Université d'Ottawa
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University of Ottawa
Faculty of Engineering,
School of Information
Technology and Engineering

ELG 4172 Digital signal processing

Professor: Miodrag Bolic

Final Exam

This exam is **180 minutes** long.

- Simple calculators are allowed
- notes and textbooks are allowed (open book exam)

Last name:

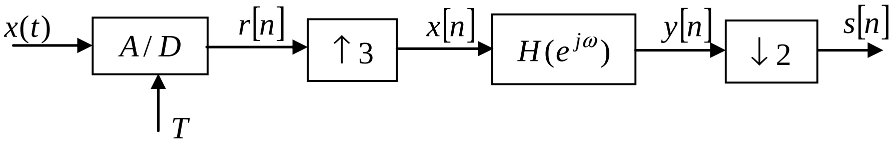
First name:

Student #:

Problem	Maximum	Score
1	7	
2	7	
3	7	
4	11	
5	5	
6	13	
Total	50	

Question 1 Downsampling and Upsampling **/7**

Consider the following system:



For the following specifications:

$X(j\Omega) = 1 \quad |\Omega| < 1000\pi$

$X(j\Omega) = 0 \quad |\Omega| > 1000\pi$

$T = 1/2000$ sec.

$H(e^{j\omega}) = 1 \quad |\omega| < \pi/3$

$H(e^{j\omega}) = 0 \quad |\omega| > \pi/3$

draw the spectrum of the different discrete time signals (i.e. $R(e^{j\omega})$, $X(e^{j\omega})$, $Y(e^{j\omega})$, $S(e^{j\omega})$.

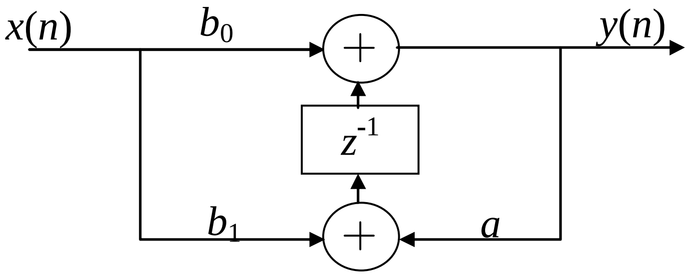
Question 2 Finite Word Effects **/7**

Consider an IIR filter shown in figure below. Multiplier coefficients are a, b_0 and b_1. Assume that the multiplications are performed in fixed point arithmetics using two's complement number format (normalized between -1 and +1), and using rounding after each multiplication operation. Assume that 10 bits are used for numbers, out of which 1 bit is used for sign and 9 bits for the magnitude.

a) Find the transfer function $H(z)$ between the input and the output of the filter shown in Figure below.

b) Find the total noise PSD $P_y(z)$ found at the output $y(n)$, caused by the rounding operations on the multiplications.

c) Compute the output roundoff noise variance (output power).

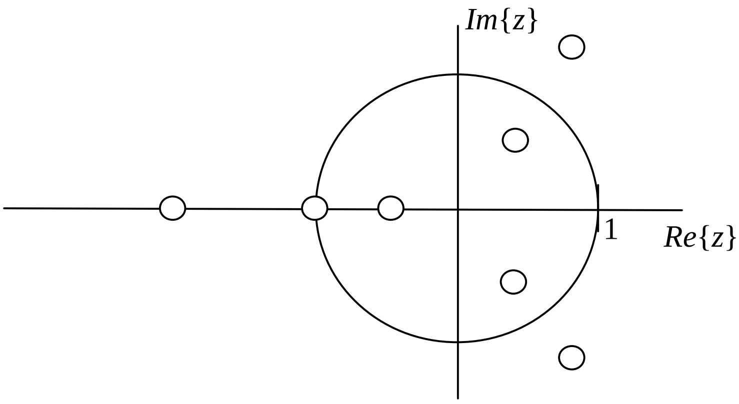


Question 3 Quick Theory Questions /7

- name an advantage of decomposing and processing a signal in M sub-bands

- when decomposing a signal into M sub-bands, explain why the overall complexity is not increased, even though there are then M signals to process instead of only one.

- is the filter described by the following zeros in the figure below a linear phase filter ? Explain why.



- explain which product quantization method (rounding, truncation) is normally preferred, and why

- what is the best window to use if we need to window a block of measured data to detect two components closely located in frequency (like two sinusoidal components) ?

- what are the main benefits (name two of them) of the Chebyshev/Parks-McLellan design method for linear phase FIR filters, over the use of a simple window-based method ?

- discuss the main advantages and drawbacks (name one of each) of the floating point number representation over the fixed point number representation.

Question 4 Filter Structures **/11**

For the following transfer function:

$$H(z) = \frac{\left(1 - \frac{1}{5}z^{-1}\right)\left(1 + z^{-1} + \frac{1}{2}z^{-2}\right)}{\left(1 - \frac{1}{3}z^{-1}\right)\left(1 - z^{-1} + \frac{1}{2}z^{-2}\right)}$$

draw the resulting a) canonical direct form (i.e. direct form II), b) the cascade form and c) the IIR lattice (i.e. lattice-ladder) form.

Question 5 IIR Design **/5**

The following filter is a low-pass Butterworth filter of order 3, with a normalized passband cutoff frequency $\Omega'_p = 1.0$:

$$H(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

Design a stopband discrete time filter $H(z)$ with cutoff frequencies $\omega_{p1} = \frac{\pi}{4}$ and $\omega_{p2} = \frac{\pi}{2}$. For the conversion from continuous time to discrete time, use the bilinear transform with $T = 2$. In each step, you do not need to find the numerical values of the coefficients and simplifications at each step are not necessary.

Question 6 FIR design **/13**

Consider the following specifications for a linear phase FIR bandpass filter with real coefficients:

$$\left.\begin{array}{ll} |H(e^{j\omega})| \le 0.01 & |\omega| < 0.2\pi \\ 0.95 \le |H(e^{j\omega})| \le 1.05 & 0.3\pi < |\omega| < 0.7\pi \\ |H(e^{j\omega})| \le 0.02 & |\omega| > 0.8\pi \end{array}\right\} \text{ specified over the interval 0 to } \pi .$$

a) Use the basic windowing method to design the filter (i.e. computing the impulse response of an ideal filter, then windowing it). Use a *Hanning* window and compute the order M (i.e. $M+1$ samples) required for the window. Use the following table:

Type of window	Approximate transition width of main lobe
Rectangular	$4\pi/(M+1)$
Bartlett	$8\pi/(M+1)$
Hanning	$8\pi/(M+1)$
Hamming	$8\pi/(M+1)$
Blackman	$12\pi/(M+1)$

b) For the same specifications, now use the procedure of the Kaiser window, based on the requirements for the ripple levels.