Spin Networks and Anyonic Topological Quantum Computing

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Spin Networks and Anyonic Topological Computing

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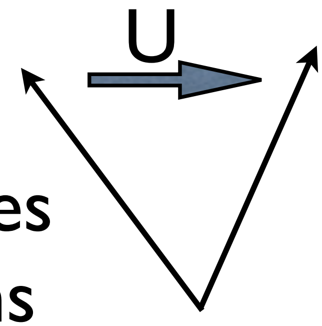
Quantum Mechanics in a Nutshell

0. A state of a physical system corresponds to a unit vector |S> in a complex vector space.



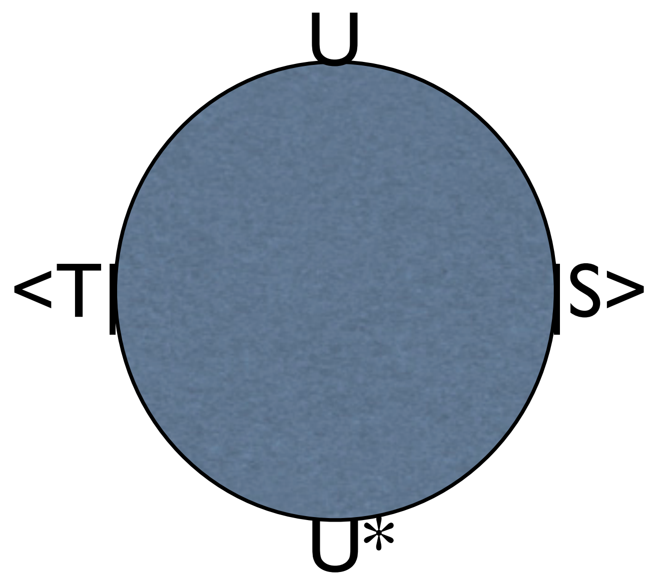
1. (measurement free) Physical processes are modeled by unitary transformations applied to the state vector: |S> -----> U|S>

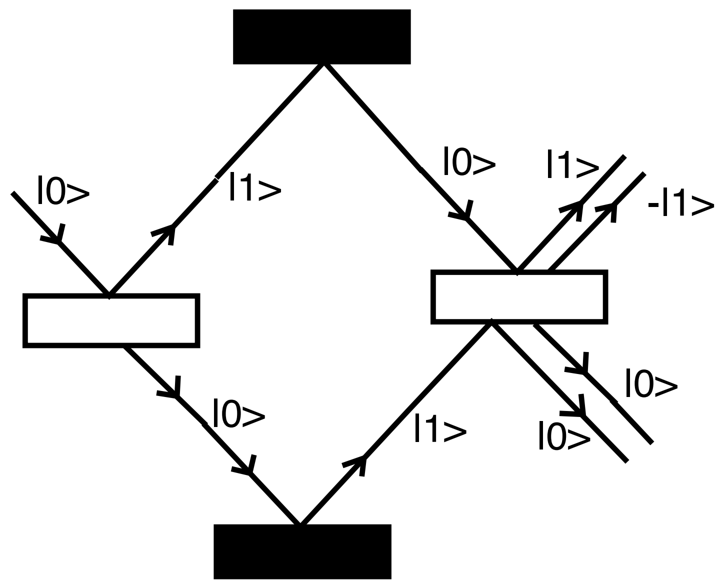
2. If |S> = z1|e1> + z2|e2> + ... + zn|en> in a measurement basis {e1,e2,...,en}, then measurement of |S> yields |ei> with probability |zi|^2.

Preparation, Transformation, Measurement.

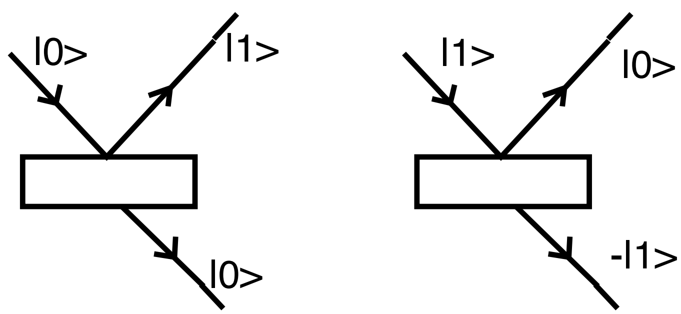
Psi = <T|U|S>

Psi*Psi = <S| U*|T> <T|U|S>





Mach-Zender Interferometer



$$H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} / \mathrm{Sqrt}(2) \qquad M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$HMH = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Quantum Gates
are unitary transformations
enlisted for the purpose of computation.

CNOT =

1	0	0	0
0	1	0	0
0	0	0	1
0	0	1	0

CNOT|00> = |00>
CNOT|01> = |01>
CNOT|10> = |11>
CNOT|11> = |10>

Universal Gates

A *two-qubit gate* G is a unitary linear mapping

$$G : V \otimes V \longrightarrow V \otimes V$$ where V is

a two complex dimensional vector space. We say that the gate G is *universal for quantum computation* (or just *universal*) if G together with local unitary transformations (unitary transformations from V to V) generates all unitary transformations of the complex vector space of dimension 2^n to itself. It is well-known [44] that $CNOT$ is a universal gate.

A gate G is universal
iff
G is entangling.

A gate G, as above, is said to be *entangling* if there is a vector

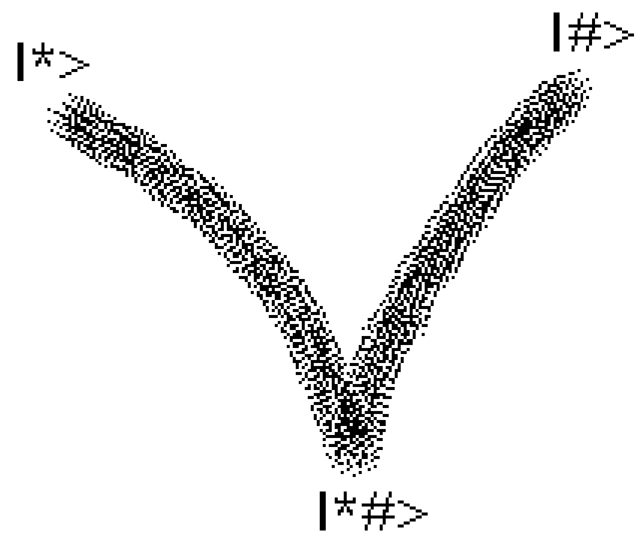
$$|\alpha\beta\rangle = |\alpha\rangle \otimes |\beta\rangle \in V \otimes V$$

such that $G|\alpha\beta\rangle$ is not decomposable as a tensor product of two qubits. Under these circumstances, one says that $G|\alpha\beta\rangle$ is *entangled.*

In [6], the Brylinskis give a general criterion of G to be universal. They prove that *a two-qubit gate G is universal if and only if it is entangling.*

An Entangled State

The EPR State



$$|{*}\#\rangle = (|01\rangle + |10\rangle)/\mathrm{Sqrt}(2)$$

An Entanglement Criterion

Remark. A two-qubit pure state

$$|\phi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

is entangled exactly when $(ad - bc) \neq 0$. It is easy to use this fact to check when a specific matrix is, or is not, entangling.

The Bell States

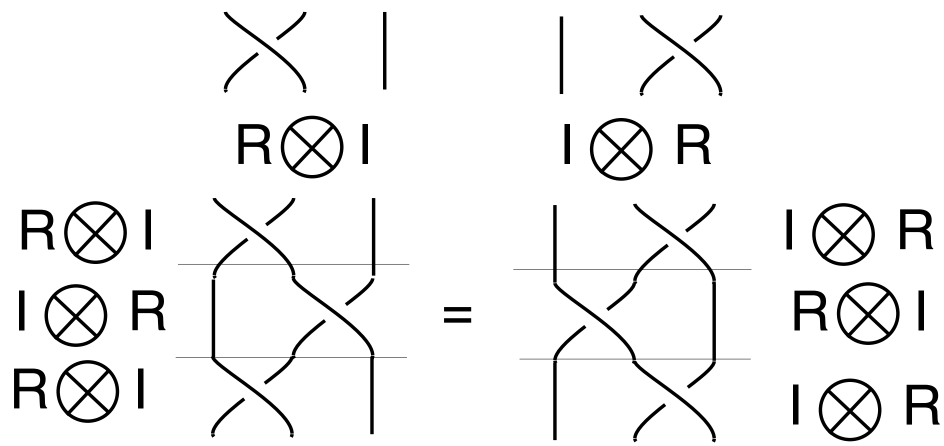
$$R|00\rangle = (1/\sqrt{2})|00\rangle - (1/\sqrt{2})|11\rangle,$$

$$R|01\rangle = (1/\sqrt{2})|01\rangle + (1/\sqrt{2})|10\rangle,$$

$$R|10\rangle = -(1/\sqrt{2})|01\rangle + (1/\sqrt{2})|10\rangle,$$

$$R|11\rangle = (1/\sqrt{2})|00\rangle + (1/\sqrt{2})|11\rangle.$$

Braiding and the Yang-Baxter Equation



$$(R \otimes I)(I \otimes R)(R \otimes I) = (I \otimes R)(R \otimes I)(I \otimes R).$$

Braiding Operators are Universal Quantum Gates

Let V be a
two complex dimensional vector space.

Universal gates can be constructed
from certain solutions to the
Yang-Baxter Equation

$$R\colon\ V \otimes V \longrightarrow V \otimes V$$

$$(R \otimes I)(I \otimes R)(R \otimes I) = (I \otimes R)(R \otimes I)(I \otimes R).$$

Representative Examples of Unitary Solutions to the Yang-Baxter Equation that are Universal Gates.

$$R = \begin{pmatrix} 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \end{pmatrix}$$

Bell Basis Change Matrix

$$R' = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & 0 & d \end{pmatrix} \qquad R'' = \begin{pmatrix} 0 & 0 & 0 & a \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ d & 0 & 0 & 0 \end{pmatrix}$$

$$R_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Swap Gate with Phase

Issues

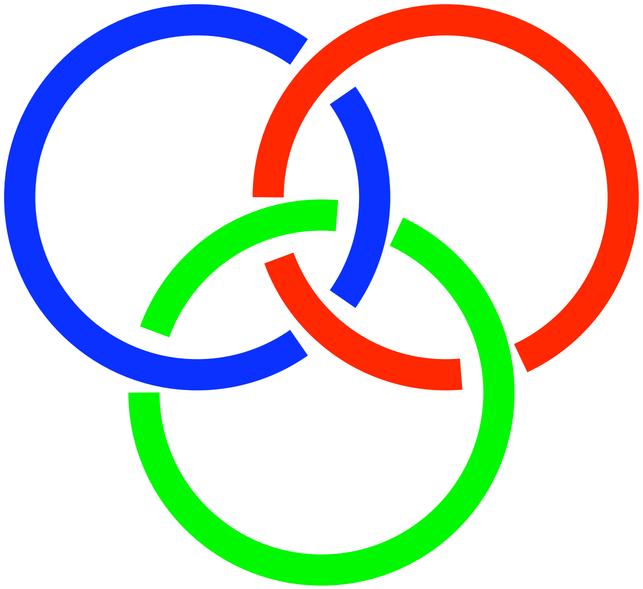
1. Giving a Universal Gate that is topological does NOT create "topological quantum computing" because the U(2) local operations have not been made topological.

2. Nevertheless, Yang-Baxter gates are interesting to construct and help to discuss Topological Entanglement versus Quantum Entanglement.

Quantum Entanglement and Topological Entanglement

An example of Aravind [1] makes the possibility of such a connection even more tantalizing. Aravind compares the Borromean rings (see figure 2) and the GHZ state

$$|\psi\rangle = (|\beta_1\rangle|\beta_2\rangle|\beta_3\rangle - |\alpha_1\rangle|\alpha_2\rangle|\alpha_3\rangle)/\sqrt{2}.$$



Is the Aravind analogy only superficial?!

Consider this state.

$$|\psi\rangle = (1/2)(|000\rangle + |001\rangle + |101\rangle + |110\rangle)$$

Observation in any coordinate
yields entangled and unentangled
states with equal probability.

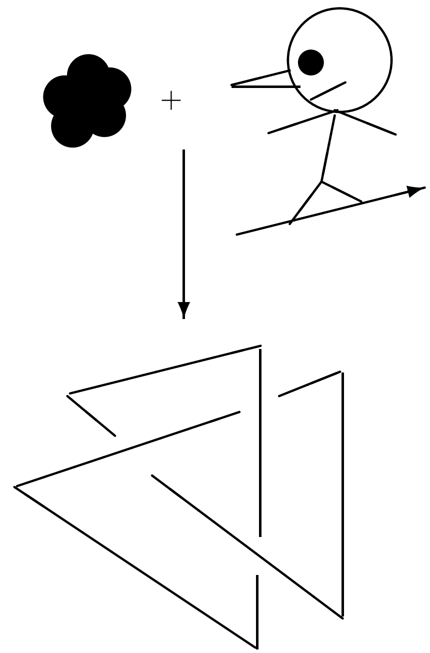
e.g.

$$|\psi\rangle = (1/2)(|0\rangle(|00\rangle + |01\rangle) + |1\rangle(|01\rangle + |10\rangle)$$

First coordinate measurement
gives
|00> + |01> and
|01> + |10>
with equal probability.

Do we need Quantum Knots?

a|K> + b|K'>



K: probability |a|^2

K':probability |b|^2

Observing a Quantum Knot

Air on the Dirac Strings

$SU(2)$ Representations of the Artin Braid Group

Theorem. If $g = a + bu$ and $h = c + dv$ are pure unit quaternions,then, without loss of generality, the braid relation $ghg = hgh$ is true if and only if $h = a + bv$, and $\phi_g(v) = \phi_{h^{-1}}(u)$. Furthermore, given that $g = a + bu$ and $h = a+bv$, the condition $\phi_g(v) = \phi_{h^{-1}}(u)$ is satisfied if and only if $u \cdot v = \frac{a^2-b^2}{2b^2}$ when $u \neq v$. If $u = v$ then then $g = h$ and the braid relation is trivially satisfied.

g = a + bu
h= a + bv

u•v = (a^2 - b^2)/2b^2

An Example. Let

$$g = e^{i\theta} = a + bi$$

where $a = cos(\theta)$ and $b = sin(\theta)$. Let

$$h = a + b[(c^2 - s^2)i + 2csk]$$

where $c^2 + s^2 = 1$ and $c^2 - s^2 = \frac{a^2-b^2}{2b^2}$. Then we can reexpress g and h in matrix form as the matrices G and H. Instead of writing the explicit form of H, we write $H = FGF^*$ where F is an element of $SU(2)$ as shown below.

$$G = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}$$

$$F = \begin{pmatrix} ic & is \\ is & -ic \end{pmatrix}$$

SU(2) Fibonacci Model

$$\tau^2 + \tau = 1.$$

$$g = e^{7\pi i/10}$$

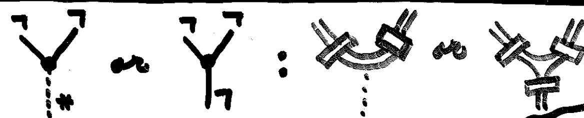
$$f = i\tau + k\sqrt{\tau}$$

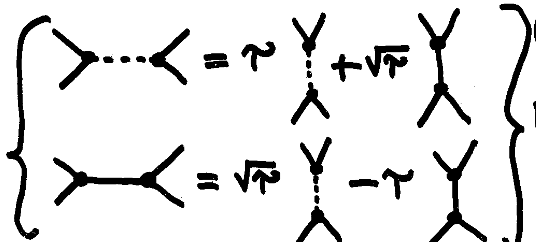
$$h = frf^{-1}$$

{g,h} represents 3-strand braids,
generating a dense subset of SU(2).

