Characterization of Revenue Monotonicity in Combinatorial Auctions

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Abstract—An auction mechanism consists of an allocation rule and a payment rule. There have been several studies on characterizing strategy-proof allocation rules; if the allocation rule satisfies a condition called weak-monotonicity, an appropriate payment rule is guaranteed to exist. One desirable property that an auction mechanism should satisfy is revenue monotonicity; a seller's revenue is guaranteed to weakly increase as the number of bidders grows. In this paper, we first identify a simple condition called summation-monotonicity for characterizing strategy-proof and revenue monotone allocation rules. To the best of our knowledge, this is the first attempt to characterize revenue monotone allocation rules. Based on this characterization, we also examine the connections between revenue monotonicity and false-name-proofness, which means a bidder cannot increase his utility by submitting multiple bids under fictitious names. In a single-item auction, we show that they are basically equivalent; a mechanism is false-name-proof if and only if it is strategy-proof and revenue monotone. On the other hand, we show these two conditions cannot coexist in combinatorial auctions under some minor condition.

Keywords-Combinatorial auctions, game theory, revenue

I. INTRODUCTION

Mechanism design of combinatorial auctions has become an integral part of Electronic Commerce and a promising field for applying AI and agent technologies. Among various studies related to Internet auctions, those on combinatorial auctions have lately attracted considerable attention.

One desirable property of an auction mechanism is that it is *strategy-proof*. A mechanism is strategy-proof if, for each bidder, reporting his true valuation is a *dominant strategy*, i.e., an optimal strategy regardless of the actions of other bidders. In theory, the revelation principle states that in the design of a mechanism, we can restrict our attention to strategy-proof mechanisms without loss of generality [2]. In other words, if a certain property, e.g., Pareto efficiency or high revenue, can be achieved using a mechanism in a dominant strategies of bidders, then the property can also be achieved using a strategy-proof mechanism.

A combinatorial auction mechanism consists of an allocation rule that defines the allocation of goods for each bidder and a payment rule that defines the payment of each winner. There have been many studies on characterizing strategy-proof social choice functions (allocation rules in auctions). This is also called the implementability of social choice functions. In particular, a family of *monotonicity* concepts have been identified to characterize implementable social choice functions. For example, Bikhchandani *et al.* [3] proposed *weak-monotonicity* and showed that it is a necessary and sufficient condition for strategy-proof mechanisms. These concepts are defined only on an allocation rule; if it satisfies such a condition, it is guaranteed that there exists an appropriate payment rule that achieves strategy-proofness. Thus, a mechanism designer can concentrate on allocation rules when developing/verifying a mechanism.

Besides these studies, maximizing a seller's revenue has also been a major research topic. For single-item auctions, Myerson [2] introduced the idea of *optimal auctions*, which maximize the expected revenue given a distribution of bidders' types. For combinatorial auctions, various mechanisms have been developed to achieve approximately optimal revenue in several different settings [4], [5].

On the other hand, revenue monotonicity is recognized as one of desirable properties a mechanism should satisfy [6]. A mechanism is revenue monotone if the seller's revenue is guaranteed to weakly increase as the number of bidders grows. This property is quite reasonable, since a growing number of bidders increases competition. However, it is shown that even the Vickrey-Clarke-Groves (VCG) mechanism does not achieve revenue monotonicity. Nevertheless, there has been virtually no work on characterizing revenue monotone mechanisms. One notable exception is Rastegari et al. [7], who proved there exists no mechanism that is revenue monotone, strategy-proof, and weakly maximal, where weak maximality is a weaker notion of Pareto efficiency. Furthermore, they mentioned a connection between revenue monotonicity and *false-name-proofness*, which is known as another desirable property of combinatorial auctions.

False-name-proofness generalizes strategy-proofness by assuming a bidder can submit multiple bids under fictitious identifiers, e.g., multiple e-mail addresses [8]. Several false-name-proof mechanisms have been developed so far [9], [10]. Also, Todo *et al.* [11] fully characterized false-name-proof allocation rules by a condition called *sub-additivity*.

To the best of our knowledge, our paper is the first attempt to characterize revenue monotone mechanisms. First,

An earlier version of this paper was published as [1] and presented at the 12th International Workshop on Agent-Mediated Electronic Commerce (AMEC). This version has rigorous proofs, illustrative examples, and results on combinatorial auctions, which are not included in the earlier version.

we identify a condition called *summation-monotonicity* and prove that weak-monotonicity and summation-monotonicity characterize strategy-proof and revenue monotone allocation rules. Then we actually verified existing combinatorial auction mechanisms and found that, since the allocation rules of many well-known mechanisms do not satisfy summationmonotonicity, these mechanisms are not revenue monotone.

Second, our characterization successfully clarifies the connections between revenue monotonicity and false-name-proofness. Summation-monotonicity and sub-additivity look quite similar, but they are different and interact in a rather complicated way. In single-item auctions, we show that they are basically equivalent; a mechanism is false-name-proof if and only if it is strategy-proof and revenue monotone. On the other hand, we show that these two conditions cannot coexist in combinatorial auctions; under some minor conditions, there exists no combinatorial auction mechanism that is simultaneously revenue monotone and false-name-proof.

This paper is organized as follows. Section II describes our model. Section III introduces a condition called *summation-monotonicity* and proves that it is necessary and sufficient for revenue monotonicity. Section IV examines whether summation-monotonicity holds in existing allocation rules. Section V provides theoretical considerations of the connections between revenue monotonicity and falsename-proofness. Section VI concludes this paper.

II. PRELIMINARIES

Assume there exists a set of potential bidders $\mathbb{N} = \{1, 2, \ldots, n\}$ and a set of goods $G = \{g_1, g_2, \ldots, g_m\}$. Let us define $N \subseteq \mathbb{N}$ as the set of bidders participating in an auction. Each bidder $i \in N$ has his preferences for each bundle or goods $B \subseteq G$. Formally, we model this by supposing that bidder i privately observes a parameter (or signal) θ_i that determines his preferences. We refer to θ_i as the *type* of bidder i and assume it is drawn from a set Θ_i .

