Power Distribution of Phase-Modulated Microwave Signals in a Dispersive Fiber-Optic Link

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***Abstract*—An optical phase-modulated signal can be converted to an intensity-modulated signal in a dispersive optical fiber. Due to the intrinsic nonlinearity of optical phase modulation, higher order harmonics will be generated. In this letter, an analytical model to describe the transmission of a phase-modulated microwave signal in a dispersive radio-over-fiber (RoF) link is presented. A compact and closed-form expression of the generated harmonics at the output of the RoF link is derived. Base on the analytical model, results on the power distributions of different orders of harmonics along the RoF link as well as the upper limit of the dynamic range of the RoF system are obtained. The presented model helps simplify the design and analysis of a phase-modulation-based microwave photonic link or system.**

***Index Terms*—Chromatic dispersion, microwave photonics, phase modulation (PM), radio-over-fiber (RoF), wireless optical link.**

I. INTRODUCTION

RECENTLY, optical phase modulation (PM) has been widely used in microwave-photonic systems, for applications such as radio-over-fiber (RoF) transmission, photonic microwave mixing, photonic microwave bandpass filtering, and photonic microwave generation. The key advantage of using an optical phase modulator in a microwave photonic system is that the phase modulator is not biased which eliminates the bias drifting problem that exists in an optical intensity-modulator-based system. In a PM-based system, the phase-modulated signal would be converted to an intensity-modulated (IM) signal through PM–IM conversion in a dispersive optical fiber. The converted IM signal can be directly detected using a photodetector (PD). This property has been recently utilized in an RoF system for subcarrier signal transmission [1], [2] and photonic microwave bandpass filtering [3]. For large signal modulation, the intrinsic nonlinearity of optical PM would lead to the generation of higher order sidebands. The nonlinearity property of PM has recently been used in photonic microwave mixing and generation [4], [5]. However, for linear applications, such as in an RoF link and a photonic microwave filter, the modulation index has to be kept small to avoid PM-induced nonlinearity.

In the design of a PM-based subcarrier system, it is essential to know analytically the distributions of the generated sidebands in the optical link. For linear applications, the small signal approximation is widely used, where only the first-order optical sidebands are considered. For nonlinear applications, however, higher order sidebands should be considered. A common way reported in literature is to consider only a small number of higher order sidebands to simplify the mathematical treatment [3], [5]. Although an analytical model for an RoF transmission link using an intensity modulator has recently been presented [6], to our knowledge, no closed-form expression incorporating all orders of sidebands for PM-based RoF transmission link has been reported. In this letter, an analytical model to describe the transmission of phase-modulated microwave signals in a dispersive RoF link is presented. The analytical model would help simplify the design and analysis of a PM-based microwave photonic link.

II. THEORETICAL DERIVATION

The complex envelope of a phase-modulated microwave signal can be written as

$$\begin{aligned} E(t) &= \exp[jm\cos(\omega_m t)] \\ &= \sum_{n=-\infty}^{\infty} j^n J_n(m)\exp(jn\omega_m t) \end{aligned} \tag{1}$$

where m is the modulation index, ω_m is the angular frequency of the modulating microwave signal, $J_n(\cdot)$ is the Bessel function of the first kind of order n.

The transfer function of a dispersive optical fiber with a length of L is given as

$$H(j\omega) = \exp\left(j\frac{1}{2}\beta_2 L\omega^2\right) \tag{2}$$

where β_2 is the dispersion coefficient of the optical fiber.

The complex envelope of the signal after transmission in the dispersive fiber can be expressed as

$$\begin{aligned} E_f(t) &= \sum_{n=-\infty}^{\infty} j^n J_n(m) \\ &\quad \times \exp(jn\omega_m t)\exp\left(j\frac{1}{2}\beta_2 L n^2\omega_m^2\right) \\ &= \sum_{n=-\infty}^{\infty} \Gamma_n. \end{aligned} \tag{3}$$

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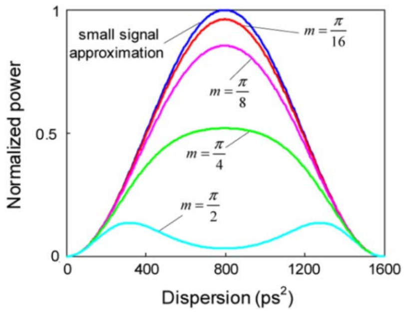

Fig. 1. Power distribution of the first-order harmonic versus the dispersion in the optical link with different modulation indexes (10-GHz signal).

