A Continuously Tunable Microwave Fractional Hilbert Transformer Based on a Photonic Microwave Delay-Line Filter Using a Polarization Modulator

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***Abstract*—A continuously tunable microwave fractional Hilbert transformer (FHT) implemented based on a photonic microwave delay-line filter is proposed and demonstrated. The photonic microwave delay-line filter with negative coefficients is realized based on polarization-modulation using a polarization modulator (PolM) and polarization-modulation to intensity-modulation conversion in an optical polarizer. The tunability of the fractional order is achieved by tuning the coefficient of the zeroth tap. An FHT with a tunable order from 0.3 to 1 is demonstrated. The accuracy of the FHT is evaluated; a phase deviation less than 5° within the passband is achieved.**

***Index Terms*—Hilbert transform, microwave photonics, optical signal processing, photonic microwave delay-line filter.**

I. Introduction

A Hilbert transformer (HT) is a fundamental operator for signal processing, which can find numerous applications such as in radar systems, communication systems and modern instrumentation systems [1]. To improve the performance of an HT and also to provide an additional degree of flexibility, a classical HT was generalized into a fractional Hilbert transformer (FHT) in [2]. Therefore, the classical Hilbert transform can be considered as a special case of the fractional Hilbert transform. In [3], [4], a discrete version of an FHT was developed which was applied to the implementations of single sideband (SSB) modulation and image edge detection.

The implementation of a classical HT based on an integrated electronic delay-line filter was demonstrated in [5]. The phase error of the HT in [5] was about 10°, which was considered large. Thanks to the advantages of broad bandwidth and low loss offered by optics, photonic implementation of microwave Hilbert transform has been a topic of interest recently [6]–[10]. In general, a photonic-assisted FHT can be realized based on a fiber Bragg grating (FBG) or a multitap photonic microwave transversal filter [6]–[10]. In [6]–[8], a photonic microwave FHT was implemented using a phase-shifted uniform fiber Bragg grating (FBG) or a sampled FBG. The sampled FBG in [8] was used as a classical HT to implement optical single-sideband modulation. The major limitation of the FBG-based FHT is that the fractional order is not tunable. Once the FBG is fabricated, the order of the FHT is fixed. In [9], [10], a classical microwave HT implemented based on a multitap photonic microwave delay-line filter was proposed, and the use of the HT to achieve two orthogonally phased microwave signals was reported [9], [10]. The negative taps of the filter were generated using two Mach–Zehnder modulators (MZMs) that are biased to operate at the complementary slopes. A classical HT with a fixed order was achieved, but the tunability of the fractional order was not proposed or demonstrated.

In this letter, we propose and demonstrate an FHT based on a microwave photonic delay-line filter using a single polarization modulator (PolM) with continuously tunable fractional order. We show that the fractional order of the FHT can be continuously tuned by simply tuning the zeroth tap coefficient of the delay-line filter. The negative taps are achieved based on polarization-modulation to intensity-modulation conversion. A tunable FHT based on a five-tap photonic microwave delay-line filter is designed and experimentally demonstrated. An FHT with an order tunable from 0.3 to 1 is implemented.

II. Principle

The frequency response of an FHT is given by [2]

$$H_P(f)=\begin{cases} e^{-j\varphi}, & f\geq 0\\ e^{j\varphi}, & f<0\end{cases} \tag{1}$$

where $\varphi = P\times\pi/2$ and P is the fractional order. As can be seen, an FHT is a φ phase shifter and the FHT becomes a classical HT when $P = 1$. The corresponding discrete-time impulse response of the FHT is given by [2], [4]

$$h_P[n]=\begin{cases} \sin(\varphi)\cdot\frac{2}{\pi}\cdot\frac{\sin^2(\frac{\pi n}{2})}{n}, & n\neq 0\\ \cos(\varphi), & n=0\end{cases} \tag{2}$$

As (2) is the discrete version of the ideal temporal impulse response of the FHT, the corresponding frequency response is periodic and bandwidth-limited. From (2) we can see that the coefficients of the even taps are all equal to zero except the zeroth

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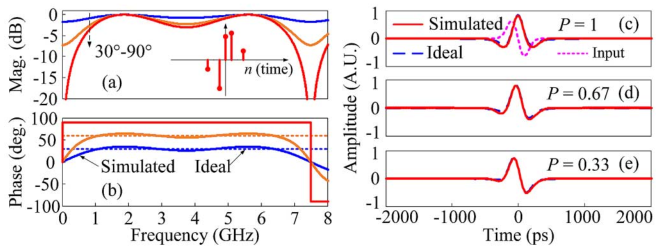

Fig. 1. (a)–(b) Normalized frequency response of a continuously tunable FHT with different phase shifts at 30°, 60°, and 90°. Inset: impulse response of the FHT. (c)–(e) Numerical simulations for the designed FHT shown in (a)–(b).