Let us denote the set of all possible type profiles as $\Theta_{\mathbb{N}} = \Theta_1 \times \ldots \times \Theta_n$ and a type profile as $\theta = (\theta_1, \ldots, \theta_n) \in \Theta_{\mathbb{N}}$. Observe that type profiles always have one entry for every potential bidder, regardless of the set of participating bidders N. We use a symbol **0** in the vector θ as a placeholder for each non-participating bidder $i \notin N$ and represent $(\theta_1, \ldots, \theta_{i-1}, \mathbf{0}, \theta_{i+1}, \ldots, \theta_n)$ as θ_{-i} , for $\theta = (\theta_1, \ldots, \theta_{i-1}, \theta_i, \theta_{i+1}, \ldots, \theta_n)$. When a set of bidders N participates in the auction, we denote the set of possible type profiles as $\Theta_N (\subseteq \Theta_{\mathbb{N}})$. That is, Θ_N is the set of all type profiles θ for which $\theta_i = \mathbf{0}$ if and only if $i \notin N$.

We assume a *quasi-linear*, *private value* model with *no* allocative externality. The utility of bidder *i*, when *i* obtains a bundle, i.e., a subset of goods $B \subseteq G$ and pays *p*, is represented as $v(\theta_i, B) - p$. We assume a valuation *v* is normalized by $v(\theta_i, \emptyset) = 0$ and satisfies *free disposal*, i.e., $v(\theta_i, B') \ge v(\theta_i, B)$ for all $B' \supseteq B$. We call each Θ_i that satisfies these conditions a *rich domain* [3]. In other words, the domain of types Θ_i is rich enough to contain all possible valuations. This assumption is required so that weak-monotonicity characterizes strategy-proofness.

A combinatorial auction mechanism \mathcal{M} consists of an allocation rule X and a payment rule p. When a set of bidders N participates, an allocation rule is defined as $X : \Theta_N \to A_N$, where A_N is a set of possible allocations over N. Similarly, a payment rule is defined as $p : \Theta_N \to \mathbb{R}^N_+$. Let X_i and p_i respectively denote the bundle allocated to bidder i and the amount bidder i must pay. We use notations $X(\theta_i, \theta_{-i})$ and $p(\theta_i, \theta_{-i})$ to represent the allocation and payment when the declared type of bidder i is θ_i and the declared type profile of other bidders is θ_{-i} .

For simplicity, we restrict our attention to *deterministic* mechanisms and assume a mechanism is *almost anonymous* across bidders and goods; obtained results are invariant under the permutation of the identifiers of bidders/goods except for the case of ties. We also assume a mechanism satisfies *consumer sovereignty* [7]; there always exists a type θ_i for bidder *i*, where bidder *i* can obtain bundle *B*. In other words, if bidder *i*'s valuation for *B* is high enough, then he can obtain *B*. Furthermore, we restrict our attention to *individually rational* mechanisms. A mechanism is individually rational if $\forall N \subseteq \mathbb{N}, \forall i \in N, \forall \theta_i, \forall \theta_{-i}, v(\theta_i, X_i(\theta_i, \theta_{-i})) - p_i(\theta_i, \theta_{-i}) \geq 0$. This means that no participant obtains negative utility by reporting his true type.

Let us introduce the notion called strategy-proofness.

Definition 1 (strategy-proofness): A combinatorial auction mechanism $\mathcal{M}(X,p)$ is strategy-proof if $\forall N \subseteq$ $\mathbb{N}, \forall i \in N, \forall \theta_{-i}, \forall \theta_i, \forall \theta'_i, v(\theta_i, X_i(\theta_i, \theta_{-i})) - p_i(\theta_i, \theta_{-i}) \geq$ $v(\theta_i, X_i(\theta'_i, \theta_{-i})) - p_i(\theta'_i, \theta_{-i}).$

A mechanism is strategy-proof if reporting true type θ_i is a (weakly) dominant strategy for any bidder *i* with type θ_i and any type profile θ_{-i} ; it maximizes his utility regardless of the other bidders' reports. A strategy-proof allocation rule is fully characterized by a simple condition called *weak-monotonicity*, assuming the type domain is rich [3].

Definition 2 (weak-monotonicity): An allocation rule X satisfies weak-monotonicity if $\forall N \subseteq \mathbb{N}, \forall i \in N, \forall \theta_{-i}, \forall \theta_i, \forall \theta'_i, v(\theta_i, X_i(\theta_i, \theta_{-i})) - v(\theta_i, X_i(\theta'_i, \theta_{-i})) \geq v(\theta'_i, X_i(\theta_i, \theta_{-i})) - v(\theta'_i, X_i(\theta'_i, \theta_{-i})).$

Bikhchandani *et al.* [3] proved that if an allocation rule is weakly monotone, we can always find an appropriate payment rule to truthfully implement the allocation rule.

Next, let us introduce the notion of *revenue monotonicity* [6], which is known as another desirable property of combinatorial auctions.

Definition 3 (revenue monotonicity): A combinatorial auction mechanism $\mathcal{M}(X,p)$ is revenue monotone if $\forall N \subseteq \mathbb{N}, \forall \theta \in \Theta_N, \forall j \in \mathbb{N},$

$$\sum_{i \in \mathbb{N}} p_i(\theta) \ge \sum_{i \in \mathbb{N} \setminus \{j\}} p_i(\theta_{-j}).$$
(1)

A mechanism is revenue monotone if a seller's revenue from an auction is guaranteed to weakly increase as the number of bidders grows. The left-hand side of Eq. 1 indicates the seller's revenue from the auction when the set of bidders Nparticipates in the auction. The right-hand side indicates the seller's revenue when bidder j drops out. In other words, a combinatorial auction is revenue monotone if the seller's revenue does not increase by dropping a bidder.

