin & Lambek):

Conjecture: *For all odd natural numbers $n > 4$, we can write* $4/n = 1/x + 1/y + 1/z$ for distinct positive integers x, y, z.

Verify this for $n = 5, 7, 9$.

Problems to Hand In:

1. Read my typed notes on representation for different bases from the webpage.

   (i) Consider the number 6657 in ordinary base 10. Give the base 5, 8, and 12 representation of 6657. For base 12, use digits: $\{0, 1, ..., 9, t, e\}$, where t is 10 and e is 11.

   (ii) Write 6657 in sexagesimal notation using ordinary (Hindu-Arabic) numbers and commas.

   (iii) Write 6657 in sexagesimal cuneiform notation.

   (iv) Write 6657 in Hieroglyphic Egyptian notation (recall they wrote in reverse base 10).

2. Do the following calculations:

   (i) Convert the following sexagesimal fractions to ordinary decimal fractions in lowest terms: 0;22,30 , 0;08,06 , 0; 05,33,22.

(ii) Convert 67.57 (in decimal notation) to sexagesimal.

(iii) Find the reciprocal 1/54 in sexagesimal.

(iv) In the Babylonian system, multiply 18 times 1,21 . In that system, compute $50 \div 18$

3. The Rhind Papyrus (Problem 4) asks to divide 7 loaves of bread among 10 men. Show that each man gets $\bar{\bar{3}}\ \overline{30}$ loaves. Use the method in class to check the answer, writing a table, as follows (in Egyptian notation):

	1	$\bar{\bar{3}}\ \overline{30}$
$\checkmark$	2	?
	4	?
$\checkmark$	8	?

4. (i) For what base b is $3{\cdot}3 = 10$?

(ii) For what base b is $3{\cdot}3 = 12$?

(iii) Find a base b where $(72)_{10} = (2200)_b$

(iv) If $b > 2$, prove that $(121)_b$ is a square integer.

(v) What is the smallest base $b > 1$ such that $(301)_b$ represents a square integer?

5. (i) Show that $z/pq = 1/pr + 1/qr$, where $r = (p + q)/z$. This formula was found on a papyrus written in Greek, sometime between the 6th and 8th century, in the city Akhmim on the Nile.

(ii) Look at the Rhind papyrus table for $2/n$ on the webpage. Does the above formula give the cases $n = 5, 7, 9, 15$? [Hint: sometimes the Egyptians seem to use the identity $2/n = 2x/nx$, for $x > 0$. You may have to pick an appropriate choice of x.]

6. Recall, in Babylonian mathematics: n is *regular* if and only if $1/n = a_0 + \frac{a_1}{60} + \frac{a_2}{60^2} + \cdots + \frac{a_r}{60^r}$, for some integer $r > 0$. In class we mentioned the following theorem:

Theorem: *n is regular if and only if n has the form $2^i 3^j 5^k$ for some non-negative integers i, j, k.*

Write up the proof carefully in both directions.

7. (i) Find two ways of writing 1/4 as the sum of two distinct unit fractions.

(ii) Anglin-Lambek (p. 10) have a problem: prove that $4/(4m + 3)$ can be written as the sum of three distinct unit fractions, for all positive integers m. Please verify this result for $m = 1, 2, 3$.

(iii) Extra Credit: prove the Anglin-Lambek problem above.