We shall see that the representation
labeled "SU(2) Fibonacci Model"
in the last slide
extends beyond SU(2) to
representations of many-stranded
braid groups rich enough
to generate quantum computation.

Fibonacci Model in a Nutshell

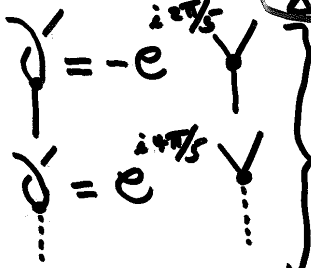




$R + FRF$ gen rep of B_3

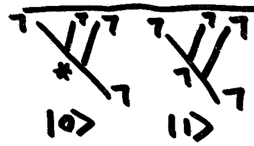
$$F = \begin{pmatrix} \tau & \sqrt{\tau} \\ \sqrt{\tau} & -\tau \end{pmatrix}$$

$\tau^2 + \tau = 1$, $\tau = \frac{1}{\Delta}$, $\Delta^2 = 1 + \Delta$
$\Delta = (1+\sqrt{5})/2$.



$$R = \begin{pmatrix} e^{4\pi i/5} & 0 \\ 0 & -e^{2\pi i/5} \end{pmatrix}$$

$A = e^{3\pi i/5}$, $\delta = -A^2 - A^{-2} = -2\cos(6\pi/5) = \frac{1+\sqrt{5}}{2}$

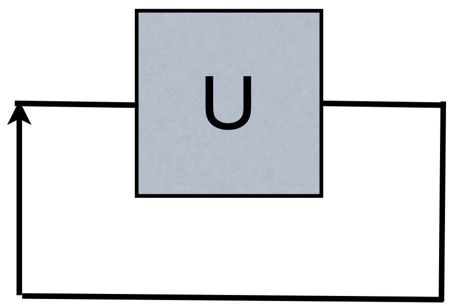


$V_\tau^{\tau\tau\tau} = V$ gives

rep: $B_3 \longrightarrow U(2)$

rep(χ|) = R
rep(|χ) = FRF } gens dense subset of $U(2)$.

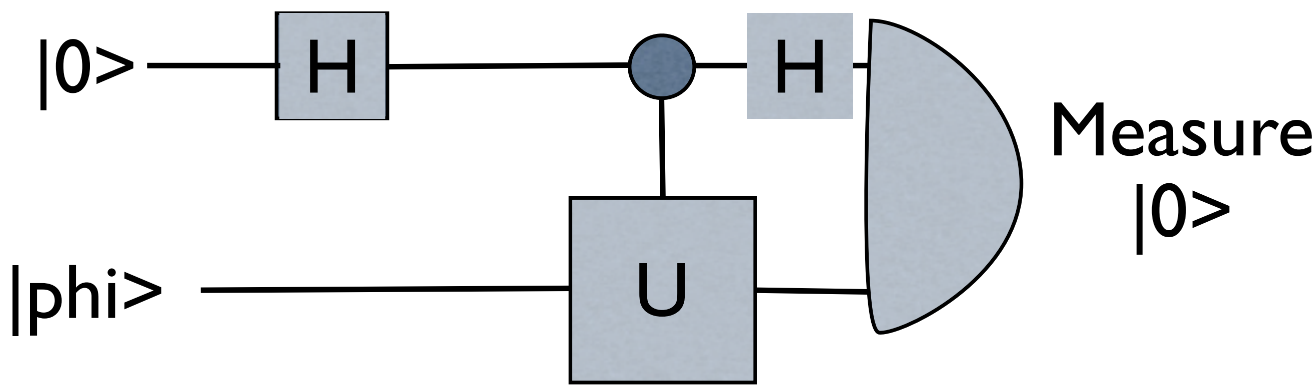
Quantum Computation of the Trace of a Unitary Matrix



1. A good example of a quantum algorithm.

2. Useful for the quantum computation of knot polynomials such as the Jones polynomial.

Hadamard Test



|0> occurs with probability
1/2 + Re[<phi|U|phi>]/2

Quantum Hall Effect

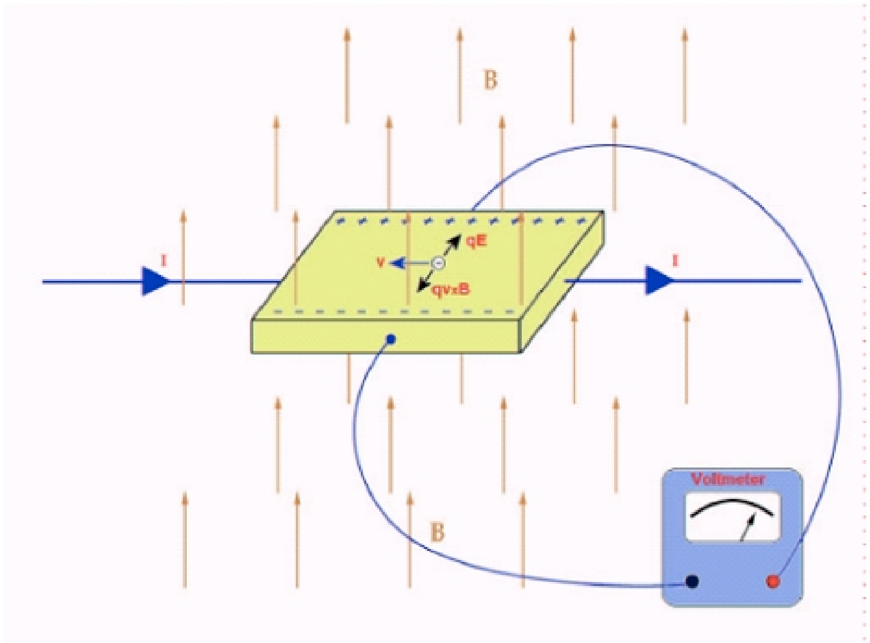
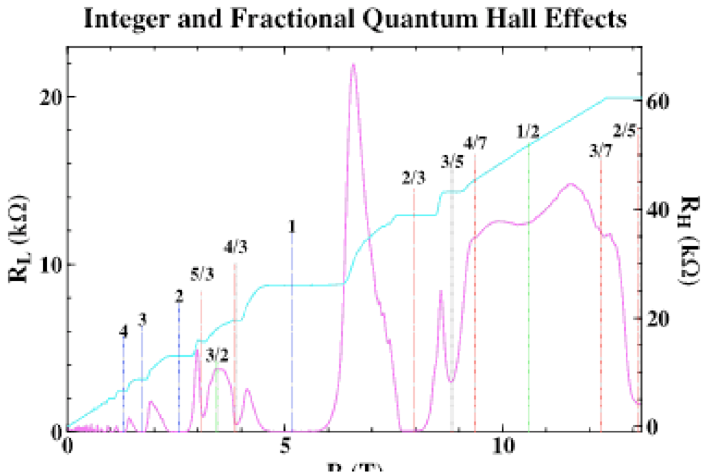


Figure 1: A schematics of the experimental setup of the Hall effect. A current driven through the conductor, drawn as a prism, leads to the emergence of voltage in the perpendicular direction. This is the Hall voltage, which Maxwell erroneously predicted to be zero.

Fractional Quantum Hall Effect (Cambridge Univ Website)

The fractional quantum Hall effect (FQHE) is a fascinating manifestation of simple collective behaviour in a two-dimensional system of strongly interacting electrons. At particular magnetic fields, the electron gas condenses into a remakable state with liquid-like properties. This state is very delicate, requiring high quality material with a low carrier concentration, and extremely low temperatures. As in the integer Quantum Hall Effect, a series of plateaux forms in the Hall resistance. Each particular values of magnetic field corresponds to a filling factor (the ratio of electrons to magnetic flux quanta) nu=*p*/*q*, where *p* and *q* are integers with no common factors). *q* always turns out to be an odd number. The principal series of such fractions are 1/3, 2/5, 3/7 etc, and 2/3, 3/5, 4/7, etc.



There are two main theories of the FQHE:

- **Fractionally-charged quasiparticles**. This theory, proposed by Laughlin, hides the interactions by constructing a set of quasiparticles with charge $e^*=e/q$, where the fraction is p/q as above.
- **Composite Fermions**. This theory was proposed by Jain, and Halperin, Lee and Read. In order to hide the interactions, it attaches two (or, in general, an even number) flux quanta *h*/*e* to each electron, forming integer-charged quasiparticles called Composite Fermions. The fractional states are mapped to the Integer QHE. This makes electrons at a filling factor 1/3, for example, behave in the same way as at filing factor 1. A remarkable result is that filling factor 1/2 corresponds to zero magnetic field. Experiments support this.

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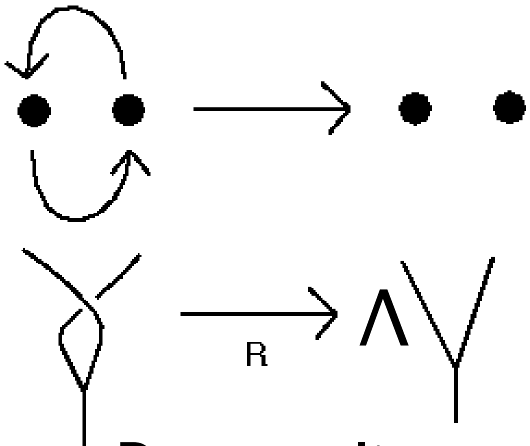
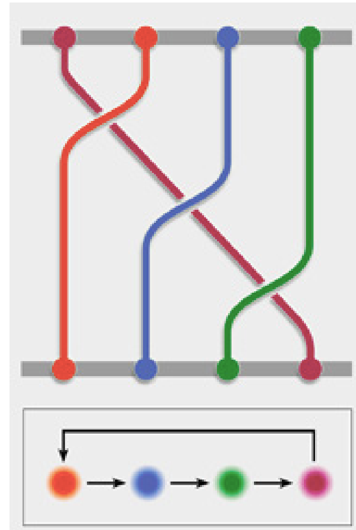
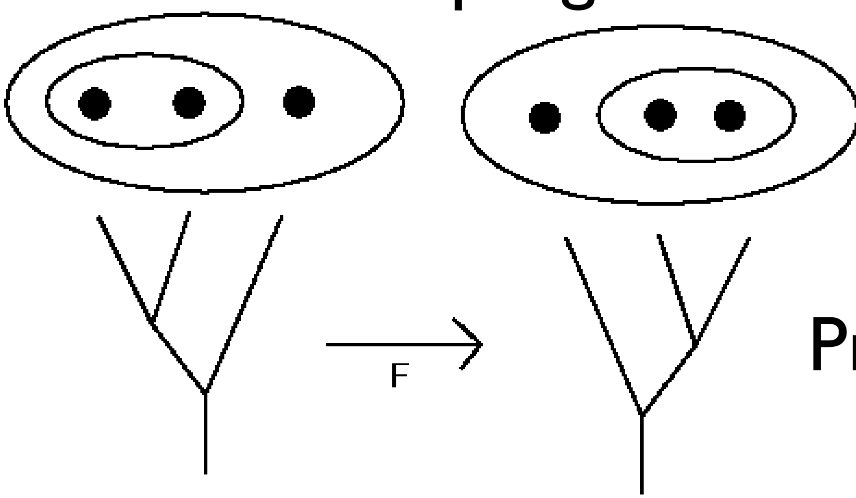
The quasi-particle theory is connected with Chern-Simons Theory and it explains the FQHE on the basis of "anyons": particles that have non-trivial (not +1 or -1) phase change when they exchange places in the plane.

Braiding Anyons



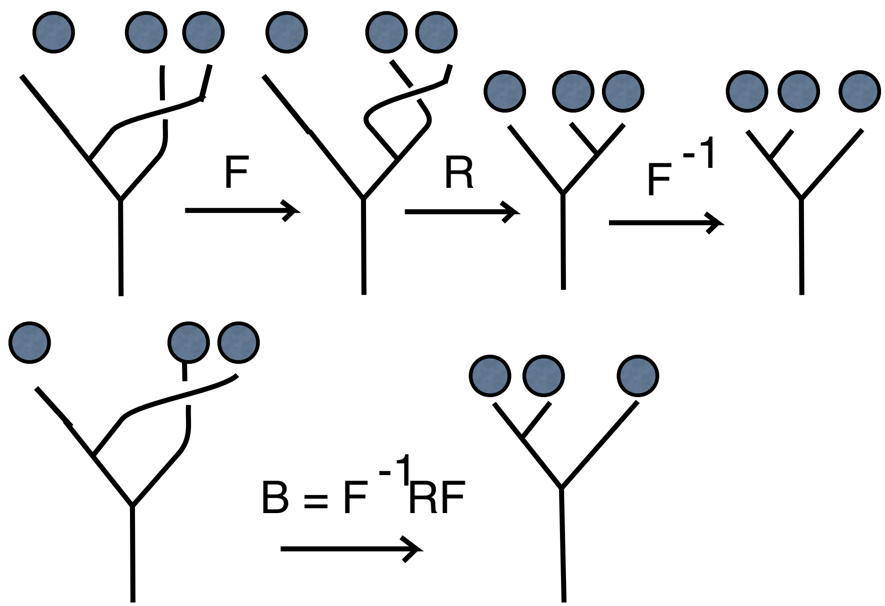


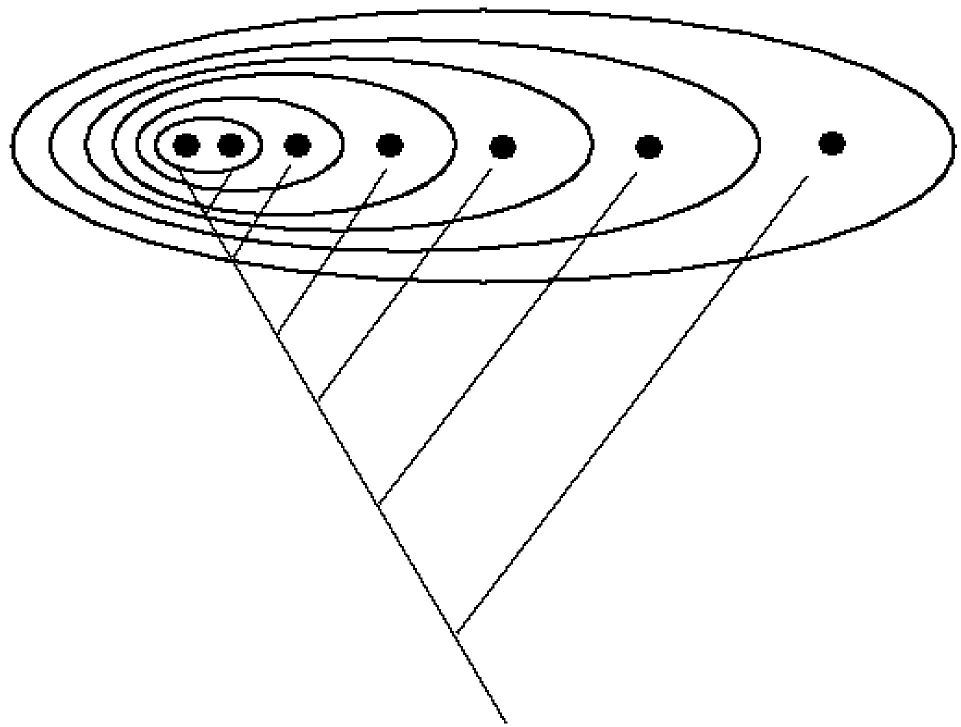
Recoupling



Process Spaces

Non-Local Braiding is Induced via Recoupling





Process Spaces Can be Abitrarily Large.
With a coherent recoupling theory, all transformations are in the representation of one braid group.