Rastegari *et al.* [7], a seminal work on revenue monotonicity, shows the impossibility result that there exists no deterministic strategy-proof, weakly maximal combinatorial auction mechanism that is revenue monotone. Roughly speaking, a combinatorial auction mechanism is *weakly maximal* if its allocation cannot be augmented to cause a losing bidder to win without hurting winning bidders. It is a weaker notion of Pareto efficiency.

Furthermore, Rastegari *et al.* [7] mentioned a connection between revenue monotonicity and *false-name-proofness*. A mechanism is *false-name-proof* if for each bidder, reporting his true valuations using a single identifier (although the bidder can use multiple identifiers) is a dominant strategy.

To introduce this property along with our model in this paper, we add several notations. Let us consider a situation where bidder *i* uses *s* false identifiers id_1, \ldots, id_s and define a mapping function ϕ such that $\phi(i) = \{id_1, \dots, id_s\};$ i.e., $\phi(i)$ represents a set of identifiers owned by bidder *i*. Observe that $|\phi(i)| \geq 1$. Let us represent a type profile as $\theta = (\theta_{id_1}, \dots, \theta_{id_s}, \theta_{s+1}, \dots, \theta_n)$, and similarly represent a type profile reported by the set of bidders $\phi(i)$ as $\theta_{\phi(i)} = (\theta_{id_1}, \dots, \theta_{id_s})$. Here we use $\theta_{id_1}, \dots, \theta_{id_s}$ instead of $\theta_1, \ldots, \theta_s$ for convenience. On the other hand, let $(\theta_i, \mathbf{0}, \dots, \mathbf{0})$ denote the type profile when bidder *i* reports θ_i with only one identifier although he can use s identifiers. That is, **0** means that the identifier is not used by bidder *i*. Note that these notations can be introduced w.l.o.g., since we assume almost anonymous mechanisms. Furthermore, to consistently address false-name-proofness, we represent a type profile reported by the set of participating bidders other than $\phi(i)$ as $\theta_{-\phi(i)} = (\mathbf{0}, \dots, \mathbf{0}, \theta_{s+1}, \dots, \theta_n).$

 $\begin{array}{ll} \textit{Definition 4 (False-name-proofness): A combinatorial auction mechanism $\mathcal{M}(X,p)$ is false-name-proof if $\forall N \subseteq \mathbb{N}, \forall i \in N, \forall \phi(i), \forall \theta_{-\phi(i)}, \forall \theta_i, \forall \theta_{\phi(i)}, $ \end{array}$

$$v(\theta_{i}, X_{i}((\theta_{i}, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)})) - p_{i}((\theta_{i}, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)}) \\ \geq v(\theta_{i}, \bigcup_{l \in \phi(i)} X_{l}(\theta_{\phi(i)}, \theta_{-\phi(i)})) - \sum_{l \in \phi(i)} p_{l}(\theta_{\phi(i)}, \theta_{-\phi(i)})$$

$$(2)$$

Note that when $|\phi(i)| = 1$ the definition becomes equivalent to strategy-proofness. It has been shown that VCG is not false-name-proof and that there exists no false-name-proof, Pareto efficient mechanism [8]. In addition, Todo *et al.* [11] identified a condition called *sub-additivity* and fully characterized false-name-proof allocation rules.

III. CHARACTERIZING REVENUE MONOTONICITY

This section introduces a simple condition called summation-monotonicity that characterizes revenue monotone allocation rules when coupled with weak-monotonicity.

Definition 5 (Summation-monotonicity): An allocation rule X satisfies summation-monotonicity if $\forall N \subseteq \mathbb{N}$, $\forall \theta \in \Theta_N, \forall j \in \mathbb{N}$,

$$\begin{aligned} \forall \theta'_{i} \quad \text{s.t.} & \left\{ \begin{array}{l} X_{i}(\theta'_{i}, \theta_{-i}) \supseteq X_{i}(\theta) \text{ and} \\ v(\theta'_{i}, X_{i}(\theta'_{i}, \theta_{-i})) = v(\theta'_{i}, X_{i}(\theta)), \\ \forall \theta''_{i} \quad \text{s.t.} \quad v(\theta''_{i}, X_{i}(\theta''_{i}, \theta_{-\{i,j\}})) = 0, \\ \sum_{i \in \mathbb{N}} v(\theta'_{i}, X_{i}(\theta)) \geq \sum_{i \in \mathbb{N} \setminus \{j\}} v(\theta''_{i}, X_{i}(\theta_{-j})). \end{aligned} \right. \end{aligned}$$
(3)

Note here that $\theta_{-\{i,j\}}$ denotes a type profile that excludes bidder *i* and *j*. This condition implies for any set of participating bidders and any type profile, the revenue, which is the sum of the critical values of the bidders in an auction, weakly decreases when any bidder drops out from the auction.

The following is an intuitive explanation why summationmonotonicity holds for a strategy-proof, revenue monotone mechanism. Let us consider a combinatorial auction with two goods g_1 and g_2 . Assume that it allocates g_1 to bidder 1 and g_2 to bidder 2 when a set of bidders N participates. On the other hand, assume that it allocates g_1 to bidder 3 and g_2 to bidder 4 when bidder j drops out. The top two rectangles of Fig. 1 represent the total payments for bidder 1 and 2 and the bottoms for bidder 3 and 4. If the mechanism is revenue monotone, $p_1(\theta) + p_2(\theta) \ge p_3(\theta_{-j}) + p_4(\theta_{-j})$ holds.

The arrows at the top of Fig. 1 indicate the left-hand side of Eq. 3. θ'_1 means the minimal type where bidder 1 obtains g_1 or any superset, fixing other bidders' types than bidder 1. Under the mechanism, $v(\theta'_1, \{g_1\})$ must be greater than $p_1(\theta)$. Otherwise, bidder 1 has an incentive not to participate in the auction and individual rationality is violated. Similarly, θ'_2 means the minimal type where bidder 2 obtains g_2 or any superset, fixing other bidders' types than bidder 2 and $v(\theta'_2, \{g_2\})$ must be greater than $p_2(\theta)$.

