Ultrafast All-Optical Wavelet Transform Based on Temporal Pulse Shaping Incorporating a 2-D Array of Cascaded Linearly Chirped Fiber Bragg Gratings

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***Abstract*— An all-optical wavelet transformer used to analyze a high-speed and broadband electrical signal is proposed and demonstrated for the first time to our knowledge. The wavelet transform is implemented based on a temporal pulse shaping system incorporating a 2-D array of cascaded linearly chirped fiber Bragg gratings (LCFBGs). An electrical signal to be analyzed is applied to a Mach–Zehnder modulator to modulate the optical spectrum of a time-stretched optical pulse from a mode-locked laser. Each individual LCFBG in the 2-D array functions as a wavelet filter to filter a specific range of the spectrum, which is equivalent to applying a window function to the corresponding section of the temporal signal, and at the same time, as a dispersive element to perform real-time Fourier transform. A Mexican Hat wavelet is employed to implement the optical wavelet transform, which can be achieved based on a specially designed LCFBG. A theoretical analysis is performed which is verified by a proof-of-concept experiment. The key feature of this technique is that it can perform multiresolution time-frequency analysis of an ultrahigh-speed electrical signal, which can give good time resolution for high-speed signals, and good frequency resolution for low-speed signals.**

***Index Terms*— Linearly chirped fiber Bragg grating, photonic-assisted microwave signal processing, wavelet transform.**

I. Introduction

WAVELET transform is an important technique for local analysis of non-stationary and fast transient wide-band signals. The wavelet transform is a mapping of a time signal to the time-scale joint representation that is similar to the short time Fourier transform and Wigner distribution [1]. Wavelet transform is usually implemented in the electrical domain using digital electronics, but the processing speed is slow due to the limited sampling rate of the state-of-the-art digital circuits. Thanks to the inherent high speed and large bandwidth offered by optics, many fundamental signal processing operations [2]–[4] including Wavelet transform can be implemented in the optical domain with a much higher speed and broader bandwidth.

The implementation of an optical wavelet transform could be realized based on free space optics, which can be used for two-dimensional (2-D) signal processing, such as image processing and pattern recognition [5], [6]. For temporal signal processing, however, a one-dimensional (1-D) optical wavelet transform should be employed. Very recently, we proposed an all-optical short-time Fourier transformer (STFT) based on temporal pulse shaping (TPS) with an array of cascaded linearly chirped fiber Bragg gratings (LCFBGs) [7]. Since the time-frequency resolution of an all-optical STFT is fixed, the designed windowing function (spectra of the cascaded LCFBGs) can not represent the time-frequency information of an original signal with a high resolution [8].

Recently, we have proposed a technique to implement all-optical wavelet transform based on a TPS system. Some preliminary results have been obtained [9]. In this letter, a more detailed investigation of the technique is performed. A basic TPS system usually consists of a pair of complementary dispersive elements and the output signal is the Fourier transform of the input microwave signal [10], [11]. In the proposed system, an electrical signal to be analyzed is applied to a Mach–Zehnder modulator (MZM) to modulate the optical spectrum of a time-stretched optical pulse from a mode-locked laser (MLL). Each individual LCFBG in the 2-D array functions as a wavelet filter to filter a specific range of the spectrum, which is equivalent to applying a window function to the corresponding section of the temporal signal, and at the same time, as a dispersive element to perform real-time Fourier transform. A Mexican Hat wavelet is employed to implement the optical wavelet transform, which can be achieved based on a specially designed LCFBG. The key feature of this technique is that it can implement multiresolution time-frequency analysis of an ultrahigh speed electrical signal, which can give a good time resolution for high-speed signals, and a good frequency resolution for low-speed signals. A theoretical analysis is performed which is verified by a proof-of-concept experiment.

