CSI 2101 Discrete Structures | Winter 2012
Prof. Lucia Moura | University of Ottawa

Homework Assignment #3 (100 points, weight 5%)
Due: Thursday, March 22, at 1:00pm (in lecture)

Induction and program correctness

1. (20 points) **Mathematical Induction**
   Use induction to prove that for very positive integer n,
   $$\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2.$$

2. (25 points) **Strong induction**
   Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of the integers $2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8$, and so on.

   Hint: for the inductive step, separately consider the case where $k+1$ is even and where it is odd. When it is even, note that $(k+1)/2$ is an integer.

3. (25 points) **Structural Induction**
   (a) Give a recursive definition of the function $ones(s)$, which counts the number of ones in a bit string s (a bitstring is a string over the alphabet $\Sigma = \{0, 1\}$).
   (b) Use structural induction to prove that $ones(s \cdot t) = ones(s) + ones(t)$; where the symbol "·" denotes concatenation of strings.
   Hint: in some of your steps you need to rely on the recursive definition of strings and of concatenation given in the textbook, as well as on the definition of $ones(s)$ given by you.

4. (30 points) **Correctness of recursive algorithms**
   Prove that Algorithm 6 (recursive binary search algorithm) in page 314 (Section 4.4) is correct, as follows. Consider the following statement:

   $P(k)$: " If n is an integer and $a_1, a_2, \ldots, a_n$ are integers in increasing order, and i, j, x are integers such that $1 \leq i \leq n, 1 \leq j \leq n$ and $j - i = k$, then procedure $binarysearch(i, j, x)$ calculates $location$, where $location = 0$ if there exists no l, $i \leq l \leq j$, with $a_l = x$, or $location = m$ and $a_m = x$ with $i \leq m \leq j$, otherwise."

   Use strong induction to prove that $P(k)$ is true for all $k \geq 0$.