The arrows at the bottom of Fig. 1 indicate the right-hand side of Eq. 3. θ''_3 means the maximal type where bidder 3 cannot obtain g_1 ; he obtains nothing, fixing other bidders' types than bidder 3. Under the mechanism, $v(\theta''_3, \{g_1\})$ must be smaller than $p_3(\theta_{-j})$. Otherwise, a bidder with θ''_3 as his true type has an incentive to pretend that his type is θ_3 to obtain g_1 , and thus strategy-proofness is violated. Similarly, θ''_4 means the maximal type where bidder 4 cannot obtain g_2 , fixing other bidders' types than bidder 4 and $v(\theta''_4, \{g_2\})$ must be smaller than $p_4(\theta_{-j})$.

From these facts, summation-monotonicity must hold for strategy-proof, revenue monotone mechanisms (Lemma 1). Lemma 2 shows that, as long as summation-monotonicity and weak-monotonicity hold, we can find an appropriate payment rule p so that $p_1(\theta) + p_2(\theta) \ge p_3(\theta_{-j}) + p_4(\theta_{-j})$ holds. Thus, we derive the following theorem:



Figure 1. Summation-monotonicity

Theorem 1: There exists an appropriate payment rule p such that a combinatorial auction mechanism $\mathcal{M}(X, p)$ is strategy-proof and revenue monotone if and only if X satisfies weak-monotonicity and summation-monotonicity.

Lemma 1: If a combinatorial auction mechanism $\mathcal{M}(X,p)$ is strategy-proof and revenue monotone, then the allocation rule X satisfies weak-monotonicity and summation-monotonicity.

Proof: Bikhchandani *et al.* [3] proved that if \mathcal{M} is strategy-proof, X satisfies weak-monotonicity. To prove this lemma, it suffices to show that if \mathcal{M} is strategy-proof and revenue monotone, X satisfies summation-monotonicity.

Let W_N denote the set of winners when a set of bidders N participates, and let $W_{N\setminus\{j\}}$ denote the set of winners when bidder j drops out. Since $\mathcal{M}(X,p)$ is revenue monotone, from Eq. 1, we derive $\forall N \subseteq \mathbb{N}, \forall \theta \in \Theta_N, \forall j \in \mathbb{N}$,

$$\sum_{i \in W_N} p_i(\theta) \ge \sum_{i \in W_N \setminus \{j\}} p_i(\theta_{-j}).$$
(4)

Each term $p_i(\theta)$ of the left-hand side must be smaller than the minimum bid in which bidder $i(\in N)$ still wins; otherwise \mathcal{M} violates individual rationality. Thus we obtain

$$\forall i \in W_N, \forall \theta'_i \\ \text{s.t. } X_i(\theta'_i, \theta_{-i}) \supseteq X_i(\theta), v(\theta'_i, X_i(\theta'_i, \theta_{-i})) = v(\theta'_i, X_i(\theta)), \\ v(\theta'_i, X_i(\theta)) \ge p_i(\theta).$$
 (5)

On the other hand, each term $p_i(\theta_{-j})$ of the right-hand side must be greater than the maximum bid in which bidder $i (\in N \setminus \{j\})$ loses; otherwise bidder i with type θ''_i has an incentive to pretend his type is θ_i . Thus we obtain

$$\forall j, \forall i \in W_{N \setminus \{j\}}, \forall \theta_i'' \text{ s.t. } v(\theta_i'', X_i(\theta_i'', \theta_{-\{i,j\}})) = 0,$$
$$p_i(\theta_{-j}) \ge v(\theta_i'', X_i(\theta_{-j})). \tag{6}$$

As a result, from Eqs. 4, 5, and 6, we obtain

$$\sum_{i \in \mathbb{N}} v(\theta'_i, X_i(\theta)) = \sum_{i \in W_N} v(\theta'_i, X_i(\theta))$$

$$\geq \sum_{i \in W_N} p_i(\theta)$$

$$\geq \sum_{i \in W_{N \setminus \{j\}}} p_i(\theta_{-j})$$

$$\geq \sum_{i \in W_{N \setminus \{j\}}} v(\theta''_i, X_i(\theta_{-j}))$$

$$= \sum_{i \in \mathbb{N} \setminus \{j\}} v(\theta''_i, X_i(\theta_{-j}))$$

and Eq. 3 holds.

Lemma 2: If an allocation rule X satisfies weakmonotonicity and summation-monotonicity, there exists a payment rule p such that a combinatorial auction mechanism $\mathcal{M}(X, p)$ is strategy-proof and revenue monotone.

Proof: Bikhchandani *et al.* [3] proved that if X satisfies weak-monotonicity, there exists a payment rule p such that $\mathcal{M}(X,p)$ is strategy-proof. To prove this lemma, we show that if X satisfies weak-monotonicity and summation-monotonicity, we can choose p such that $\mathcal{M}(X,p)$ also satisfies revenue monotonicity.

We are going to derive a contradiction by assuming Eq. 1 does not hold, although $\mathcal{M}(X, p)$ is strategy-proof and Xsatisfies weak-monotonicity and summation-monotonicity. More specifically, we assume that for any p with which $\mathcal{M}(X, p)$ is strategy-proof, the following condition holds:

$$\forall p, \exists N \subseteq \mathbb{N}, \exists \theta, \exists j, \sum_{i \in \mathbb{N}} p_i(\theta) < \sum_{i \in \mathbb{N} \setminus \{j\}} p_i(\theta_{-j}).$$
(7)

Now let us choose $\gamma(>0)$ such that

$$\sum_{i \in \mathbb{N}} p_i(\theta) + \gamma = \sum_{i \in \mathbb{N} \setminus \{j\}} p_i(\theta_{-j}).$$
(8)

Then, choose a small enough ϵ such that $0 < \epsilon < \frac{\gamma}{2|N|-1}$ holds. Also, let us define a type θ'_i for each $i \in N$ as

$$v(\theta'_i, B_i) = \begin{cases} p_i(\theta) + \epsilon & \text{if } B_i \supseteq X_i(\theta), \\ 0 & \text{otherwise.} \end{cases}$$

These types satisfy the preconditions of Eq. 3: $X_i(\theta'_i, \theta_{-i}) \supseteq X_i(\theta)$ and $v(\theta'_i, X_i(\theta'_i, \theta_{-i})) = v(\theta'_i, X_i(\theta))$.