II. Principle

Figure 1 shows a TPS system incorporating a 2-D array of LCFBGs for the all-optical implementation of

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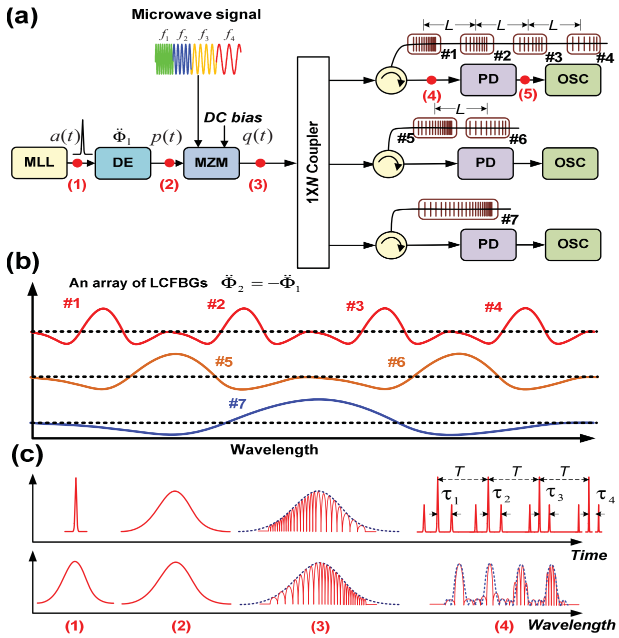

Fig. 1. (a) Schematic of an all-optical wavelet transformer based on a TPS system incorporating a 2-D array of LCFBGs. (b) Magnitude responses of the LCFBGs in the TPS system. (c) Signals at different locations of the system in the frequency and the temporal domains. LCFBG: linearly chirped fiber Bragg grating. MLL: mode-locked laser. DE: dispersive element. MZM: Mach–Zehnder modulator. OC: optical circulator. PD: photodetector. OSC: oscilloscope.

a wavelet transformer. A transform-limited Gaussian pulse is generated by an MLL, which is expressed as $a(t) = \exp(-t^2/\tau_0^2)$, where τ_0 is the half pulse width at 1/e maximum, and its Fourier transform is given by $A(\omega) = \sqrt{\pi}\tau_0 \exp(-\tau_0^2\omega^2/4)$, where ω denotes the angular frequency of the optical wave.

As shown in Fig. 1(a), a dispersive element (DE), connected before an MZM, is used to stretch the ultrashort optical pulse. It is assumed that the DE has a broad enough bandwidth to cover the entire spectral range of the optical pulse. The ultrashort optical pulse is temporally stretched and spectrally dispersed by passing through the DE. At the output of the DE, we have a stretched optical pulse, which is expressed as $P(\omega) = A(\omega) \times \exp\left(j\ddot{\Phi}_1\omega^2/2\right)$ [12], where $\ddot{\Phi}_1$ is the group velocity dispersion (GVD) of the DE. The stretched optical pulse is then directed into the MZM and is modulated by an electrical signal $x(t)$. The optical signal at the output of the MZM is given by $q(t) = p(t) \times m(t)$ or in the frequency domain

$$Q(\omega) = \frac{1}{2\pi}P(\omega) * M(\omega) = \frac{1}{2\pi}A(\omega) \times \exp\left(j\frac{\ddot{\Phi}_1}{2}\omega^2\right) * M(\omega) \quad (1)$$

where $M(\omega)$ is the Fourier transform of $m(t)$ which is the modulation function of the MZM when an electrical signal $x(t)$ is used to modulate the stretched optical pulse. When the MZM is biased at the minimum transmission point, the modulation function is given by $m(t) = -2j\sin[\beta x(t)]$. When the time-to-frequency mapping condition given by $\ddot{\Phi}_1 >> 8\pi/\Delta\omega_m^2$ is satisfied [13], where $\Delta\omega_m$ is the bandwidth of the electrical signal, we have $\exp\left(j\ddot{\Phi}_1\omega^2/2\right) * M(\omega) \propto \exp\exp\left(j\ddot{\Phi}_1\omega^2/2\right) \times m(t)|_{t=\ddot{\Phi}_1\omega}$, or

$$Q(\omega) \propto A(\omega) \times \exp\left(j\frac{\ddot{\Phi}_1}{2}\omega^2\right) \times m(t)|_{t=\ddot{\Phi}_1\omega}. \quad (2)$$