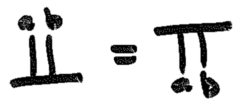
Mathematical Models for Recoupling Theory with Braiding come from a Combination of
Penrose Spin Networks and
Knot Theory.

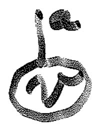
See “Temperley Lieb Recoupling Theory and Invariants of Three-Manifolds” by L. Kauffman and S. Lins, PUP, 1994.

Penrose Spin Networks

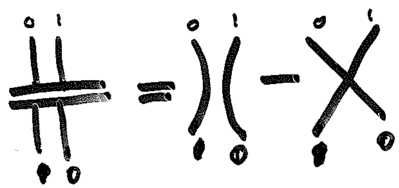
$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$SL(2,\mathbb{C}) = \left\{ A \,\middle|\, \begin{array}{l} A \text{ 2x2 matrix } \& \\ A\epsilon A^T = \epsilon \end{array} \right\}$$

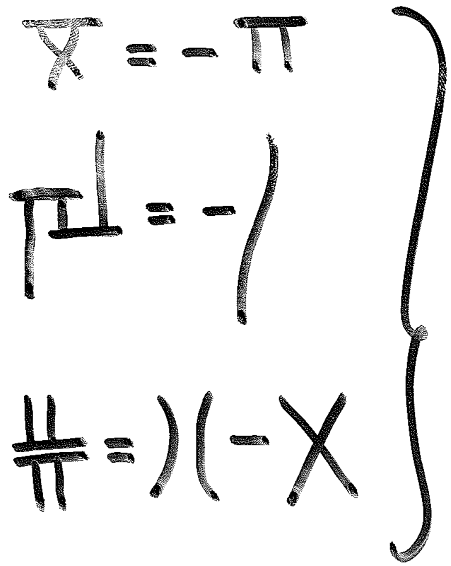
Let  $= \epsilon_{ab}$

 = vector v^a

$\langle v, w \rangle =$  $= \epsilon_{ab} v^a w^b$

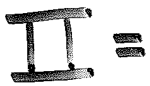
 : $\epsilon^{ab}\epsilon_{cd} = \delta^a_c \delta^b_d - \delta^a_d \delta^b_c$

Wanted diagrammatic representation that is convenient and topologically invariant in the plane.

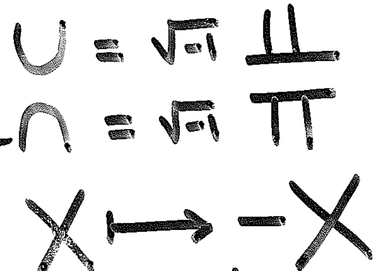


$\Pi_{ab} = \varepsilon_{ab}$

$)^a_b = \delta^a_b$

 $= \sum_{a,b} \varepsilon_{ab}\varepsilon^{ab} = 2$

Penrose: adjusts the tensors

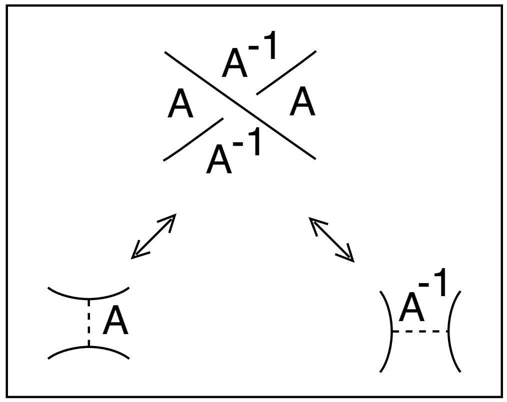


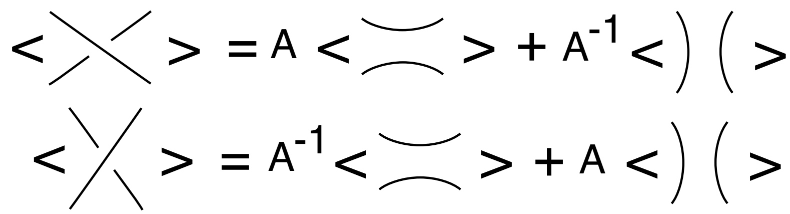
(i.e.  $= -\delta^a_d\,\delta^b_c$)

Then: 

$O = -2$

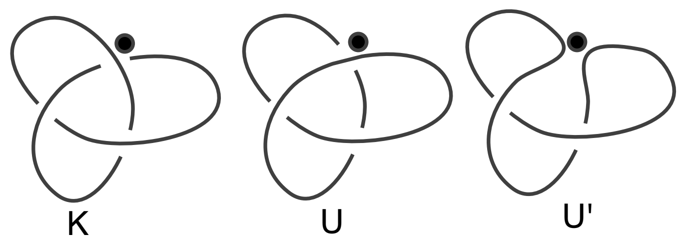
Bracket Polynomial Model for Jones Polynomial





$$< K > = \sum_{S} < K|S > \delta^{||S||-1}.$$

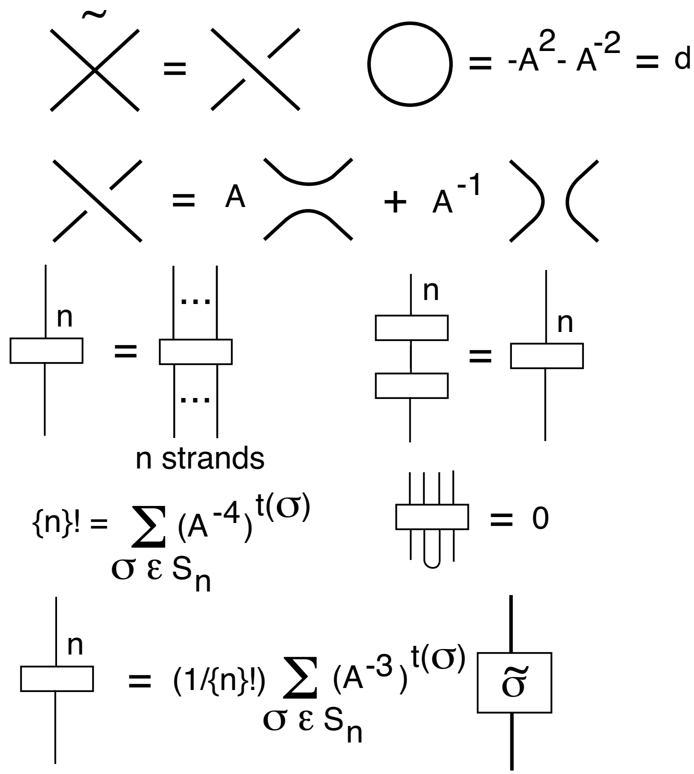
Trefoil Calculation

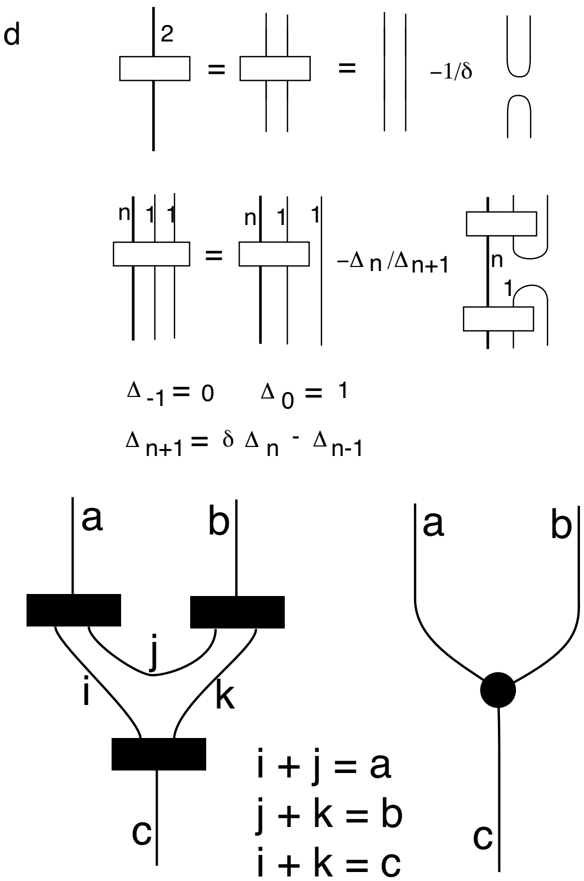


$$A^{-1} < K > -A < U >= (A^{-2} - A^2) < U' >$$

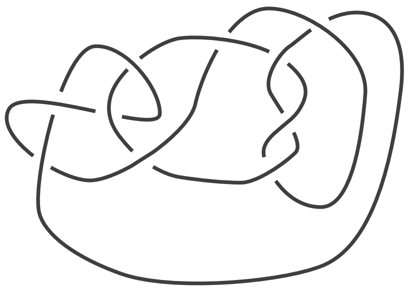
$$< U >= -A^3$$
$$< U' >= (-A^{-3})^2 = A^{-6}$$
$$< K >= -A^5 - A^{-3} + A^{-7}.$$

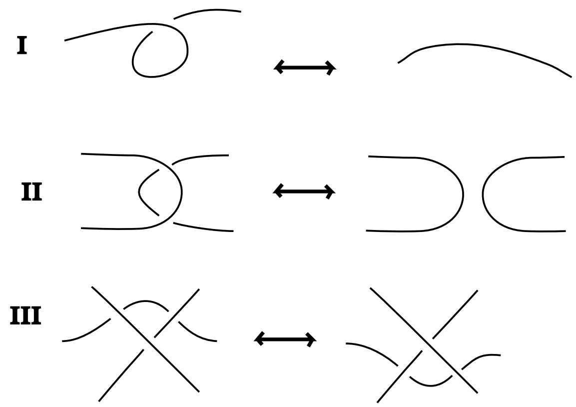
q-Deformed Spin Networks

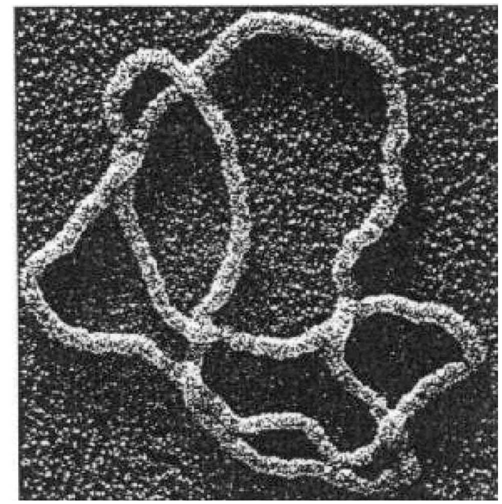


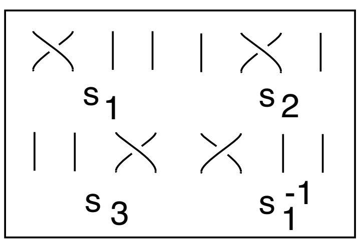


Knots and Links

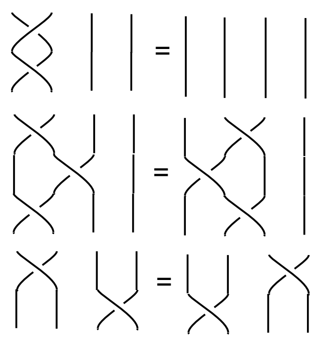








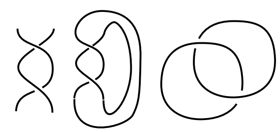
Braid Generators



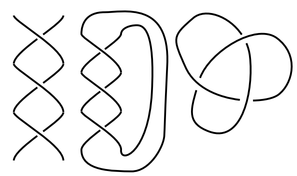
$s_1^{-1} s_1 = 1$

$s_1 s_2 s_1 = s_2 s_1 s_2$

$s_1 s_3 = s_3 s_1$



Hopf Link



Trefoil Knot

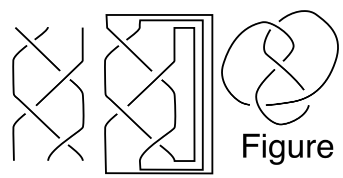
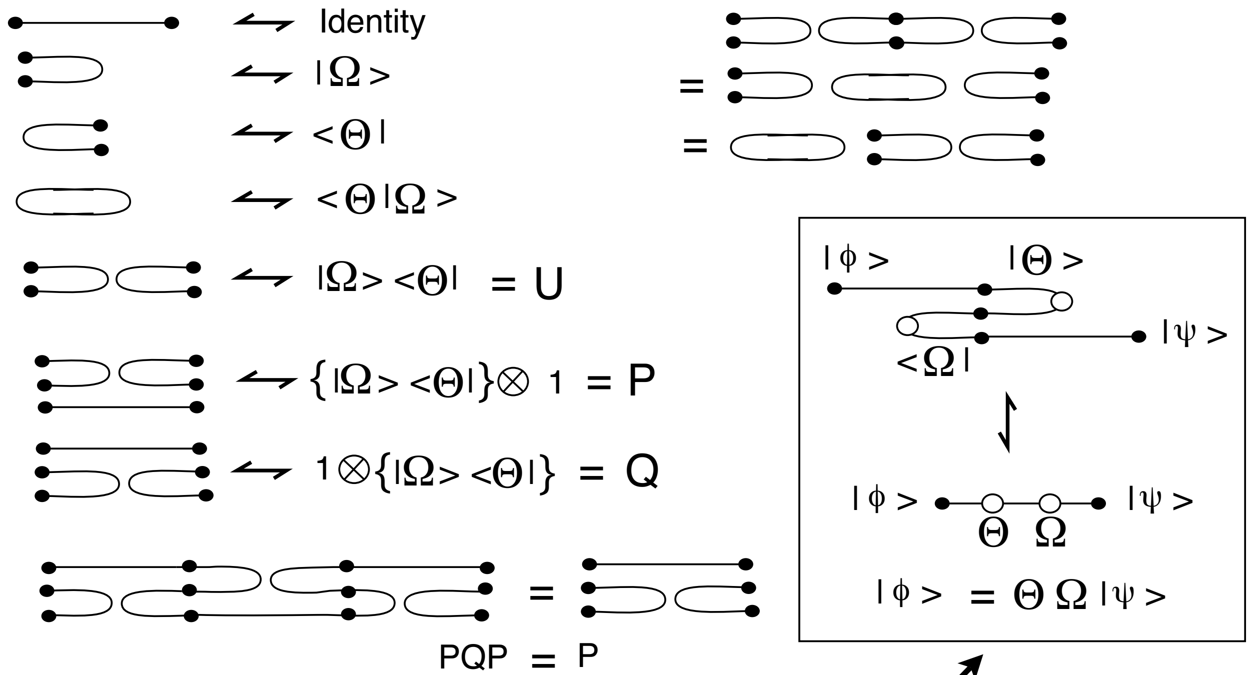


Figure Eight Knot

Temperley Lieb Category



The Key to Teleportation

Any two one-dimensional
projectors generate a
Temperley-Lieb algebra.

P = ><, Q =][

PP = ><>< = <> >< = <> P

QQ = [] Q

PQP = ><][>< = <][> >< = <][> P

QPQ = [><] Q

This trick can be used to manufacture
unitary representations of the three-strand
braid group.