Furthermore, let us define θ_i'' for each $i \in N \setminus \{j\}$ as

$$v(\theta_i'', B_i) = \begin{cases} p_i(\theta_{-j}) - \epsilon & \text{if } B_i \supseteq X_i(\theta_{-j}), \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, these types satisfy the preconditions of Eq. 3: $v(\theta_i'', X_i(\theta_i'', \theta_{-\{j,i\}})) = 0.$

As a result, from Eq. 3, the following inequality holds:

$$\sum_{i \in N} p_i(\theta) + |N| \cdot \epsilon \ge \sum_{i \in N \setminus \{j\}} p_i(\theta_{-j}) - (|N| - 1) \cdot \epsilon.$$
(9)

By substituting Eq. 8 into Eq. 9, we obtain $\gamma \leq (2|N|-1) \cdot \epsilon$. This contradicts the assumption of $\epsilon < \frac{\gamma}{2|N|-1}$.

IV. VERIFYING EXISTING MECHANISMS

This section verifies whether summation-monotonicity is satisfied in three allocation rules to show how our characterization works. First, we focus on a Pareto efficient allocation rule and then examine two inefficient allocation rules.

Claim 1: A Pareto efficient allocation rule does not satisfy summation-monotonicity.

Proof: Assume there are three bidders 1, 2, and 3 and two goods g_1 and g_2 for sale. Consider the following situation where their reported types are given as follows:

	$\{g_1\}$	$\{g_2\}$	$\{g_1,g_2\}$
bidder 1:	7	0	7
bidder 2:	0	0	8
bidder 3:	0	7	7

A Pareto efficient allocation rule allocates g_1 to bidder 1 and g_2 to bidder 3. In this case, it allocates g_1 to bidder 1 with θ'_1 such that $v(\theta'_1, \{g_1\}) = 1 + \epsilon$. Similarly, it allocates g_2 to bidder 3 with θ'_3 such that $v(\theta'_3, \{g_2\}) = 1 + \epsilon$. Thus, we obtain $2 \cdot (1 + \epsilon)$ as the left-hand side of Eq. 3.

On the other hand, let us consider the situation where bidder 3 drops out from the auction. In the Pareto efficient allocation rule, bidder 2 obtains $\{g_1, g_2\}$ if he has a value greater than 7 on $\{g_1, g_2\}$. If he has a type θ_2'' such that $v(\theta_2'', \{g_1, g_2\}) = 7 - \epsilon$ and $v(\theta_2'', \{g_1\}) = v(\theta_2'', \{g_2\}) = 0$, he obtains no good, i.e., $X(\theta_2'') = \emptyset$. For bidder 1, since he obtains no good under the Pareto efficient allocation rule, $v(\theta_1'', \emptyset) = 0$ holds. Thus, the right-hand side of Eq. 3, which is the maximum bid where bidder 2 loses, is $7 - \epsilon$.

Finally we obtain $2 \cdot (1+\epsilon) < 7-\epsilon$ and the Pareto efficient allocation rule does not satisfy summation-monotonicity. In fact, by bidder 3's dropping out, the seller's revenue increases from 2 to 7, and revenue monotonicity fails.

The fact that there exists no revenue monotone, Pareto efficient mechanism has already been proved [6]. However, note that our proof is much simpler, since we can ignore the payment rule and concentrate on the allocation rule.

Second, our characterization also reconfirms that the Set (Set) mechanism, which is trivial and false-name-proof, is revenue monotone. Set is a very simple mechanism that allocates all goods to a single bidder with the highest valuation for the grand bundle, i.e., a bundle of all goods. Effectively, it sells the grand bundle as a single good using the Vickrey/second-price auction. The allocation rule in Set is described as follows:

$$X_{i}(\theta_{i}, \theta_{-i}) = \begin{cases} G & \text{if } v(\theta_{i}, G) \geq \max_{l \in N \setminus \{i\}} v(\theta_{l}, G) \\ \emptyset & \text{otherwise.} \end{cases}$$

Claim 2: The allocation rule in Set satisfies summationmonotonicity.

Proof: Assume that bidder *i* wins when a set of bidders N participates. Since there is only one winner *i*, the left-hand side of Eq. 3 equals to $v(\theta'_i, G)$, i.e., the valuation where bidder *i* still wins. Clearly, $v(\theta'_i, G) \ge \max_{l \in N \setminus \{i\}} v(\theta_l, G)$ must hold.

Next, let us consider the situation where bidder j drops out. When $j \neq i$, bidder i remains the winner. Since there is only one winner i, the right-hand side of Eq. 3 equals to $v(\theta_i'', G)$, i.e., the valuation where bidder i becomes a loser. Clearly, $v(\theta_i'', G) \leq \max_{l \in N \setminus \{i,j\}} v(\theta_l, G)$ holds. Thus, we obtain $v(\theta_i', G) \geq v(\theta_i'', G)$ and Eq. 3 holds.

When j = i, winner *i* drops out. Let us denote the new winner as *k*. Since there is only one winner *k*, the right-hand side of Eq. 3 equals to $v(\theta_k'', G)$, i.e., the valuation

where bidder k becomes a loser. Clearly, $v(\theta_k'', G) \leq \max_{l \in N \setminus \{i,k\}} v(\theta_l, G)$ holds. Thus, we obtain $v(\theta_i', G) \geq v(\theta_k'', G)$ and Eq. 3 holds.

Finally, let us examine a non-trivial false-name-proof mechanism called the Leveled Division Set (LDS) mechanism [9]. Rastegari *et al.* [6] claims that this mechanism is revenue monotone. However, surprisingly, our characterization reveals that it is not.