It can be seen from (2) that the amplitude of the optical spectrum of the input ultrashort pulse is modulated by a scaled version of the modulation signal $m(t)$ at the MZM, as shown in Fig. 1(c). The modulated signal is then directed into the 2-D array of LCFBGs through a $1\times N$ optical coupler. The transfer function of the ith LCFBG is given by $S_i(\omega) = G_i(\omega)\exp\left(j\ddot{\Phi}_i\omega^2/2\right)$, where $\ddot{\Phi}_i = -\ddot{\Phi}_1$ is the GVD of the LCFBG, and $G_i(\omega)$ is the amplitude response of the ith LCFBG. The output signal from the ith LCFBG in the time domain is given by

$$y_i(t) \propto \int_{\omega_i-\delta\omega_i/2}^{\omega_i+\delta\omega_i/2} m\left(\ddot{\Phi}_1\omega_i\right) \times G_i'(\omega_i) \times \exp(-j\omega t)\,d\omega \quad (3)$$

where $G_i'(\omega_i) = A(\omega) \times G_i(\omega_i)$ is the product between the optical spectrum of the ultrashort pulse $A(\omega)$ and the amplitude response $G_i(\omega_i)$ of the ith LCFBG, $\delta\omega_i$ is the bandwidth of the LCFBG, and ω_i represents the central frequency of the ith LCFBG. It worth noting that the bandwidth of the ith LCFBG determines the time-frequency resolution of the wavelet transform. Since the bandwidth of the LCFBG in a different branch of the N channels is different, the all-optical wavelet transform can be used to implement a multiresolution time-frequency analysis of an ultrahigh speed electrical signal. The LCFBG can be designed using a discrete layer peeling algorithm [14] and fabricated using a state-of-the-art FBG writing techniques. The output signal is detected at a photodetector (PD) and measured by an oscilloscope (OSC). The spectrum of the modulation signal locally in time can be retrieved by using the wavelet transformer.

III. Proof-of-Concept Experiment

A proof-of-concept experiment is then implemented to validate the proposed technique. An MLL generating an ultrashort pulse train with a repetition rate of 48.6 MHz and a pulse width of 550 fs is directed into the DE, which is a dispersion compensating fiber (DCF) having a value of dispersion of 784.24 ps^2/rad, to stretch the ultrashort pulse from the MLL, as shown in Fig. 1(a). The pulse from the MLL has a full-width at half-maximum (FWHM) of 8 nm with a central wavelength of 1557.4 nm. Since a phase mask that can be used to fabricate the 2-D array of LCFBGs with a dispersion value of 784.24 ps^2/rad and a bandwidth of about 20 nm is not available, the magnitude response of the LCFBG, which is a Mexican Hat wavelet, is equivalently implemented using a programmable optical filter. A single-mode fiber (SMF) with a value of dispersion of -784.24 ps^2/rad is connected after the programmable optical filter, and an LCFBG is thus equivalently realized through a joint use of the programmable optical filter and the SMF.

The stretched pulse is modulated by two consecutive electrical pulses with a carrier frequency of 1.5 GHz and

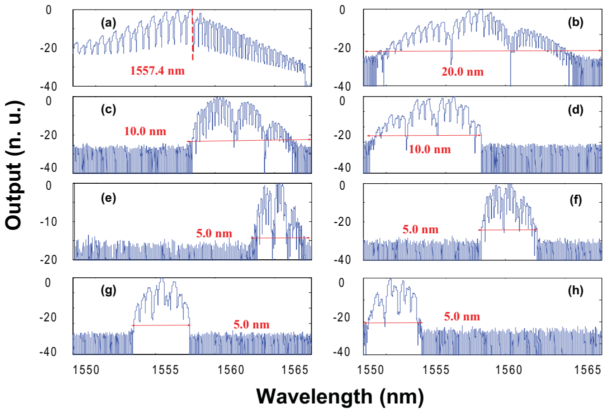

Fig. 2. Measured optical spectra of the optical signal at the output of the programmable optical filter when the programmable optical filter with a Mexican Hat wavelet shape has a different central wavelength and bandwidth of (a) 1557.4 and 20 nm, (b) 1557 and 15 nm, (c) 1562 and 10 nm, (d) 1555 and 10 nm, (e) 1564 and 5 nm, (f) 1560 and 5 nm, (g) 1555 and 5 nm, and (h) 1552 and 5 nm.