Untying Knots by NMR: first experimental implementation of a quantum algorithm for approximating the Jones polynomial

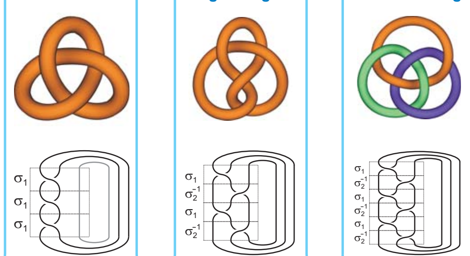
Raimund Marx[1], Andreas Spörl[1], Amr F. Fahmy[2], John M. Myers[3], Louis H. Kauffman[4], Samuel J. Lomonaco, Jr.[5], Thomas-Schulte-Herbrüggen[1], and Steffen J. Glaser[1]

[1]Department of Chemistry, Technical University Munich, Lichtenbergstr. 4, 85747 Garching, Germany
[2]Harvard Medical School, 25 Shattuck Street, Boston, MA 02115, U.S.A.
[3]Gordon McKay Laboratory, Harvard University, 29 Oxford Street, Cambridge, MA 02138, U.S.A.
[4]University of Illinois at Chicago, 851 S. Morgan Street, Chicago, IL 60607-7045, U.S.A.
[5]University of Maryland Baltimore County, 1000 Hilltop Circle, Baltimore, MD 21250, U.S.A.

roadmap of the quantum algorithm

roadmap	example #1 Trefoil	example #2 Figure-Eight	example #3 Borromean rings

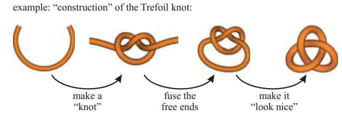
knot or link



"trace-closed" braid

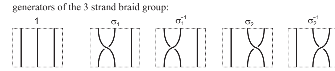
A knot is defined as a closed, non-self-intersecting curve that is embedded in three dimensions.

example: "construction" of the Trefoil knot:



J. W. Alexander proved, that any knot can be represented as a closed braid (polynomial time algorithm)

generators of the 3 strand braid group:



unitary matrix

$$U_{Trefoil} = (U_1)^3 \qquad U_{Figure-Eight} = (U_2^{-1}\cdot U_1)^2 \qquad U_{Borrom.R.} = (U_2^{-1}\cdot U_1)^3$$

$$U_1 = \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & -e^{i\theta}\frac{\sin(4\theta)}{\sin(2\theta)} + e^{-i\theta} \end{pmatrix}$$

$$U_2 = \begin{pmatrix} -e^{i\theta}\frac{\sin(6\theta)}{\sin(4\theta)} + e^{-i\theta} & -e^{i\theta}\frac{\sqrt{\sin(6\theta)\sin(2\theta)}}{\sin(4\theta)} \\ -e^{i\theta}\frac{\sqrt{\sin(6\theta)\sin(2\theta)}}{\sin(4\theta)} & -e^{i\theta}\frac{\sin(2\theta)}{\sin(4\theta)} + e^{-i\theta} \end{pmatrix}$$

It is well known in knot theory, how to obtain the unitary matrix representation of all generators of a given braid goup (see "Temperley-Lieb algebra" and "path model representation"). The unitary matrices U_1 and U_2, corresponding to the generators σ_1 and σ_2 of the 3 strand braid group are shown on the left, where the variable "θ" is related to the variable "A" of the Jones polynomial by: $A = e^{-i\theta_{rad}}$. The unitary matrix representations of σ_1^{-1} and σ_2^{-1} are given by U_1^{-1} and U_2^{-1}.

The knot or link that was expressed as a product of braid group generators can therefore also be expressed as a product of the corresponding unitary matrices.

controlled unitary matrix

Step #1:
from the 2x2 matrix U to the 4x4 matrix cU:

$$cU = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & U \end{pmatrix}$$

Step #2:
application of cU on the NMR product operator I_{1x}:

$$cU\ I_{1x}\ cU^\dagger = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & U \end{pmatrix} \frac{1}{2} \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ 0 & U \end{pmatrix}^\dagger = \frac{1}{2}\begin{pmatrix} 0 & U^\dagger \\ U & 0 \end{pmatrix}$$

Step #3:
measurement of I_{1x} and I_{1y}:

$$tr\left\{ I_{1x}\ \frac{1}{2}\begin{pmatrix} 0 & U^\dagger \\ U & 0 \end{pmatrix}\right\} = \frac{1}{2}\Re(tr\{U\})$$

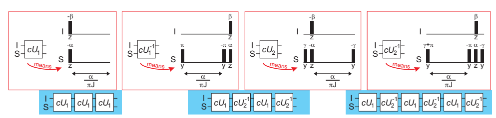
$$tr\left\{ I_{1y}\ \frac{1}{2}\begin{pmatrix} 0 & U^\dagger \\ U & 0 \end{pmatrix}\right\} = \frac{1}{2}\Im(tr\{U\})$$

Instead of applying the unitary matrix U, we apply it's controlled variant cU. This matrix is especially suited for NMR quantum computers [4] and other thermal state expectation value quantum computers: you only have to apply cU to the NMR product operator I_{1x} and measure I_{1x} and I_{1y} in order to obtain the trace of the original matrix U.

Independent of the dimension of matrix U you only need ONE extra qubit for the implementation of cU as compared to the implementation of U itself.

The measurement of I_{1x} and I_{1y} can be accomplished in one single-scan experiment.

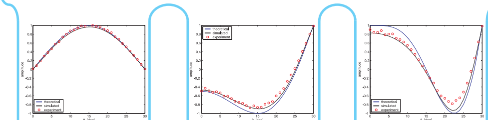
NMR pulse sequence



All knots and links can be expressed as a product of braid group generators (see above). Hence the corresponding NMR pulse sequence can also be expressed as a sequence of NMR pulse sequence blocks, where each block corresponds to the controlled unitary matrix cU of one braid group generator.

This modular approach allows for an easy optimization of the NMR pulse sequences: only a small and limited number of pulse sequence blocks have to be optimized.

NMR experiment



Comparison of experimental results, theoretical predictions, and simulated experiments, where realisitic inperfections like relaxation, B_1 field inhomogeneity, and finite length of the pulses are included.

For each data point, four single-scan NMR experiments have been performed: measurement of I_{1x}, measurement of I_{1y}, reference for I_{1x}, and reference for I_{1y}. If necessary each data point can also be obtained in one single-scan experiment by measuring amplitude and phase in a referenced setting.

Jones polynomial

Jones Polynomial "Trefoil":
$(A^{-4} + A^{-12} - A^{-16})\cdot(-A^2 - A^{-2})$

Jones Polynomial "Figure-Eight":
$+A^8 - A^4 + A^{-8} - A^{-4} + A^0$

Jones Polynomial "Borromean rings":
$-A^3 + 3A^2 - 2A^1 - A^{-3} + 3A^{-2} - 2A^{-1} + 4A^0$

The Jones Polynomials can be reconstructed out of the NMR experiments by:

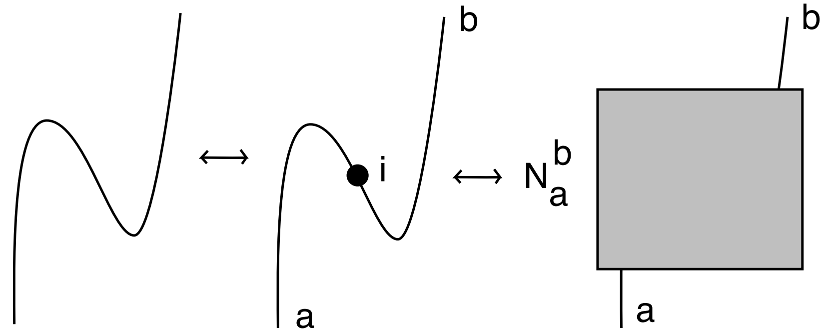
$$V_L(A) = (-A^3)^{-w(L)}\left(tr\{U\} + A^{I(L)}[(-A^2 - A^{-2})^2 - 2]\right)$$

where:
$w(L)$ is the writhe of the knot or link L
$tr\{U\}$ is determined by the NMR experiments
$I(L)$ is the sum of exponents in the braid word corresponding to the knot or link L

References:

1) 1) L. Kauffman, AMS Contemp. Math. Series, 305, edited by S. J. Lomonaco, (2002), 101-137 (math.QA/0105255)
2) R. Marx, A. Spörl, A. F. Fahmy, J. M. Myers, L. H. Kauffman, S. J. Lomonaco, Jr., T. Schulte-Herbrüggen, and S. J. Glaser: paper in preparation
3) Vaughan F. R. Jones, Bull. Amer. Math. Soc., (1985), no. 1, 103-111
4) J. M. Myers, A. F. Fahmy, S. J. Glaser, R. Marx, Phys. Rev. A, (2001), 63, 032302 (quant-ph/0007043)
5) D. Aharonov, V. Jones, Z. Landau, Proceedings of the STOC 2006, (2006), 427-436 (quant-ph/0511096)
6) M. H. Freedman, A. Kitaev, Z. Wang, Commun. Math. Phys., (2002), 227, 587-622

Diagrammatic Matrices, Knots and Teleportation



$$N_a^b = M_{ai} M^{ib}$$

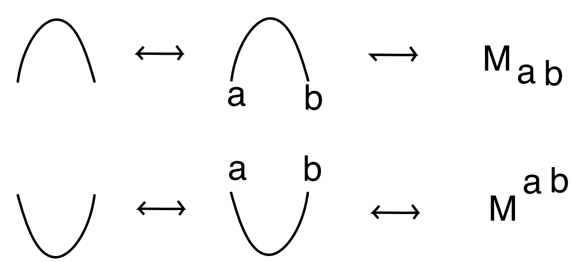
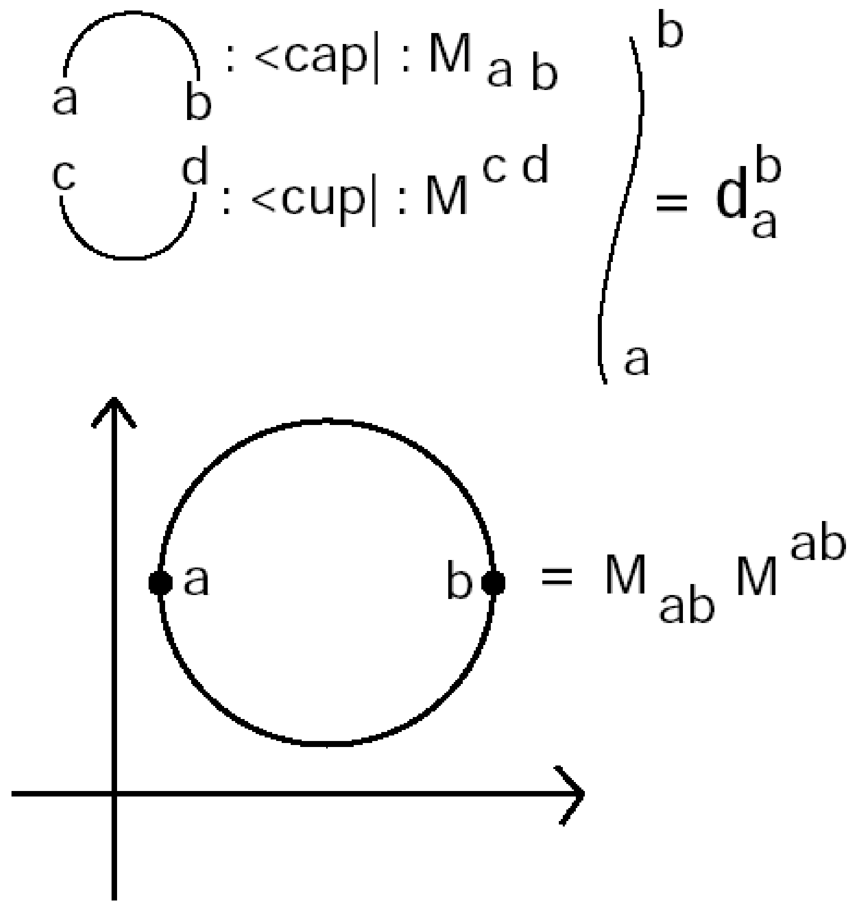
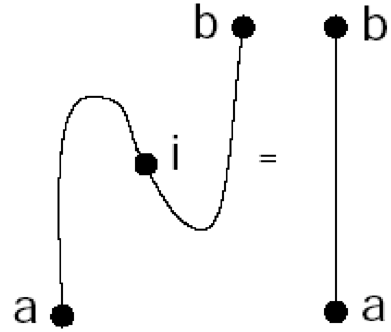
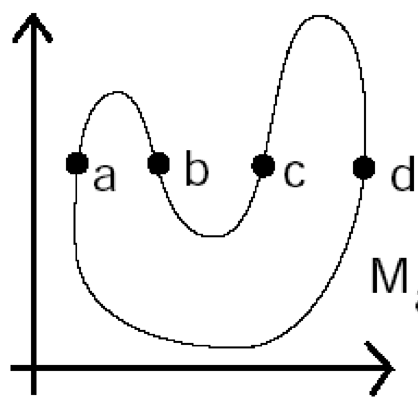


Figure 5 - Matrix Composition

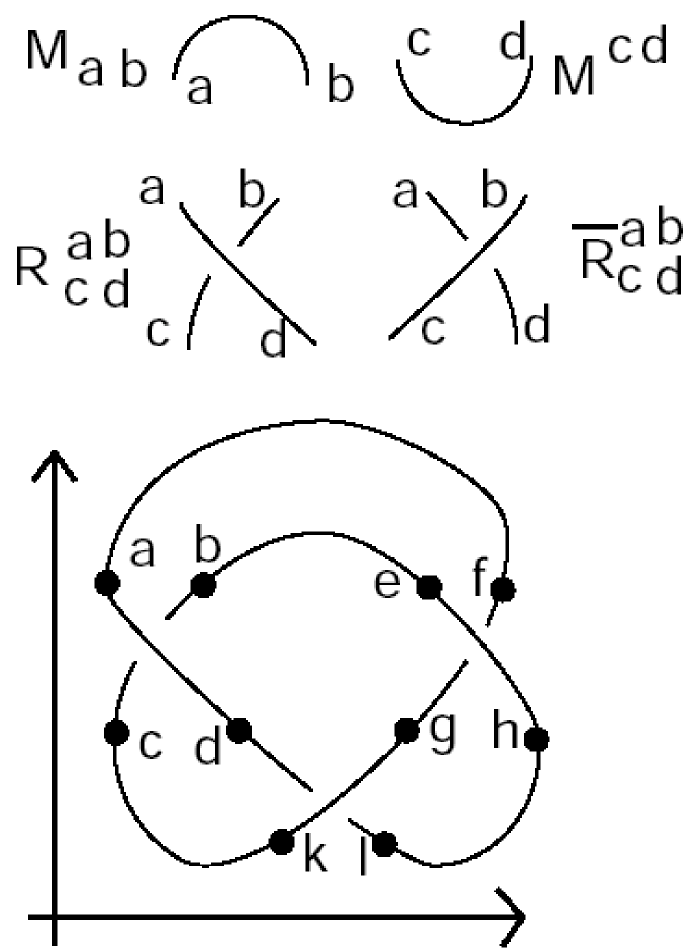

a
b
: <cap| : M a b
c
d
: <cup| : M c d
b
= d_a^b
a
a
b
= $M_{ab} M^{ab}$



$M_{ai} M^{ib} = d_a^b$

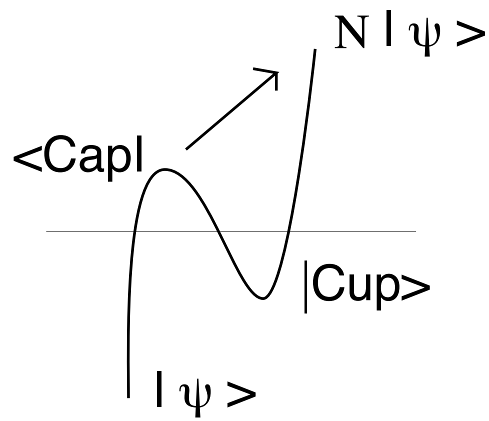


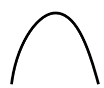
$M_{ab} M^{bc} M_{cd} M^{ad}$

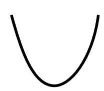


$$Z_K = M_{af} M_{be} M^{ck} M^{lh} R^{ab}_{cd} R^{ef}_{gh} \overline{R}^{dg}_{kl}$$