LDS intuitively predetermines a leveled division set and reserve prices for each single good. This leveled division set describes a possible way for dividing goods among different bidders, e.g., at level 1, all goods are sold in one bundle and divided into smaller bundles as the level increases. LDS chooses the level based on the declared types and uses VCG within the level to determine the allocation and payments. For two goods, LDS is identical to Set if the reserve prices are 0.

Claim 3: The allocation rule in LDS does not satisfy summation-monotonicity.

Proof: Assume that there are two goods g_1 and g_2 for sale and the reserve prices for each bundle are defined as follows: $r_{\{g_1\}} = 3$, $r_{\{g_2\}} = 3$, $r_{\{g_1,g_2\}} = r_{\{g_1\}} + r_{\{g_2\}} = 6$. Also assume that there are five bidders 1-5, whose types are given as follows:

	$\{g_1\}$	$\{g_2\}$	$\{g_1,g_2\}$
bidder 1:	5	0	5
bidder 2:	4	0	4
bidder 3:	0	5	5
bidder 4:	0	4	4
bidder 5:	0	0	$6 + \epsilon$

In this case, LDS allocates $\{g_1, g_2\}$ to bidder 5. Since the reserve price on $\{g_1, g_2\}$ is 6, the minimum bid where bidder 5 still wins is $6 + \epsilon$. Thus, the right-hand side of Eq. 3 becomes $6 + \epsilon$.

On the other hand, let us consider the situation where bidder 5 drops out. In this case, LDS allocates g_1 and g_2 to bidders 1 and 3, respectively. The maximum bid for g_1 (g_2) where bidder 1 (bidder 3) loses is $4 - \epsilon$. Thus, the left-hand side of Eq. 3 becomes $2 \cdot (4 - \epsilon)$.

Therefore, we obtain $2 \cdot (4 - \epsilon) > 6 + \epsilon$ and the allocation rule in LDS does not satisfy summation-monotonicity. In fact, by bidder 5's dropping out, the seller's revenue increases from 6 to 8, and revenue monotonicity fails.

V. REVENUE MONOTONICITY AND FALSE-NAME-PROOFNESS

As it was considered that there is a connection between revenue monotonicity and false-name-proofness [6], [7], the example in Claim 1 provides a common example where VCG is neither revenue monotone nor false-name-proof. Let us consider a situation where bidder 1', who values 14 only on $\{g_1, g_2\}$, uses two identifiers 1 and 3. Since VCG allocates g_1 and g_2 to identifiers 1 and 3, respectively, bidder 1' obtains $\{g_1, g_2\}$ and pays 2. On the other hand, when only two bidders 1' and 2 participate in the auction, i.e., when bidder 1' does not use false identifiers, bidder 1' obtains $\{g_1, g_2\}$ and pays 8. As this example shows, increasing the number of participating bidders by or not by false identifiers can reduce the seller's revenue.

Therefore, a sub-additive allocation rule apparently always coincides with a summation-monotone allocation rule, and vice versa. However, this is not true, although it is certain that sub-additivity looks quite similar to summation-monotonicity (Definition 5). Recall that summation-monotonicity implies that the sum of the critical values of bidders in an auction is guaranteed to weakly decrease when some of the bidders drop out from the auction. On the contrary, sub-additivity implies that the critical value of a bidder when he uses a single identifier is guaranteed to be smaller than or equal to that when he uses multiple false identifiers. In fact, existing false-nameproof mechanisms, such as LDS, are not revenue monotone.

A. Single-item Auctions

We stated that, in general, revenue monotonicity cannot coexist with false-name-proofness. Nevertheless, in singleitem auctions, we show that they are equivalent under the following condition.

Assumption 1: For any set of participating bidders N and for any bidder $j (\in N)$, if a mechanism allocates a good to a bidder when $N \setminus \{j\}$ participates, it always allocates the good to some bidder when N participates.

We believe that introducing Assumption 1 is quite natural. From a seller's viewpoint, it is undesirable that a good is no longer allocated when more bidders join the auction. Under this assumption, the following theorem holds.

Theorem 2: Under Assumption 1, a single-item auction mechanism is false-name-proof if and only if it is strategy-proof and revenue monotone.

To prove this theorem, let us separately prove Lemmas 4 and 5. Before proving Lemma 4, we introduce Lemma 3 for strategy-proof and revenue monotone single-item auctions.

Lemma 3: Let us consider strategy-proof and revenue monotone single-item auctions that sell good g. If bidder k wins when the set of bidders N participates, bidder k also wins when any bidder $j \neq k \in N$ drops out.

Proof: First, since we assume almost anonymous and strategy-proof mechanisms, a bidder can win a good only when his bid is higher than those of other participants. Formally, assume that bidder k wins when he reports θ_k . Then the left-hand side $v(\theta'_k, g)$ of Eq. 3 satisfies

$$v(\theta'_k, g) \le v(\theta_k, g). \tag{10}$$

This intuitively means that the critical value to obtain the good is lower than $v(\theta_k, g)$.

Second, bidder k still has the largest valuation when bidder $j \neq k$ drops out. Now, assume that bidder k doesn't win in this situation. The critical value $cv_k^{N\setminus\{j\}}$ for k to win the good g is strictly greater than $v(\theta_k, g)$. Therefore, we can choose γ such that

$$v(\theta_k, g) = c v_k^{N \setminus \{j\}} - \gamma \tag{11}$$

holds. Let us also choose a small enough $\epsilon(0 < \epsilon < \gamma)$ and define a type θ_k'' such that $v(\theta_k'',g) = cv_k^{N\setminus\{j\}} - \epsilon$. Bidder k loses when he reports θ_k'' . Then, the type θ_k'' satisfies $v(\theta_k'', X_k((\theta_k'', 0, \dots, 0), \theta_{-\{k,j\}})) = 0$. Since $\epsilon < \gamma$ holds, from Eq. 11, we obtain

$$v(\theta_k, g) < v(\theta_k'', g). \tag{12}$$

Finally, from Eqs. 10 and 12, $v(\theta''_k, g) > v(\theta_k, g) \ge v(\theta'_k, g)$ holds and this violates summation-monotonicity. Now, we are ready to prove Lemma 4.