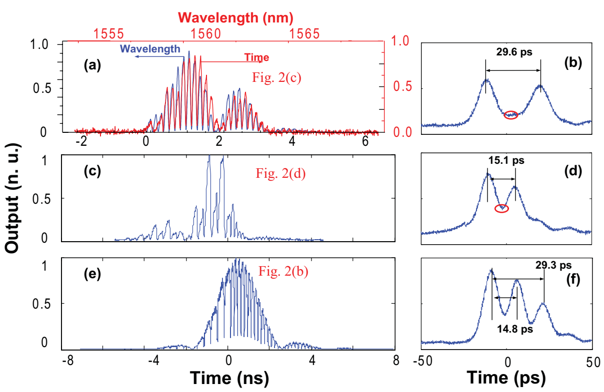

Fig. 3. Measured temporal waveforms at the output of the MZM [3(a), 3(c), and 3(e)] and at the output of the SMF [3(b), 3(d), and 3(f)], corresponding to the spectra shown in Fig. 2(c), 2(d), and 2(b). The spectrum shown in Fig. 2(c) is also shown in Fig. 3(a) to verify the time-to-frequency analogy.

3 GHz at the MZM. The MZM is biased to operate at the double-sideband with suppressed carrier (DSB-SC) modulation mode. Fig. 2 shows the spectrum of the optical signal at the output of the programmable optical filter. By tuning the central wavelength and bandwidth of the programmable optical filter, a different Mexican Hat wavelet function with different central wavelength and bandwidth is applied to the modulation signal, as shown in Fig. 2. Fig. 3(a) shows a measured temporal waveform at the output of the programmable optical filter. Its spectrum is also shown. As can be seen the spectrum has a shape that is a scaled version of the temporal waveform, which verifies the time-to-frequency analogy given by (2).

Fig. 3(a), (c) and (e) shows the temporal waveforms at the output of the programmable optical filter, and Fig. 3(a), (c) and (e) shows the temporal waveforms at the output of the SMF, which correspond to the spectra shown in Fig. 2(c), (d) and (b). The time spacing in Fig. 3(b) between the two peaks is 29.6 ps, corresponding to a microwave frequency of 3.01 GHz. By tuning the central wavelength of the programmable optical filter, the 1.5-GHz signal is Fourier transformed, with the result show in Fig. 3(d). The time spacing between the two peaks is 15.1 ps, corresponding to a microwave frequency of 1.53 GHz. When the bandwidth of the programmable optical filter covers the whole spectrum of the MLL, both the low and high frequency microwave signals are Fourier transformed. The output pulses should be a summation of the output pulses shown in Fig. 3(b) and (d). However, due to the existence of the third-order dispersion of the SMF and the DCF and the mismatch of the third-order dispersion, the temporal position of the pulse pair shown in Fig. 3(d) is shifted such that the left pulse in Fig. 3(d) is located in the same location as the left pulse in Fig. 3(b), leading to three pulses at the output. To solve the problem, LCFBGs with no or very low third-order dispersion should be employed.

IV. Conclusion

We have proposed and demonstrated a novel technique to implement all-optical wavelet transform based on a TPS system incorporating a 2-D array of LCFBGs. The key feature of this technique is that it can be used to implement multiresolution time-frequency analysis of an ultrahigh speed electrical signal, which gives a good time resolution for high-speed signals, and a good frequency resolution for low-speed signals. A theoretical analysis was performed which was verified by a proof-of-concept experiment.

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