State and Matrix Duality



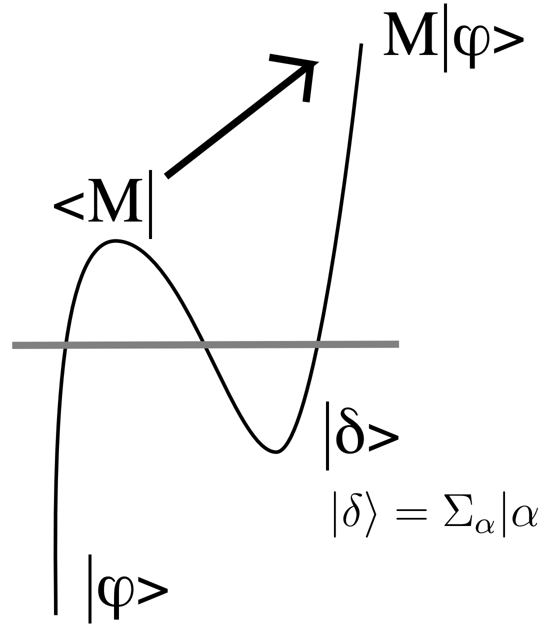
 $\longleftrightarrow \quad \langle Cap| = \sum_{a,i} M_{a,i} \langle a| \langle i|$

 $\longleftrightarrow \quad |Cup\rangle = \sum_{i,b} M^{i,b} |i\rangle |b\rangle$

$$N_a^b = \sum_i M_{ai} \, M^{ib}$$

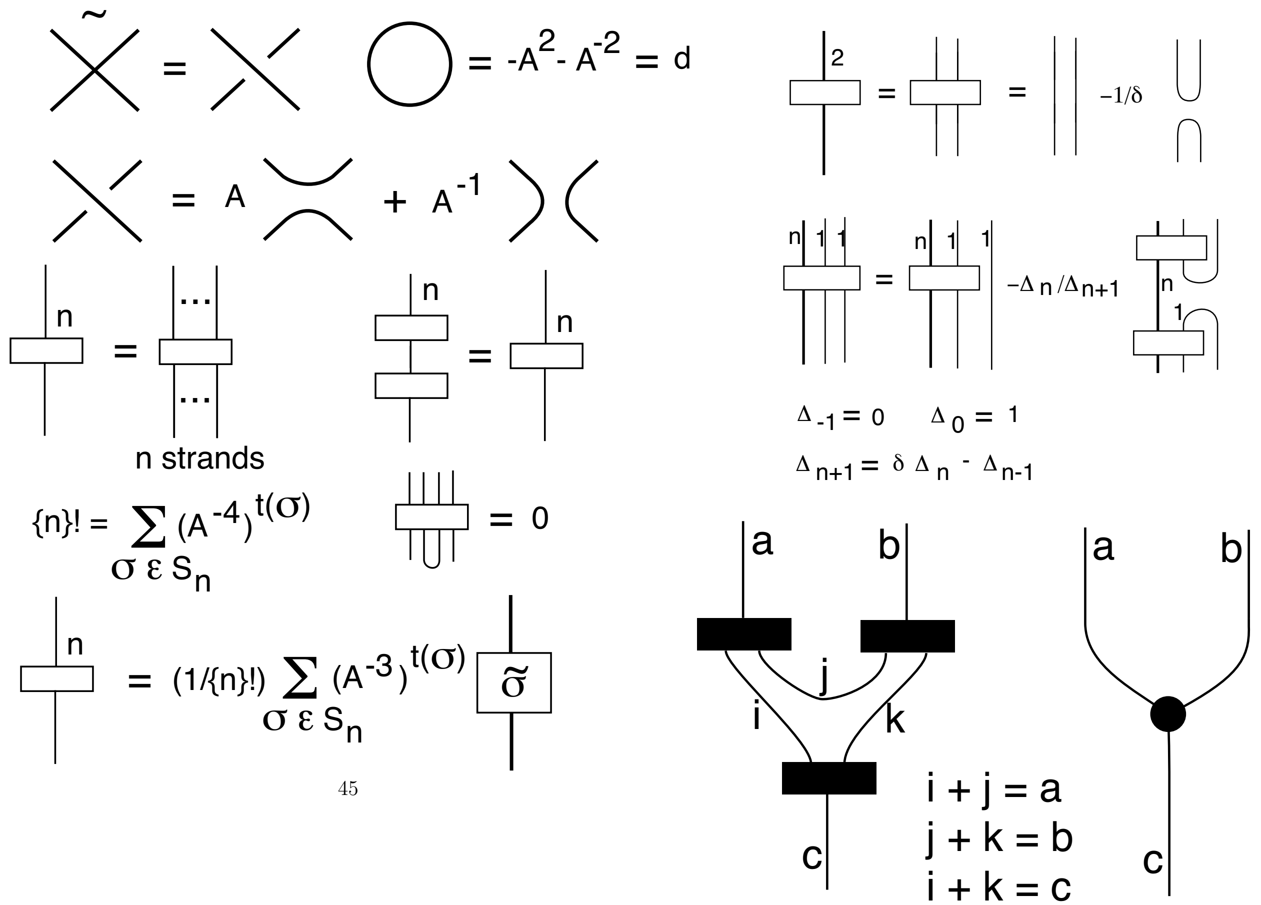
The Topology of Teleportation

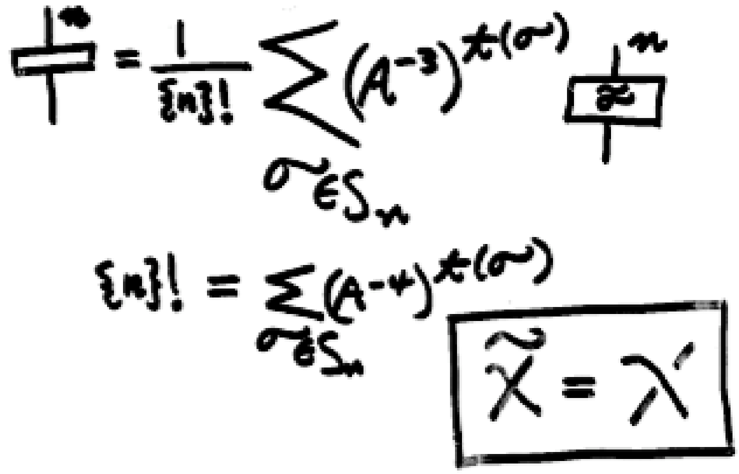
|00> + |11>   <---->   1 0
                                  0 1

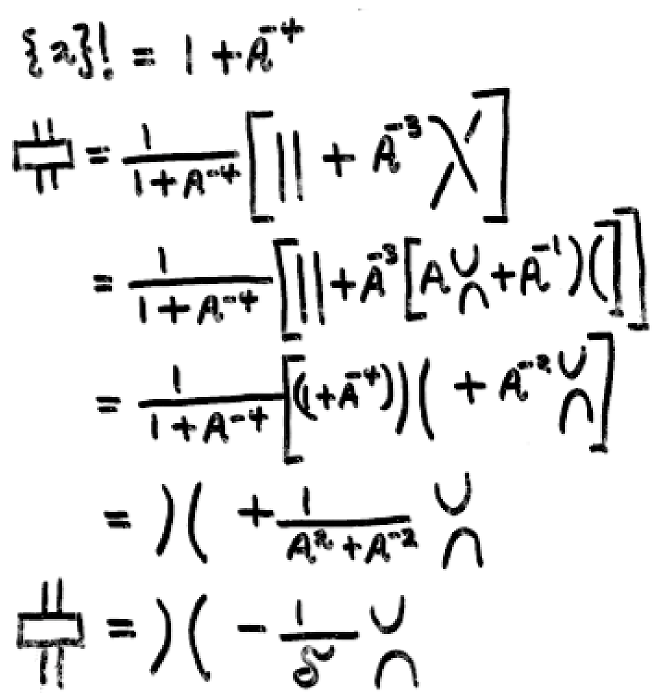


$$|\delta\rangle = \Sigma_\alpha |\alpha, \alpha\rangle \in H \otimes H.$$

q-Deformed Spin Networks

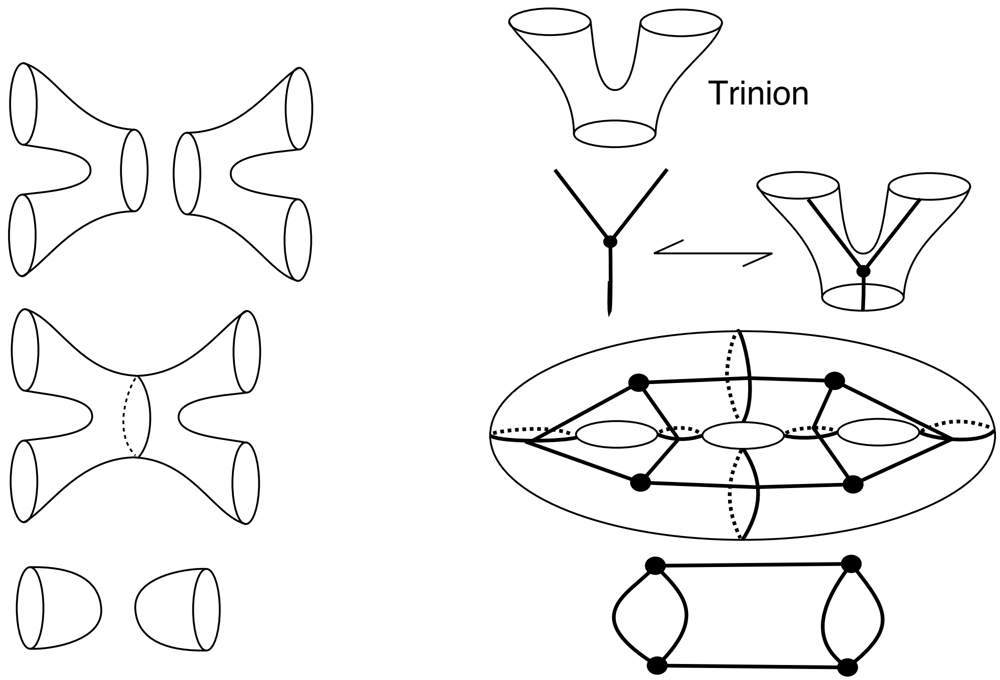






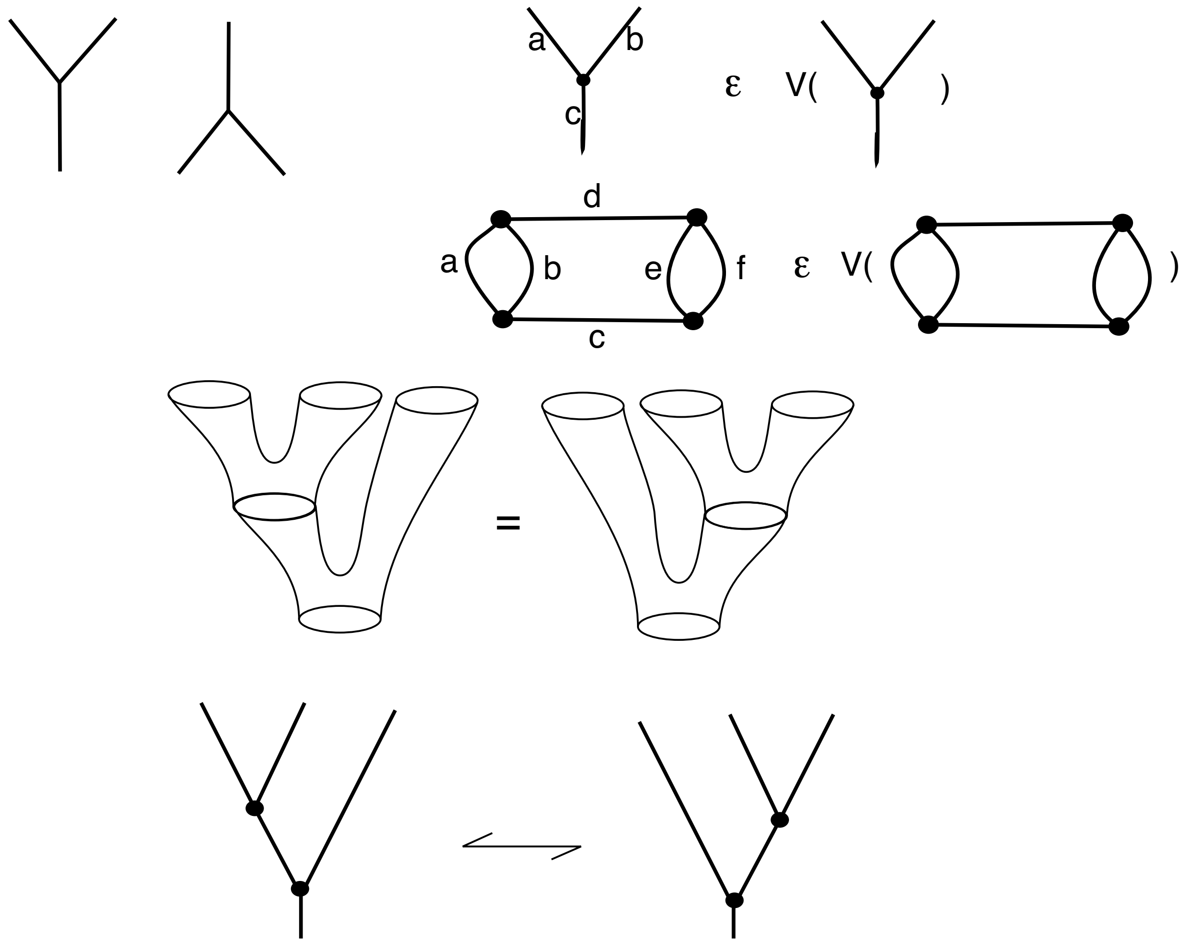
Projectors are Sums over Permuations, Lifted to Braids and Expanded via the Bracket into the Temperley Lieb Algebra