Lemma 4: Any strategy-proof, revenue monotone singleitem auction mechanism satisfies false-name-proofness.

Proof: To prove this lemma, we are going to derive a contradiction assuming that a single-item auction mechanism, which is strategy-proof and revenue monotone, is not false-name-proof. Specifically, we assume that for some $\theta_{-\phi(i)}$, there exists bidder *i* with type θ_i who can increase his profit using false identifiers $\phi(i)$:

$$v(\theta_{i}, X_{i}((\theta_{i}, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)})) - p_{i}((\theta_{i}, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)})$$

$$< v(\theta_{i}, \bigcup_{l \in \phi(i)} X_{l}(\theta_{\phi(i)}, \theta_{-\phi(i)})) - \sum_{l \in \phi(i)} p_{l}(\theta_{\phi(i)}, \theta_{-\phi(i)}).$$
(13)

Since we consider the case where bidder i can increase his utility, the winner k must be in $\phi(i)$ when a set of bidders N participates. Let θ_k denote the type reported by bidder k. From Lemma 3, bidder i wins when he reports θ_k with only one identifier. We obtain

$$X_k(\theta_{\phi(i)}, \theta_{-\phi(i)}) = X_i((\theta_k, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)}).$$
(14)

Next, from Eq. 1, we obtain

$$p_i((\theta_k, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)}) \le p_k(\theta_{\phi(i)}, \theta_{-\phi(i)}).$$
(15)

Furthermore, from strategy-proofness, we obtain

$$v(\theta_{i}, X_{i}((\theta_{i}, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)})) - p_{i}((\theta_{i}, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)}) \\ \geq v(\theta_{i}, X_{i}((\theta_{k}, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)})) - p_{i}((\theta_{k}, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)})$$

$$(16)$$

As a result, from Eqs. 14, 15, and 16,

$$v(\theta_i, X_i((\theta_i, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)})) - p_i((\theta_i, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)}) \\\geq v(\theta_i, X_k(\theta_{\phi(i)}, \theta_{-\phi(i)})) - p_k(\theta_{\phi(i)}, \theta_{-\phi(i)})$$

holds. Thus, this contradicts Eq. 13.

Lemma 5: Under Assumption 1, any false-name-proof single-item auction mechanism satisfies strategy-proofness and revenue monotonicity.

Proof: If every bidder uses only one identifier, falsename-proofness is equivalent to strategy-proofness. To prove this lemma, we show that if a mechanism is false-nameproof, then it is also revenue monotone. The model of revenue monotonicity assumes that the set of types of bidders who always participate is fixed. Thus, we can concentrate on the case where a bidder with θ_i uses s identifiers and submits $(\theta_i, \theta_{id_2}, \ldots, \theta_{id_s})$, in which he still submits his true type θ_i . Here, if there is no winner when a set of bidders $N \setminus \phi(i) \cup \{i\}$ participates, revenue monotonicity always holds, regardless of the allocation when N participates. Then, let us consider the case where the good is allocated to some bidder i when $N \setminus \phi(i) \cup \{i\}$ participates. From Assumption 1, if a good is allocated to bidder i when $N \setminus \phi(i) \cup \{i\}$ participates, then the good is also allocated to bidder k when N participates.

Let us consider that bidder k belongs to $N \setminus \phi(i)$. From strategy-proofness, a bidder wins if he submits the highest bid. For the winning bidder i when $N \setminus \phi(i) \cup \{i\}$ participates,

$$v(\theta_i, g) \ge \max_{l \in N \setminus \phi(i)} v(\theta_l, g) \ge v(\theta_k, g).$$
(17)

Since the winning bid θ_i still exists when N participates, for the winning bidder $k \in N \setminus \phi(i)$ when N participates, $v(\theta_k, g) \ge v(\theta_i, g)$ holds. Here, since Eq. 17 is violated if $v(\theta_k, g) > v(\theta_i, g)$ holds, $v(\theta_k, g)$ always equals to $v(\theta_i, g)$. Accordingly, the payment when $N \setminus \phi(i) \cup \{i\}$ participates and *i* wins equals to that when N participates and *k* wins. In fact, *i*'s payment $p_i((\theta_i, 0, \dots, 0), \theta_{-\phi(i)})$ is $v(\theta_k, g)$, while *k*'s payment $p_k(\theta_{\phi(i)}, \theta_{-\phi(i)})$ is $v(\theta_i, g)$. Therefore, the mechanism satisfies revenue monotonicity.

On the other hand, let us consider that bidder k belongs to $\phi(i)$. We obtain $v(\theta_i, X_i((\theta_i, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)})) =$ $v(\theta_i, X_k(\theta_{\phi(i)}, \theta_{-\phi(i)}))$. By substituting this into Eq. 2, $p_i((\theta_i, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi(i)}) \leq p_k(\theta_{\phi(i)}, \theta_{-\phi(i)})$ holds, so the mechanism satisfies revenue monotonicity.

B. Combinatorial Auctions

This subsection reveals that false-name-proofness and revenue monotonicity cannot coexist in combinatorial auctions. To provide a clear proof, we introduce the following two assumptions.

Assumption 2 (Independence of irrelevant good):

Assume bidder i is winning all goods. If we add an additional good that is wanted only by bidder i, and his valuation for all goods is larger than or equal to some constant c, then he still wins all goods.

The independence of irrelevant good (IIG) condition [12] is intuitively reasonable and is satisfied in almost all wellknown mechanisms, in particular, in all existing false-nameproof mechanisms (to the best of our knowledge). This is true for a mechanism that uses predefined reserve prices, such as LDS, assuming that c is large enough compared to the reserve price. Note that the IIG condition is different from the typical Independence of Irrelevant Alternatives (IIA) conditions, which are often quite strong and apply to a wide variety of situations. Since the IIG condition applies only to very specific situations, we consider it quite mild.

Assumption 3: In a combinatorial auction, if there exists no bid on multiple goods, then for each good, a mechanism allocates the good to its highest bidder, as long as the highest bid is larger than or equal to some constant c.