tap and the odd taps are negative for $n < 0$. Considering that the impulse response of the classical HT is [1]

$$h_1[n] = \begin{cases} \frac{2}{\pi} \cdot \frac{\sin^2(\frac{\pi n}{2})}{n}, & n \neq 0 \\ 0, & n = 0 \end{cases} \tag{3}$$

thus, by using (3), the impulse response of the FHT can be expressed as [2], [4]

$$\begin{aligned} h_P[n] &= \cos(\varphi)\delta[n] + \sin(\varphi)h_1[n] \\ &= \sin(\varphi)\left\{\frac{\cos(\varphi)}{\sin(\varphi)}\delta[n] + h_1[n]\right\} \end{aligned} \tag{4}$$

where $\delta[n]$ is the unit impulse function. It can be seen from (4) that the fractional Hilbert transform of a signal is a weighted sum of the original signal and its classical Hilbert transform. In addition, the order of the FHT is continuously tunable by adjusting the coefficient of the zeroth tap only, which makes the tuning easy to achieve. Note that the output microwave power is not constant for different fractional orders. To solve this problem, the sum of all the tap coefficients can be kept constant using an optical amplifier with a constant output power, and only the ratio between the zeroth tap coefficient and other tap coefficients is adjusted.

Since the impulse response of the FHT given by (2) extends to infinity in time, it should be truncated for practical implementation. Usually, a proper time-domain window function should be applied to the tap coefficients to obtain an optimal trade-off between the ripples of the spectral response and the bandwidth of the filter. In addition, the impulse response is anti-causal; therefore, for practical implementations, a proper time delay should be introduced to the truncated impulse response to make the system causal. A time-delayed impulse response is $h_d[n] = h_P[n - n_0]$, where n_0 is a prescribed time delay [11].

Fig. 1(a) and (b) show the normalized frequency response of an FHT based on a seven tap delay line filter with different phase shifts (φ) at 30°, 60° and 90°, where the time delay difference between two adjacent taps is 66.66 ps, which corresponds to a free spectral range (FSR) of 15 GHz (-7.5 to 7.5 GHz) [11]. The nonzero tap coefficients are $[-0.33, -1, h_P[0], 1, 0.33]$, where $h_P[0]$ is the tap coefficient of the zeroth tap. Here, $h_P[0]$ is 2.72, 0.91 or 0 when φ is 30°, 60° or 90°. The inset of Fig. 1(a) shows $h_P[n]$ for $\varphi = 60°$, which is normalized to $(2/\pi)\sin(\phi)$. As can be seen from Fig. 1(a) and (b), the operating bandwidth of the FHT is between 0.89 GHz and 6.66 GHz and the phase variations are less than 5° within the operating bandwidth. To evaluate the performance of the designed FHT, the temporal response of the FHT to an input signal of the first-order derivative of a transform-limited 160-ps (full-width half-maximum) Gaussian pulse is simulated. The temporal envelope of the output waveform is shown in Fig. 1(c)–(e). As can be seen, there is an excellent agreement between the simulated output waveforms and the waveforms from an ideal (calculated) fractional Hilbert transform.

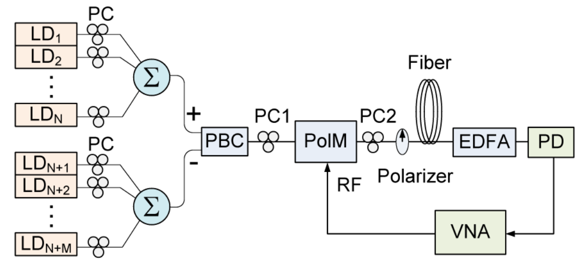

Fig. 2. Experimental setup for a continuously tunable FHT.

III. Experimental Results and Discussion

The proposed continuously tunable microwave FHT based on a five-tap delay-line filter is then experimentally demonstrated. The delay-line filter has both positive and negative coefficients, which are realized using a PolM [12].

The experimental setup is shown in Fig. 2. The "+" and "−" groups of light waves from the laser diodes (LDs) are combined and then sent to the two input ports of a polarization beam combiner (PBC) to ensure that the two groups of light waves are orthogonally polarized. The polarization-combined light waves at the output of the PBC are sent to a PolM through a polarization controller (PC1), which is used to adjust the polarization directions of the two groups of light waves to be 45° or 135° relative to one principal axis of the PolM. An optical polarizer is connected at the output of the PolM to perform the polarization-modulation to intensity-modulation. The polarization axis of the polarizer is aligned at an angle of 45° to the same principal axis of the PolM using a second PC (PC2). With the polarization direction of the input light wave at an angle of 45° or 135° to the principal axis of the PolM, an inverted or non-inverted intensity-modulated optical signal, corresponding to a negative or positive coefficient, would be obtained. The intensity-modulated signals at the output of the polarizer are sent to a dispersive fiber to introduce time delays, and then applied to a photodetector (PD). An erbium-doped fiber amplifier (EDFA) with a constant output power is used at the output of the fiber to compensate for the system loss and keep the optical power at the input of the PD constant, which means the sum of all the tap coefficients is kept constant. A delay-line filter with both positive and negative tap coefficients is thus achieved.