The complex conjugate of $E_f(t)$ is

$$\begin{aligned} E_f^*(t) &= \sum_{p=-\infty}^{\infty} j^{-p} J_p(m) \exp(-jp\omega_m t) \\ &\quad \times \exp\left(-j\frac{1}{2}\beta_2 L p^2 \omega_m^2\right) \\ &= \sum_{p=-\infty}^{\infty} \Gamma_p^*. \end{aligned} \tag{4}$$

The PM signal after the dispersive fiber has an infinite number of optical components, with a specific phase shift introduced to each optical component due to the fiber dispersion. The electrical signal at the output of the PD is proportional to the optical intensity, which is given by

$$i(t) = \Re \times E_f(t) \times E_f^*(t) \tag{5}$$

where $\Re$ is the responsivity of the PD. The photodetection at the PD results in the beating between any two optical components, leading to the generation of different orders of electrical harmonic components. By observing (3) and (4), we find that the kth harmonic component $\exp(jk\omega_m t)$ with an angular frequency $k\omega_m$ can be expressed as $i_k(t) = \sum_{n=-\infty}^{\infty} \Gamma_{n+k}\Gamma_n^*$. Then we have

$$\begin{aligned} i_k(t) = \Re \times \sum_{n=-\infty}^{\infty} \Big\{ & j^{n+k} J_{n+k}(m) \exp\left[j(n+k)\omega_m t\right] \\ & \times \exp\left[j\frac{1}{2}\beta_2 L(n+k)^2\omega_m^2\right] \\ & \times j^{-n} J_n(m) \exp\left(-jn\omega_m t\right) \\ & \times \exp\left(-j\frac{1}{2}\beta_2 L n^2 \omega_m^2\right)\Big\}. \end{aligned} \tag{6}$$

In order to use the addition theorem for Bessel functions, we rearrange (6) as

$$\begin{aligned} i_k(t) = \Re \times j^k & \exp\left(-j\frac{1}{2}\beta_2 L k^2 \omega_m^2\right) \\ & \times \exp\left(jk\omega_m t\right) \times \sum_{n=-\infty}^{\infty} \\ & \times \left\{J_{n+k}(m) \exp\left[j\beta_2 L(n+k)k\omega_m^2\right\}\right. \\ & \times \{J_n(m) \exp(j0)\}. \end{aligned} \tag{7}$$

The addition theorem for Bessel functions is expressed as [7]

$$J_k(R)e^{jk\Omega} = \sum_{n=-\infty}^{\infty} [J_{k+n}(r)e^{j(k+n)\theta}][J_n(r_0)e^{-jn\theta_0}] \tag{8}$$

where $R \cdot e^{j\Omega} = r \cdot e^{j\theta} - r_0 \cdot e^{j\theta_0}$.

By comparing (7) with (8), we have $r = m, \theta = \beta_2 L k \omega_m^2$, and $r_0 = m, \theta_0 = 0$. Then R and Ω are calculated to be $R = 2m\sin(\theta/2)$, and $\Omega = (1/2)(\pi + \theta)$. Therefore, $i_k(t)$ can now be written as

$$\begin{aligned} i_k(t) =& \Re \times j^k \exp\left(-j\frac{1}{2}\beta_2 L k^2 \omega_m^2\right) \exp\left(jk\omega_m t\right) \\ & \times J_k\left(2m\sin\frac{\theta}{2}\right) \exp\left[j\frac{1}{2}(\pi+\theta)k\right] \end{aligned} \tag{9}$$

which can be further simplified as

$$\begin{aligned} i_k(t) = \Re \times (-1)^k J_k & \left[2m\sin\left(\frac{1}{2}\beta_2 L k \omega_m^2\right)\right] \\ & \times \exp\left(jk\omega_m t\right). \end{aligned} \tag{10}$$

Equation (10) is a closed-form expression that shows the kth order harmonic component at the output of the PD after the dispersive fiber. Note that in the theoretical treatment, no small signal approximation was used and all optical components were included in the beating process. Note also that in the mathematical treatment, the frequency response of the phase modulator is normalized to unity.

The overall electrical signal at the output of the PD is the sum of all the harmonic components, $i(t) = \sum_{k=0}^{\infty} i_k(t)$. For the first-order harmonic, if the condition for the small signal approximation $m \ll 1$ is satisfied, we have

$$i_1(t) = \Re \times (-m)\sin\left(\frac{1}{2}\beta_2 L \omega_m^2\right) \exp\left(j\omega_m t\right) \tag{11}$$

where $J_1(m) \cong m/2$ is applied. Equation (11) is the well-known formula that characterizes the frequency response of the PM-based RoF system for a dispersive optical fiber with a length L and a dispersion coefficient of β_2 under linear modulation assumption [8].