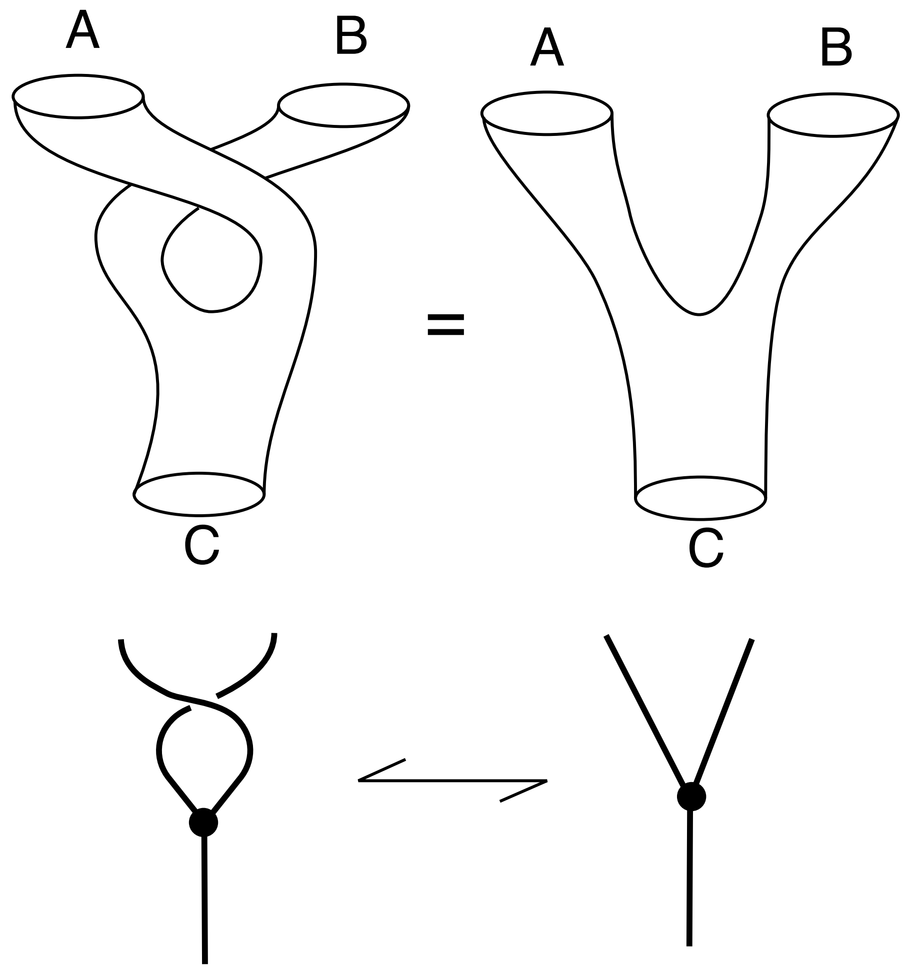
Topological Quantum Field Theory



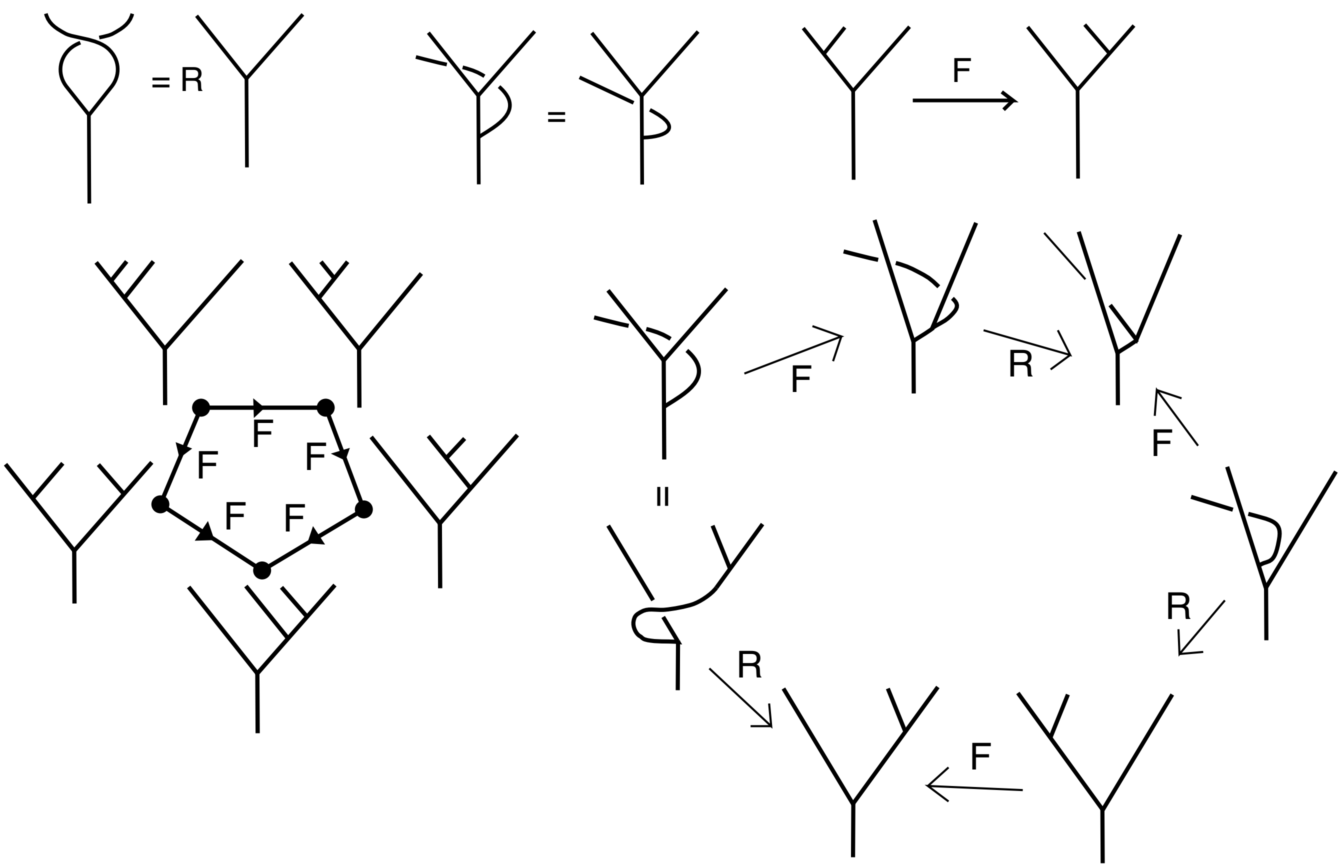
Process Spaces on Surfaces Lead to Three-Manifold Invariants.

Process Vector Spaces and Recoupling

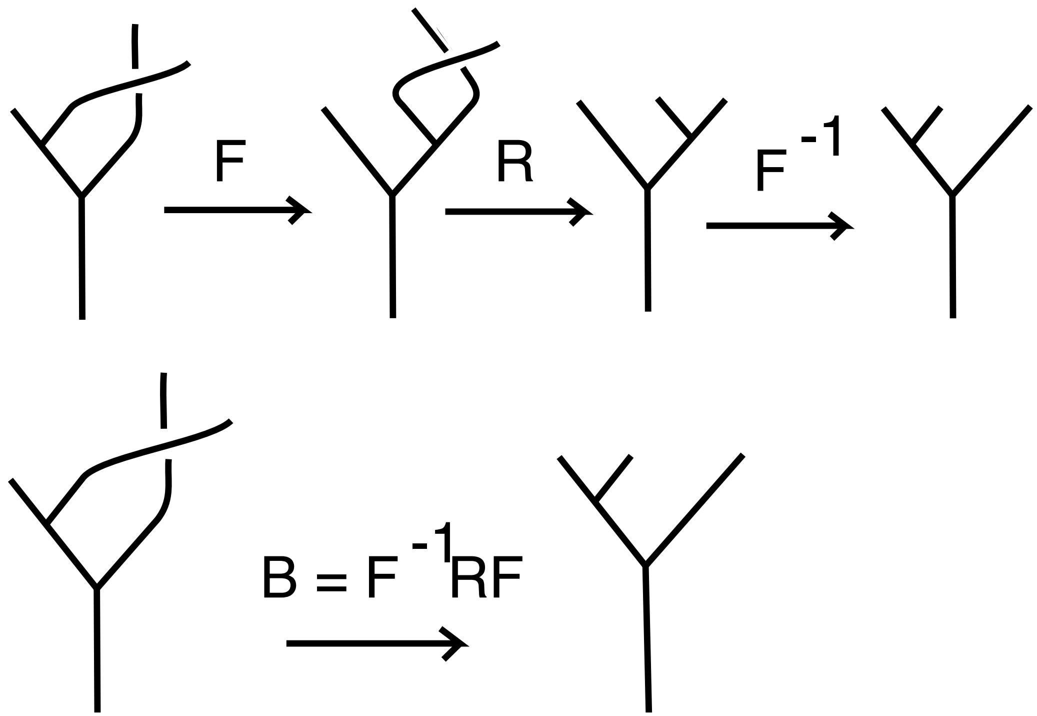



A
B
A
B
=
C
C

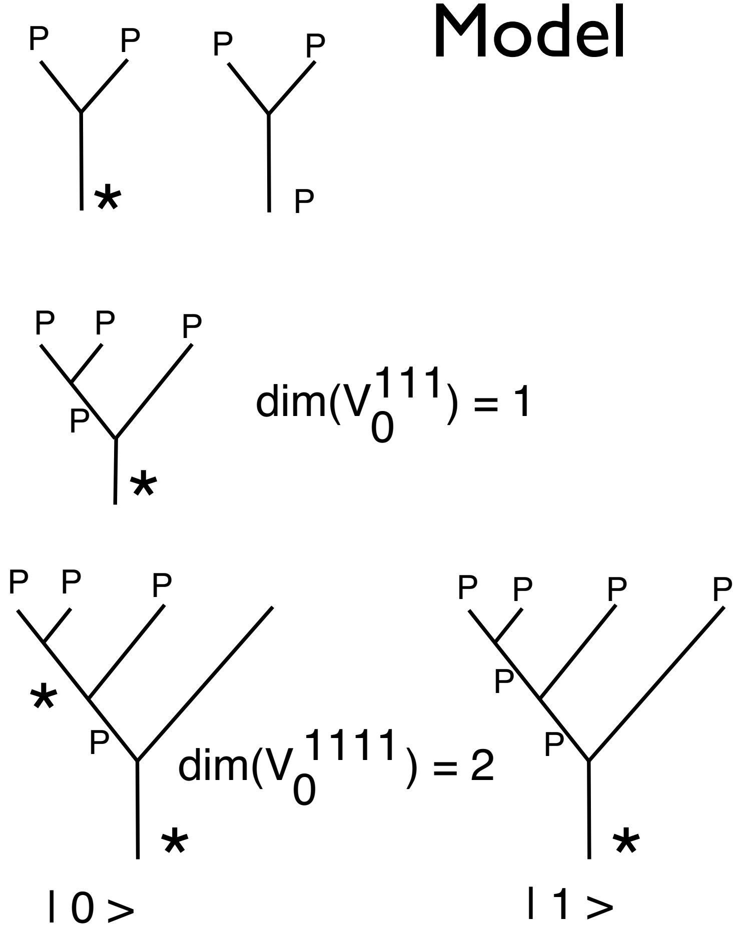
Braiding, Naturality, Recoupling, Pentagon and Hexagon -- Automatic Consequences of the Constuction



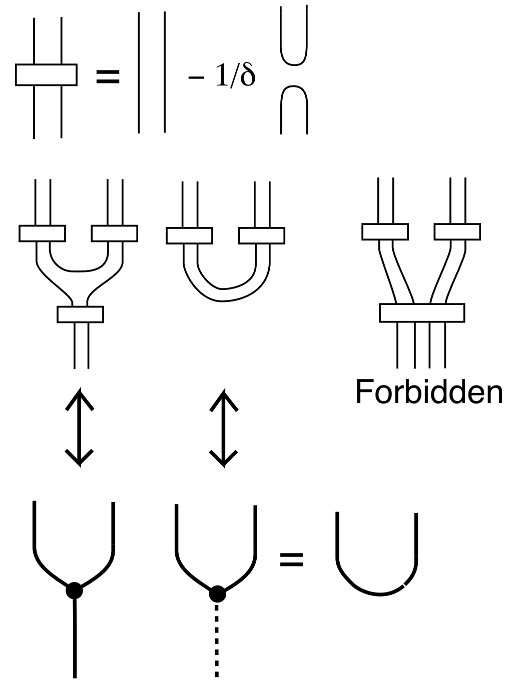
Non-Local Braiding is Induced via Recoupling



Fibonacci Model



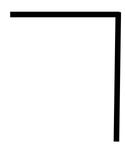
$$A = e^{3\pi i/5}.$$



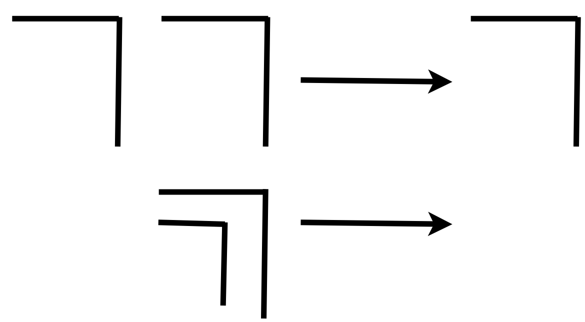
Temperley Lieb Representation of Fibonacci Model

ICONICS

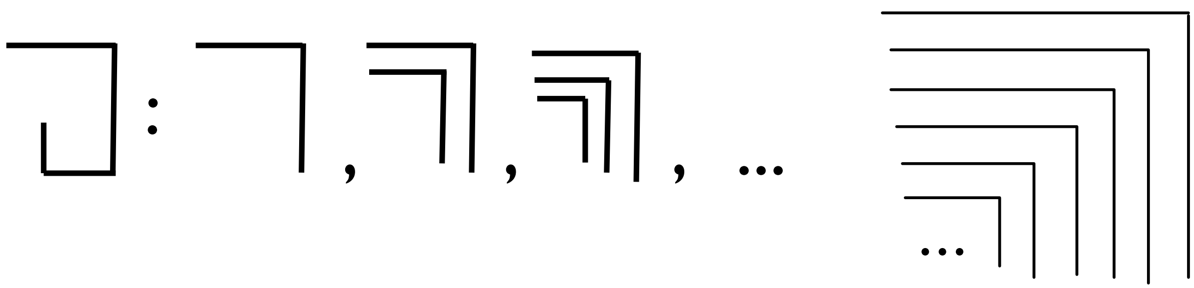
In the Fibonacci Model we have
one “particle” P that interacts
itself to produce either P or * (nothing).
This is analogous to the logical particle
representing an act of distinction
(or registration, measurement, ...)

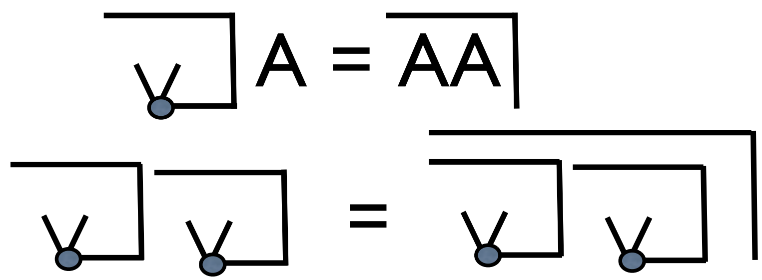


of G. Spencer-Brown
that interacts with itself in two ways:

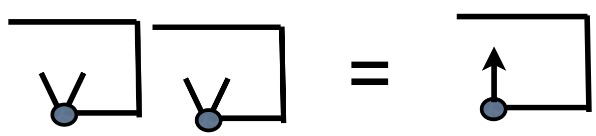


Digression: The Re-entering Mark as Iconic for Recursive Trace and Lambda Calculus Fixed Point.