This assumption is also quite natural and is satisfied in almost all well-known mechanisms.

Theorem 3: Under Assumptions 2 and 3, there exists no combinatorial auction mechanism \mathcal{M} that simultaneously satisfies revenue monotonicity and false-name-proofness.

Proof: Let us assume there exists a mechanism \mathcal{M} that satisfies Assumptions 2 and 3, revenue monotonicity, and false-name-proofness and derive a contradiction.

First, let us consider the following situation:

Case 1:

	$\{g_1\}$	$\{g_2\}$	$\{g_1,g_2\}$
bidder 1:	0	0	c
bidder 2:	$c-\epsilon$	0	$c-\epsilon$

Bidder 1 must win in Case 1. If bidder 1 is interested in $\{g_1\}$ rather than $\{g_1, g_2\}$, then bidder 1 wins from Assumption 3. Then in Case 1, bidder 1 still wins from Assumption 2.

Next, we add another bidder 3:

Case 2:

	$\{g_1\}$	$\{g_2\}$	$\{g_1, g_2\}$
bidder 1:	0	0	c
bidder 2:	$c-\epsilon$	0	$c-\epsilon$
bidder 3:	0	$c/2 - \epsilon$	$c/2 - \epsilon$

We show that bidder 1 still wins in Case 2. If no bidder wins, then the revenue becomes 0 and revenue monotonicity is violated. Also, if only bidder 3 wins, the revenue must be at most $c/2 - \epsilon$ and revenue monotonicity is violated. Thus, let us assume bidder 2 and 3 win in Case 2. Then consider the following situation:

Case 3:

	$\{g_1\}$	$\{g_2\}$	$\{g_1, g_2\}$
bidder 1:	0	0	c
bidder 2:	$c-\epsilon$	0	$c-\epsilon$
bidder 3:	0	$c-\epsilon$	$c-\epsilon$

Bidder 3 must also win in Case 3, and the payment is at most $c/2 - \epsilon$. Otherwise, bidder 3 has an incentive to underdeclare his valuation to $c/2 - \epsilon$ so that the situation becomes identical to Case 2. Also, since we assume the mechanism is almost anonymous bidder 2 also wins and pays at most $c/2 - \epsilon$. However, in Case 1, bidder 2 can submit false-name bids and make the situation identical to Case 3 and obtain $\{g_1, g_2\}$ by paying $c - 2\epsilon$. Thus, false-name-proofness is violated. In Case 2, bidder 1 must win and pays $c - \epsilon$.

Then, we add two more bidders 4 and 5: *Case 4:*

 $\{g_1\}$ $\{g_2\}$ $\{g_1, g_2\}$

bidder 1:	0	0	c
bidder 2:	$c-\epsilon$	0	$c-\epsilon$
bidder 3:	0	$c/2 - \epsilon$	$c/2 - \epsilon$
bidder 4:	$c-2\epsilon$	0	$c-2\epsilon$
bidder 5:	0	$c/2 - 2\epsilon$	$c/2 - 2\epsilon$

Adding bidders 4 and 5 will not affect the outcome in \mathcal{M} . Otherwise, revenue monotonicity is violated. Thus, in Case 4, bidder 1 still wins and pays $c - \epsilon$ for $\{g_1, g_2\}$.

Finally, let us consider the following situation:

Case 5:

	$\{g_1\}$	$\{g_2\}$	$\{g_1, g_2\}$
bidder 2:	$c-\epsilon$	0	$c-\epsilon$
bidder 3:	0	$c/2 - \epsilon$	$c/2 - \epsilon$
bidder 4:	$c-2\epsilon$	0	$c-2\epsilon$
bidder 5:	0	$c/2 - 2\epsilon$	$c/2 - 2\epsilon$

In Case 5, from Assumption 3, bidder 2 and 3 obtain g_1 and g_2 , and pay $c-2\epsilon$ and $c/2-2\epsilon$, respectively. If bidder 1 joins, the situation becomes identical to Case 4. Then the revenue decreases from $\frac{3c}{2} - 4\epsilon$ to $c - \epsilon$. Thus, revenue monotonicity is violated and this contradicts the assumption.

Let us clarify the difference between our Theorem 3 and Rastegari *et al.*'s results. They showed that there exists no deterministic strategy-proof combinatorial auction mechanism that is false-name-proof and weakly maximal. They also proved that there exists no deterministic strategy-proof combinatorial auction mechanism that is revenue monotone and weakly maximal. Thus, revenue monotonicity and false-name-proofness cannot coexist assuming the mechanism is weakly maximal. On the other hand, we proved that revenue monotonicity and false-name-proofness cannot coexist assuming the mechanism satisfies Assumptions 2 and 3. ¹

Weak maximality and these two conditions are independent; weak maximality does not mean these two conditions hold, and these conditions do not mean weak maximality holds. We believe our assumptions are very mild, since they apply only to very specific situations, while weak maximality applies to a wide variety of situations. Thus, it is more likely that a mechanism, which does not satisfy weak maximality, satisfies these two conditions.

VI. CONCLUSIONS

This paper identified a simple condition called *summation-monotonicity* for characterizing strategy-proof and revenue monotone allocation rules. To the best of our knowledge, this is the first attempt to characterize revenue monotone allocation rules. Our characterization is useful for developing/verifying mechanisms. To demonstrate the power of our characterization, we verified existing auction mechanisms and found that several non-trivial mechanisms are not revenue monotone. In addition, our characterization enables

us to examine the connections between revenue monotonicity and false-name-proofness. In a single-item auction, we showed that they are basically equivalent. Whereas, we also showed that they cannot coexist in combinatorial auctions under some minor conditions.

In future work, we hope to design a novel deterministic, revenue monotone combinatorial auction mechanism, since only a randomized mechanism has been proposed so far [13]. Furthermore, by utilizing our characterization, we hope to examine several theoretical properties of revenue monotone allocation rules, e.g., the upper bound on possible social surplus for revenue monotone mechanisms.

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¹To be precise, we also assume a mechanism is almost anonymous.