For an FHT based on a five-tap delay-line filter, since the ±2nd order coefficients are zero, only three nonzero taps are needed which are realized using three LDs. A 10.1-km standard single-mode fiber (SMF) with a value of dispersion of about 166.65 ps/nm at around 1545 nm is used as the dispersive fiber. The wavelengths of the nonzero taps are set at [1544.392,1544.792,1545.192] nm; thus, the time delay difference is 66.66 ps, corresponding an FSR of 15 GHz. The desired tap coefficients are adjusted by controlling the powers

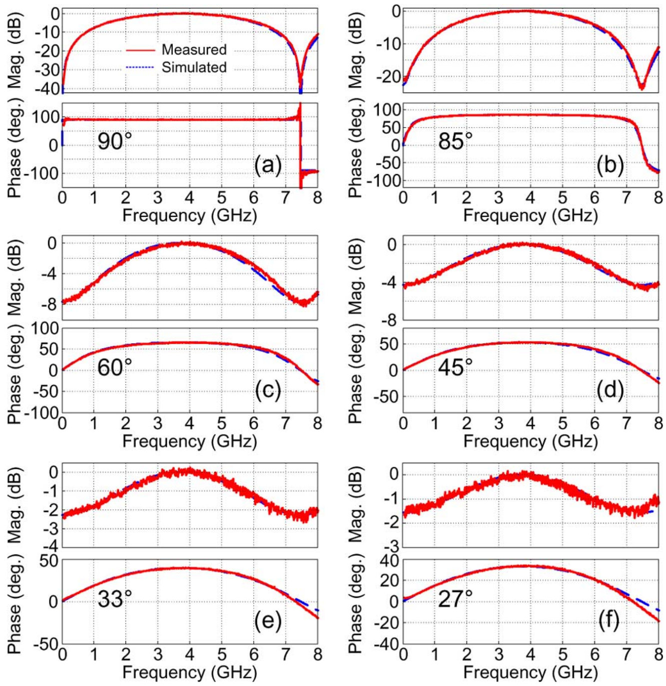

Fig. 3. Measured frequency responses of the FHT with phase shifts of 90°, 85°, 60°, 45°, 33°, and 27°. Simulated frequency responses are also shown for comparison.

of the LDs. The tap coefficients are $[1, h_p[0], 1]$. By tuning $h_p[0]$ while keeping the coefficients of the other taps unchanged, an FHT with a continuously tunable fractional order is achieved.

The normalized frequency response of the FHT with different phase shifts (φ) of 90°, 85°, 60°, 45°, 33° and 27°, corresponding to a tunable order of 1, 0.94, 0.67, 0.5, 0.37, and 0.3, is shown in Fig. 3, which is measured using a vector network analyzer (VNA, Agilent E8364A). The VNA is calibrated using the zeroth tap before the measurement of the frequency responses. As can be seen from Fig. 3, an excellent agreement is achieved between the measured and the simulated results and a phase shift between 0 and π at around zero frequency and 7.5 GHz could be clearly observed. The amplitude variations are less than 3 dB between 1.8 GHz and 5.8 GHz and the phase variations are about 5° within the 3-dB passband. Thus, an FHT with tunable fractional order is demonstrated. In the experiment, the power difference between different frequency responses with different fractional orders is less than 3 dB thanks to the employment of an EDFA with a constant output power.

It can be seen from Figs. 1(a) and (b) and 3 that the phase variations and the bandwidth of the FHT are associated with the number of taps for a given FSR. To reduce the phase variations and to increase the bandwidth, more taps should be used. In the experiment, the operating bandwidth is mainly limited by the bandwidth of the PD, which is 12 GHz. As the bandwidth of the PolM is about 40 GHz, the bandwidth of the proposed system could be increased to 40 GHz by increasing the bandwidth the PD. For wider bandwidth operation, the power fading effect resulted from the chromatic dispersion of the dispersive fiber should also be taken into consideration [13].

IV. Conclusion

We have proposed and demonstrated a continuously tunable FHT based on a photonic microwave delay-line filter with negative tap coefficients using a single PolM. We demonstrated that the fractional order can be continuously tuned by tuning the zeroth tap coefficient only. An FHT with a tunable order from 0.3 to 1 was experimentally demonstrated. The accuracy of the FHT was evaluated; a phase deviation less than 5° within the passband was achieved. The proposed FHT can find applications such as in a secure communication system, in which the fractional order is used as the secret key for demodulation [4].

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