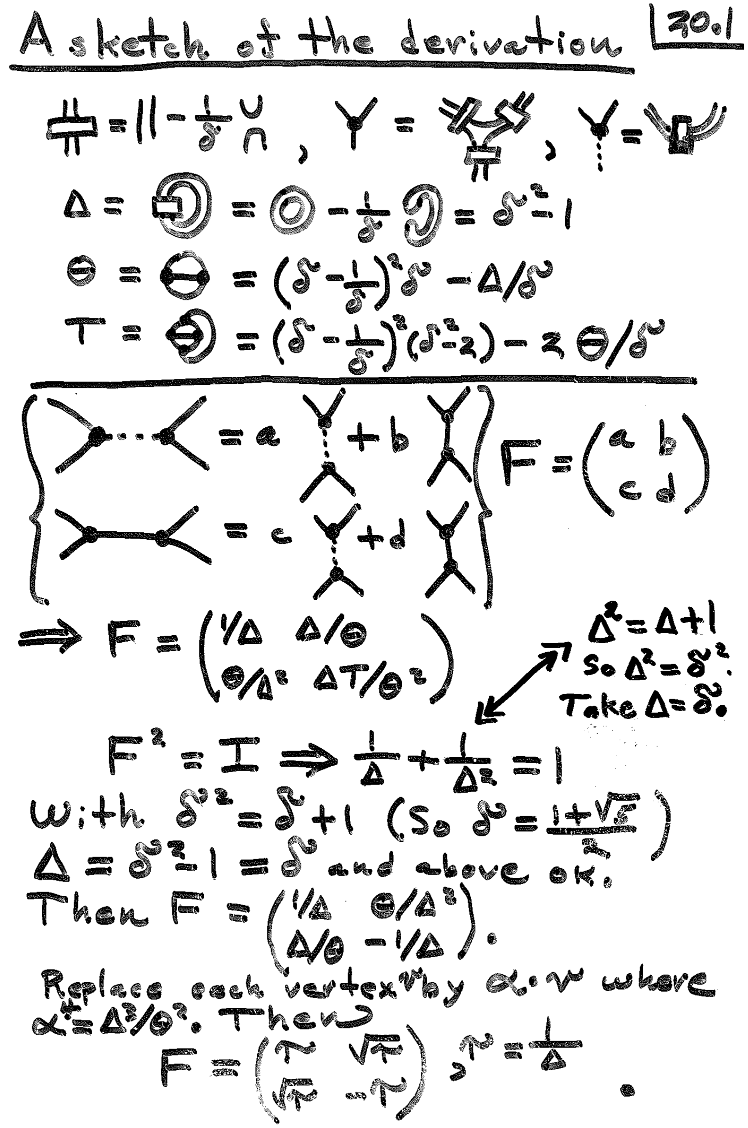




Hence



Fibonacci Model



The Simple, yet Quantum Universal, Structure of the Fibonacci Model

$$A = e^{3\pi i/5}.$$

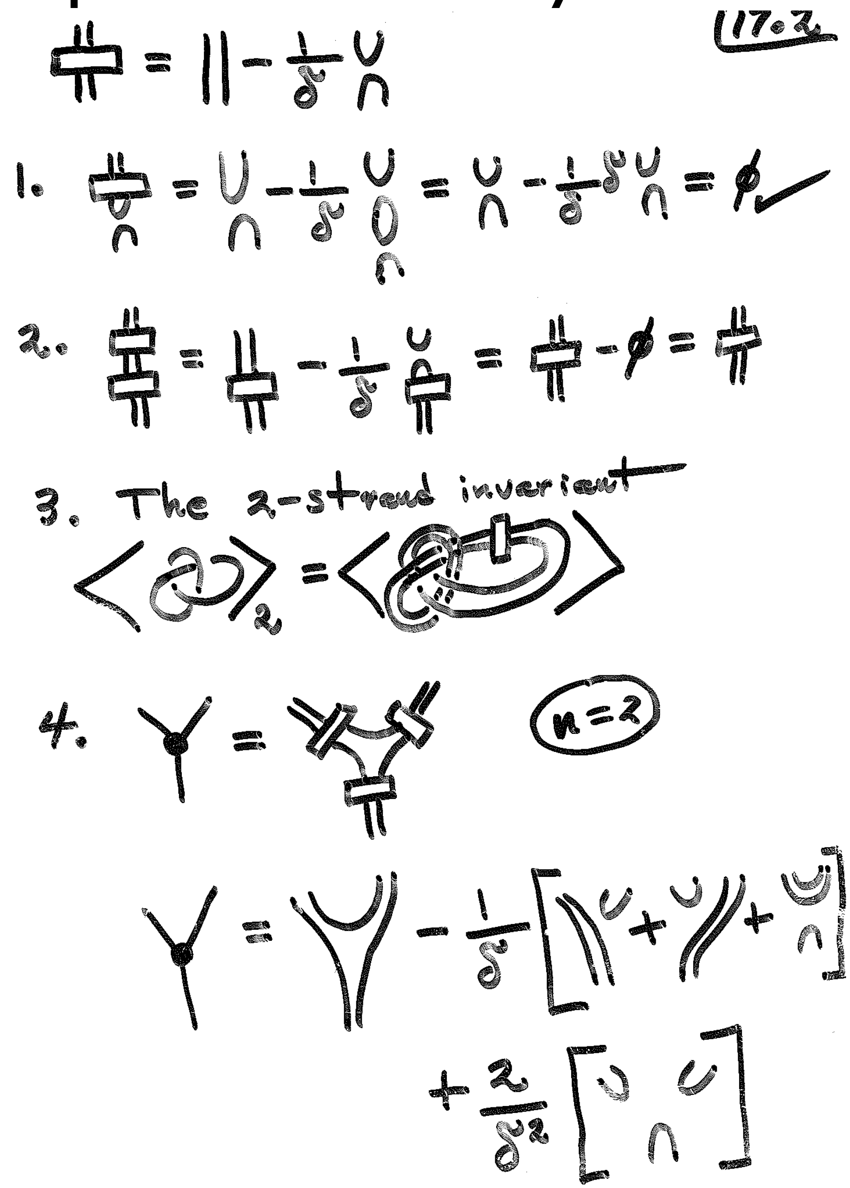
$$\delta = -A^2 - A^{-2}$$

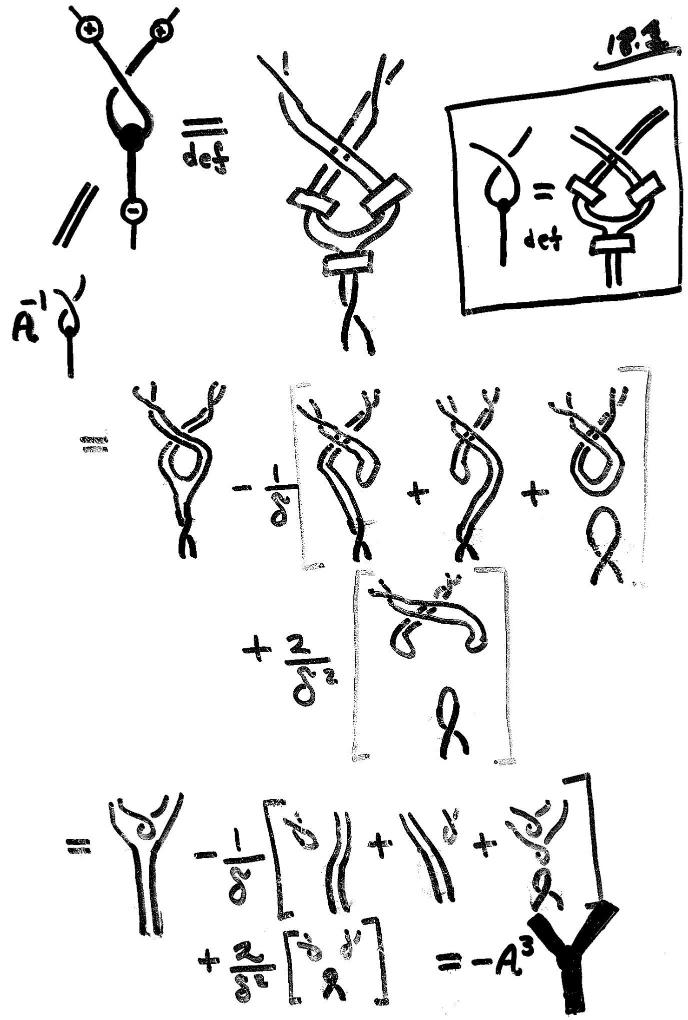
$$\Delta = \delta = (1+\sqrt{5})/2.$$

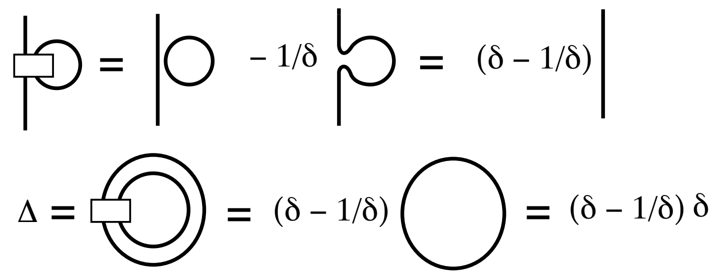
$$F = \begin{pmatrix} 1/\Delta & 1/\sqrt{\Delta} \\ 1/\sqrt{\Delta} & -1/\Delta \end{pmatrix} = \begin{pmatrix} \tau & \sqrt{\tau} \\ \sqrt{\tau} & -\tau \end{pmatrix}$$

$$R = \begin{pmatrix} -A^4 & 0 \\ 0 & A^8 \end{pmatrix} = \begin{pmatrix} e^{4\pi i/5} & 0 \\ 0 & -e^{2\pi i/5} \end{pmatrix}.$$

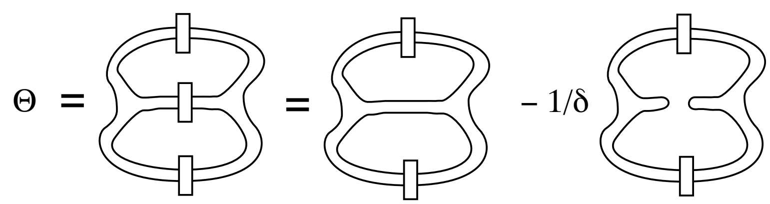
Spin Network Gymnastics



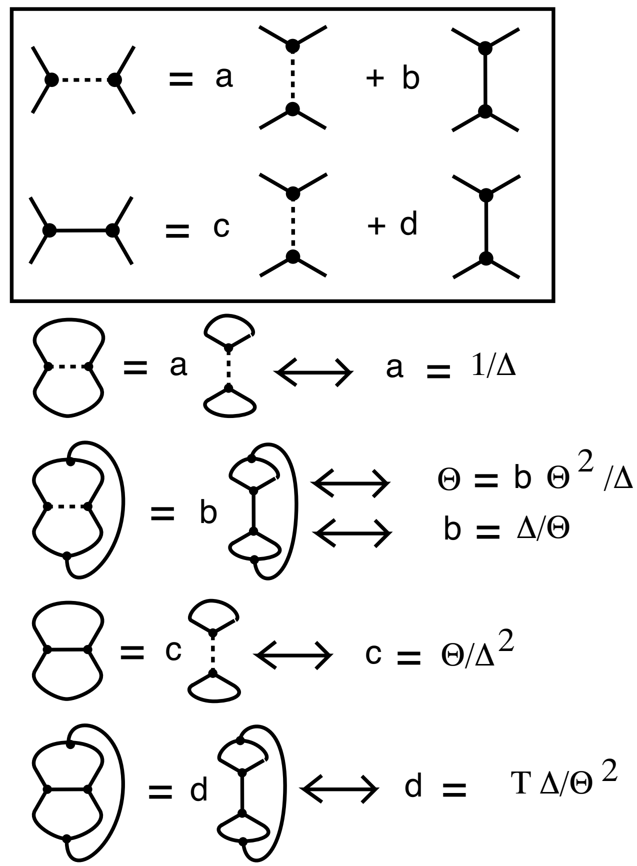

def
def
A^{-1}
$-\frac{1}{\delta}$
$+\frac{2}{\delta^2}$
$-\frac{1}{\delta}$
$+\frac{2}{\delta^2}$
$= -A^3$



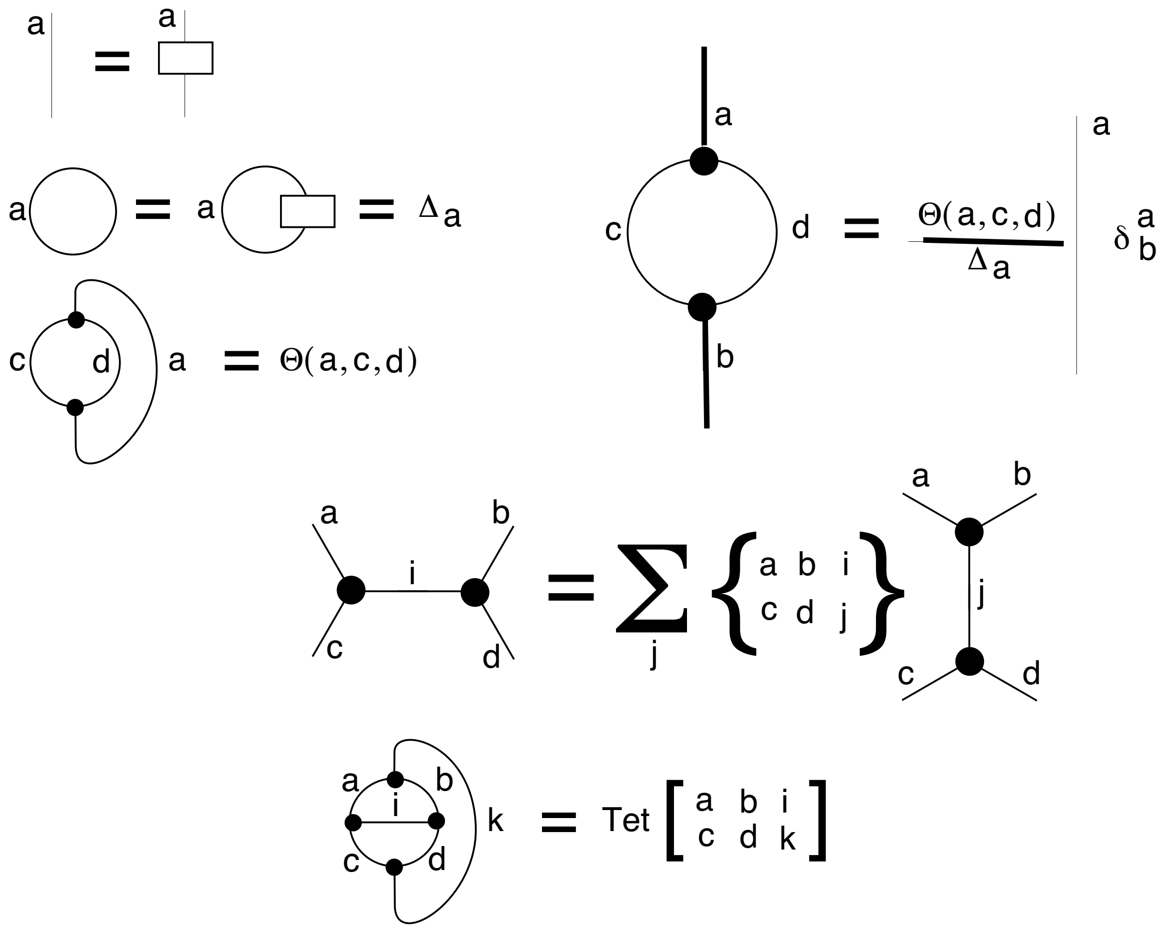
$$\Delta = \delta^2 - 1$$



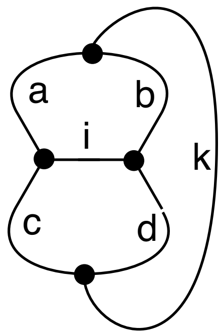
$$\Theta = (\delta - 1/\delta)^2\,\delta - \Delta/\delta$$