III. Results and Discussion

Based on the developed analytical model, the power distributions of different orders of harmonics along the dispersive RoF link as well as the upper limit of the dynamic range of the system are evaluated. Fig. 1 shows the output power of the first-order harmonic versus the dispersion with different modulation indexes in a system with 10-GHz signal input, where the harmonic power is normalized to m^2. For comparison, the result given by the small signal approximation is also plotted in the figure. It is shown that an increase in the modulation index would lead to a decrease in the normalized power (i.e., the efficiency of the transmission system is reduced) due to the stronger modulation nonlinearity. More importantly, the maximum power point along the dispersive link also changes in the case of large modulation index, as can be seen from the curve for which $m = \pi/2$ in Fig. 1. Fig. 2 shows the frequency response when $\beta_2 L = 800$ ps^2 for different modulation indexes. Due to the nonlinearity, the peak value of the transfer function with an

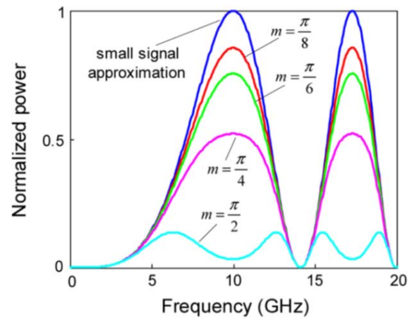

Fig. 2. Frequency response for different modulation indexes ($\beta_2 L = 800\ \mathrm{ps}^2$).

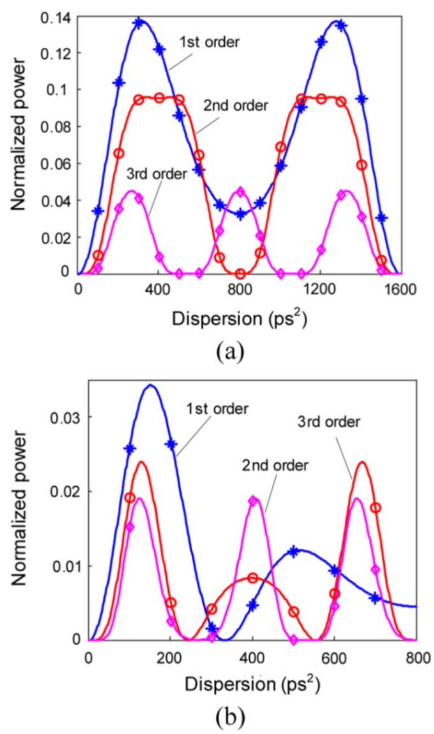

Fig. 3. Power distributions of the first-, second-, and third-order harmonics versus the dispersion in the optical link, where solid curves are theoretical predictions, the stars, circles, and diamonds denote numerical simulation results. (a) $m = \pi/2$; (b) $m = \pi$.

increased modulation index would decrease. A measure widely used to estimate the upper limit of the dynamic range is the 1-dB compression point, which indicates the RF power gain to drop by 1 dB from its small signal value. The modulation index that corresponds to the 1-dB compression point is estimated to be around $m = \pi/6.6$ at the place of the maximum response of the dispersive link. Assume that the halfwave voltage and the input impedance of the phase modulator are respectively 4.8 V and 50 Ω, the same values used in [3]; the corresponding input RF power at the phase modulator can be estimated to be 10.3 dBm, which agrees well with the result in [3, Section IV(B)].

Fig. 3 shows the power distributions of the first-, second-, and third-order harmonics along a single-mode fiber link. Numerical simulation based on fast Fourier transform (FFT) is also implemented to verify the given theoretical model, with the results also shown in Fig. 3. To achieve a high calculation precision in the simulation, we set the frequency range of the FFT as 40 times that of the input signal frequency, which means the first 20 optical sidebands are included in the beating process. It is shown that the results predicted by the theoretical model perfectly match the simulation results, which proves the correctness of the derived analytical model.

The derived analytical expression is useful for the evaluation of the performance of a PM-based microwave photonic system. It is worth noting that the closed-form expression is obtained thanks to the use of the addition theorem for Bessel functions, which was previously used in [6] to analyze the transmission of an IM signal in a dispersive optical link. For a microwave photonic link using double sideband modulation, the fiber dispersion would lead to the well-know power variations along optical link, which are usually considered a negative effect. For microwave photonic link employing PM, a dispersive device is usually used to achieve the PM to IM conversion, which can be a length of optical fiber. Therefore, the fiber in the microwave photonic system can not only serve as a transmission medium for signal distribution, but also as a dispersive device to perform PM–IM conversion. To obtain a maximum conversion efficiency, in a practical system a chirped fiber grating may be used at the receiver end to get the required total dispersion [4].

IV. Conclusion

An analytical model to describe the transmission of a phase-modulated microwave signal in a dispersive RoF link was developed. A compact and closed-form expression for the signal at the output of the fiber link was derived with all orders of optical sidebands being considered. Based on the developed analytical model, new results about the power distributions of different orders of harmonics along the fiber link as well as the upper limit of the dynamic range of the RoF system were obtained. The presented result would help simplify the design and analysis of PM-based microwave photonic links or systems.

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