= a
+ b
= c
+ d
= a
a = 1/Δ
= b
Θ = b Θ 2 /Δ
b = Δ/Θ
= c
c = Θ/Δ 2
= d
d = T Δ/Θ 2

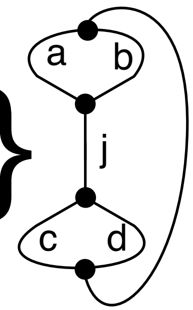
Closure, Bubble and Recoupling



The 6-j Coefficients



$$= \sum_j \left\{ \begin{matrix} a & b & i \\ c & d & j \end{matrix} \right\}$$

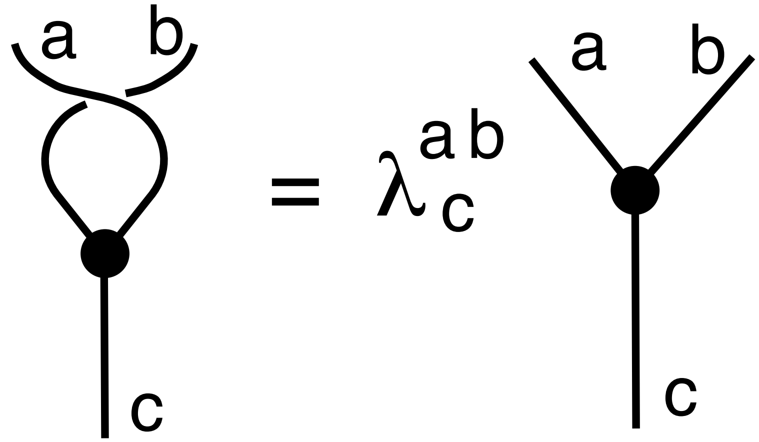


$$= \sum_j \left\{ \begin{matrix} a & b & i \\ c & d & j \end{matrix} \right\} \frac{\Theta(a,b,j)}{\Delta_j} \frac{\Theta(c,d,j)}{\Delta_j} \Delta_j \, \delta_j^k$$

$$= \left\{ \begin{matrix} a & b & i \\ c & d & k \end{matrix} \right\} \frac{\Theta(a,b,k)\, \Theta(c,d,k)}{\Delta_k}$$

$$\left\{ \begin{matrix} a & b & i \\ c & d & k \end{matrix} \right\} = \frac{\mathrm{Tet}\left[\begin{matrix} a & b & i \\ c & d & k \end{matrix} \right] \Delta_k}{\Theta(a,b,k)\;\Theta(c,d,k)}$$

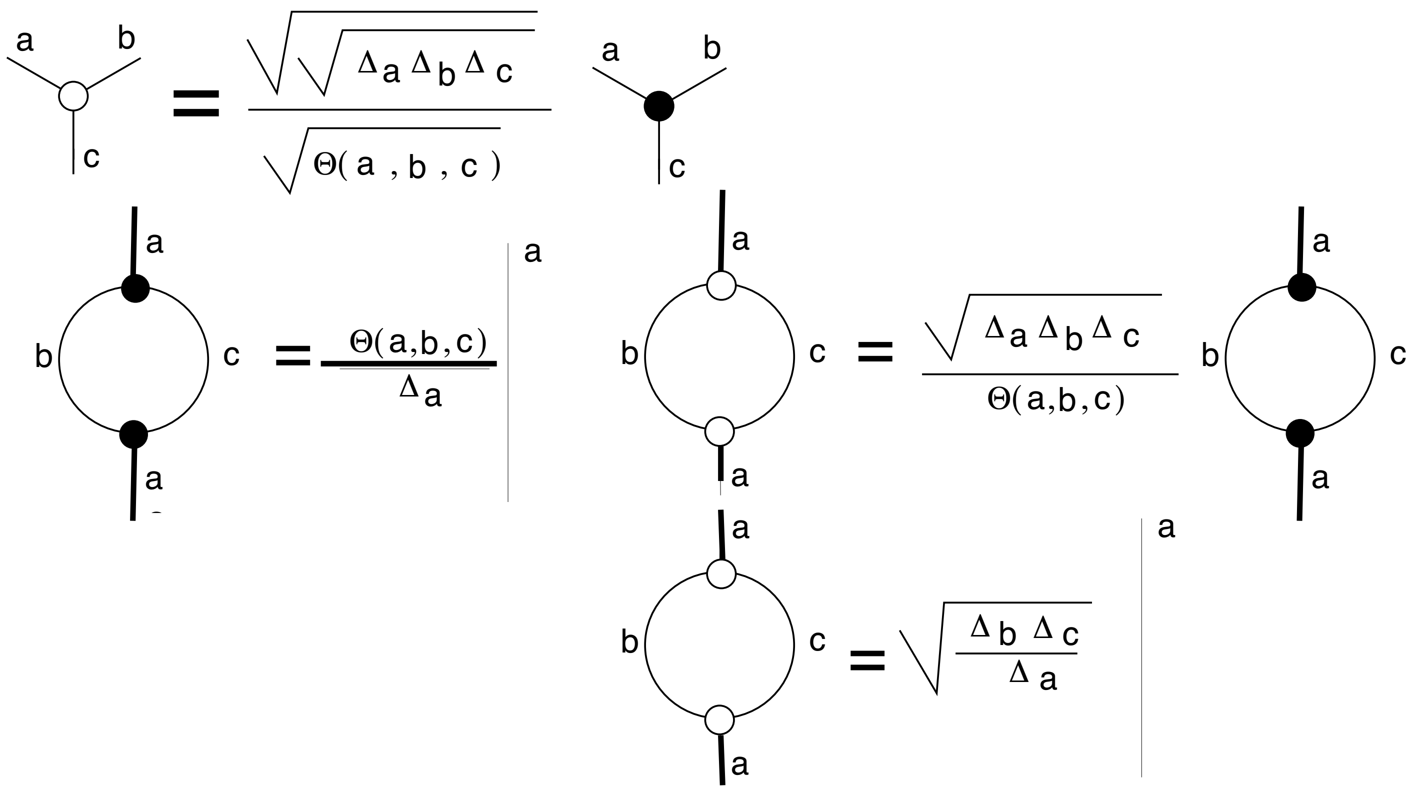
Local Braiding



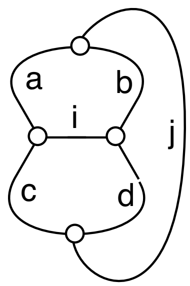
$$\lambda_c^{ab} = (-1)^{(a+b-c)/2} A^{(a'+b'-c')/2}$$

$$x' = x(x+2)$$

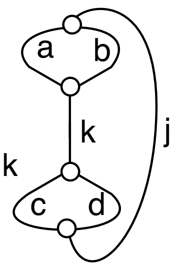
Redefining the Vertex is the key to obtaining Unitary Recoupling Transformations.



New Recoupling Formula



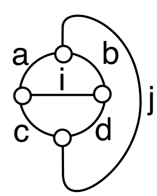
$$= \sum_k \boxed{\begin{matrix} a & b \\ c & d \end{matrix}}_{i\,k}$$



$$= \sum_k \boxed{\begin{matrix} a & b \\ c & d \end{matrix}}_{i\,k} \sqrt{\frac{\Delta_a \Delta_b}{\Delta_j}} \sqrt{\frac{\Delta_c \Delta_d}{\Delta_j}} \; \Delta_j \, \delta_j^k$$

$$= \boxed{\begin{matrix} a & b \\ c & d \end{matrix}}_{i\,j} \sqrt{\frac{\Delta_a \Delta_b}{\Delta_j}} \sqrt{\frac{\Delta_c \Delta_d}{\Delta_j}} \; \Delta_j$$

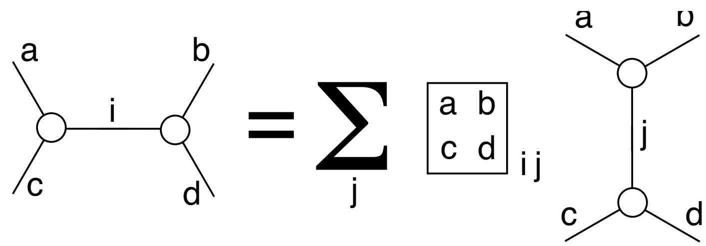
$$\boxed{\begin{matrix} a & b \\ c & d \end{matrix}}_{i\,j} =$$

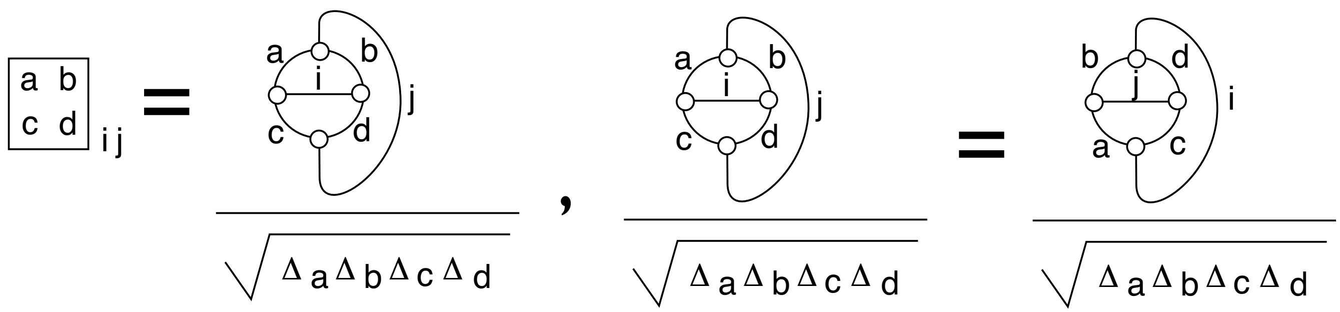


$$\overline{\sqrt{\frac{\Delta_a \Delta_b}{\Delta_j}} \sqrt{\frac{\Delta_c \Delta_d}{\Delta_j}} \; \Delta_j}$$

$$= \frac{\text{ModTet}\begin{bmatrix} a & b & i \\ c & d & j \end{bmatrix}}{\sqrt{\Delta_a \Delta_b \Delta_c \Delta_d}}$$

The Recoupling Matrix is Real Unitary at Roots of Unity.





$$M[a,b,c,d]_{ij} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{ij}$$

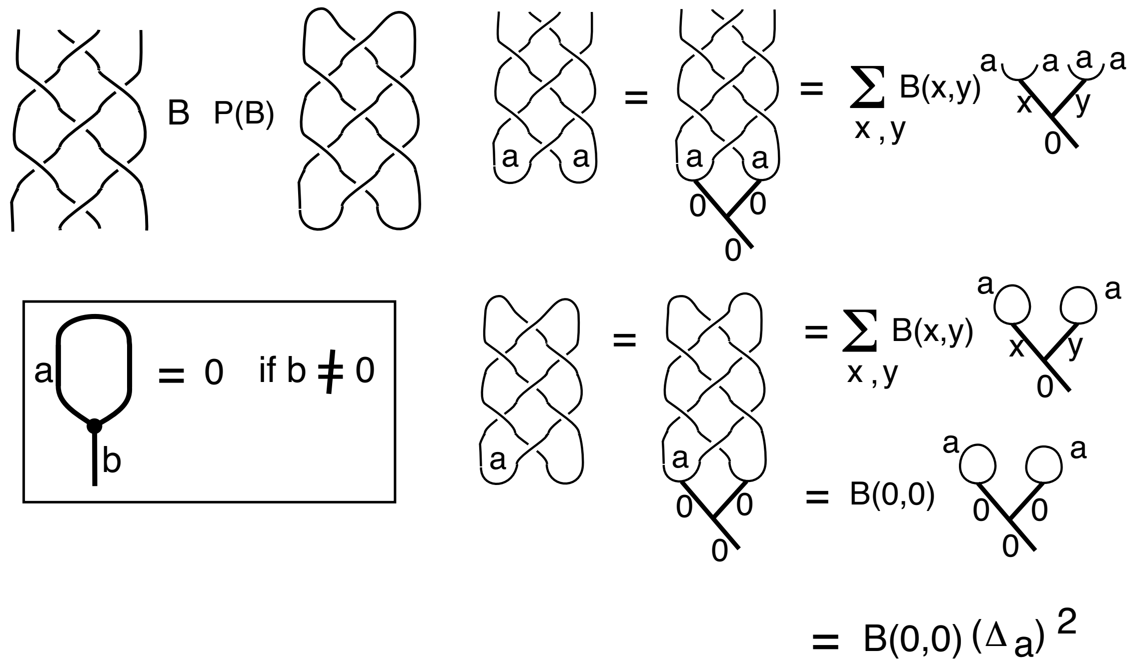
$$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{T} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$$

Theorem. Unitary Representations of the Braid Group come from Temperley Lieb Recoupling Theory at roots of unity.

$$A = e^{i\pi/2r}$$

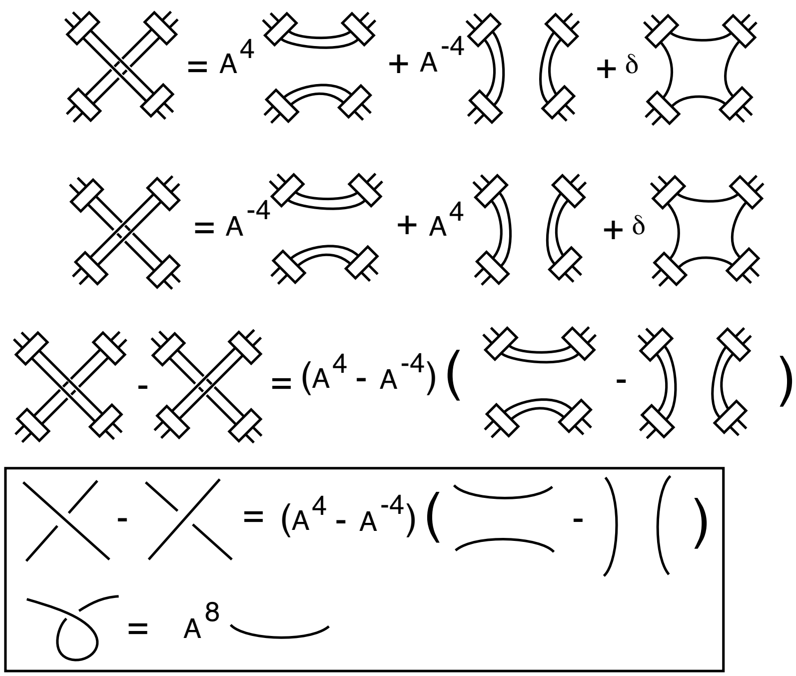
Sufficient to Produce Enough Unitary Transformations for Quantum Computing.

Quantum Computation of Colored Jones Polynomials and WRT invariants.



Need to compute a diagonal element of a unitary transformation.
Use the Hadamard Test.

Colored Jones Polynomial for n = 2 is Specialization of the Dubrovnik version of Kauffman polynomial.



Will these models actually be used
for quantum computation?
Will quantum computation actually happen?
Will topology play a key role?
Time will tell.