Arbitrary pulse shaping based on intensity-only modulation in the frequency domain

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We propose a novel technique to implement arbitrary pulse shaping of an ultrashort pulse by intensity-only modulation in the frequency domain. The intensity-only modulation is realized by nonuniformly spaced sampling in the frequency domain. By properly designing the sampling function, multiple pulses in the time domain will be generated with one of which being the desired waveform. The desired waveform is then selected by a time window. Both the optical amplitude and phase distributions of the output waveform can be controlled. Theoretical analysis is presented. An example showing the generation of a rectangular pulse is provided. © 2008 Optical Society of America

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Ultrashort pulse shaping has become increasingly important and can be found in many important applications such as in optical communications, quantum coherent control, and femtosecond spectroscopy [1]. Most of the pulse-shaping techniques are implemented in the Fourier domain to manipulate the amplitude and phase of the optical pulse in the frequency domain. Usually, the amplitude and phase of the input pulse in the frequency domain can be altered using a spatial light modulator (SLM) [1,2]. Ultrashort pulse shaping can also be implemented in the time domain using a temporal pulse-shaping (TPS) system that consists of two conjugate dispersive elements and a modulator placed in between. An ultrashort optical pulse is temporally stretched and spectrally dispersed by passing through the first dispersive element, which is then spectrum shaped by modulating the spectrum with an RF signal at the modulator. The temporal compression is realized by passing the spectrum-shaped pulse through the second dispersive element. A waveform that is the Fourier transform of the RF signal is obtained [3–5]. A thorough review of these techniques can be found in [1].

Theoretically, an ultrashort pulse-shaping system could be modeled as a linear and time-invariant (LTI) filter. Assume that the LTI filter has a frequency response $H(\omega)$, or an impulse response $h(t)$. If the input signal is $E_I(\omega)$, the output signal is then given by $E_O(\omega)=[H(\omega)]E_I(\omega)$. For a given input pulse, since the output pulse can be a waveform having any amplitude and phase distributions, the frequency response of the filter, $H(\omega)$, should have both the amplitude and phase terms, i.e., $H(\omega)=|H(\omega)|\exp[j\varphi(\omega)]$, which should be realized in a pulse-shaping system with both amplitude and phase modulations. It is challenging to implement both amplitude and phase modulations in the frequency domain, especially since an accurate synchronization between the amplitude and phase signals is required, which makes the system complicated and costly.

In this Letter, we propose a novel technique to implement arbitrary pulse shaping in the frequency domain using intensity-only modulation. In the proposed system, the required phase modulation could be equivalently realized by *nonuniformly spaced sampling*. It is known that the sampling in the frequency domain would result in multiple pulses in the time domain. In our approach, the nonuniformly spaced sampling in the frequency domain could lead to the generation of multiple pulses with one having the desired amplitude and phase profiles, which can be selected with the help of a time window. A diagram showing the proposed arbitrary pulse shaping system is illustrated in Fig. 1.

Assume $S_0(\omega)$ is a periodic sampling function with a period of Ω in the frequency domain, which can be expanded using the Fourier series expansion

$$S_0(\omega)=\sum_k F_k \exp\left(jk\frac{2\pi}{\Omega}\omega\right), \quad (1)$$

where F_k is the Fourier series coefficients. In the proposed approach, the pulse shaping is implemented

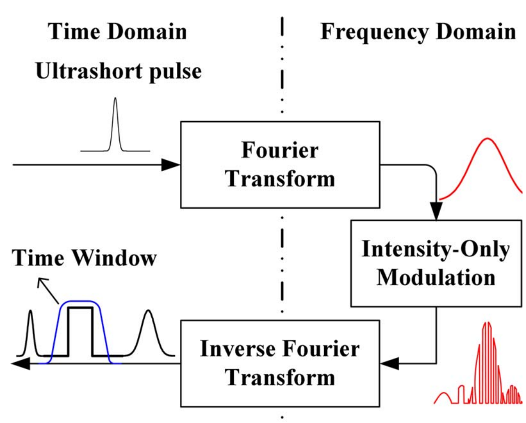

Fig. 1. (Color online) Arbitrary pulse shaping with intensity-only modulation in the frequency domain.

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using nonuniformly spaced sampling in the frequency domain. If the phase and the amplitude of the sampling function are modulated, we have

$$S(\omega) = A(\omega) S_0[\omega + f(\omega)], \qquad (2)$$

where $f(\omega)$ and $A(\omega)$ are respectively the phase and amplitude modulation functions. Substituting Eq. (1) into Eq. (2), we have

$$S(\omega) = \sum_k F_k A(\omega) \exp\left[jk \frac{2\pi}{\Omega} f(\omega) \right] \exp\left(jk \frac{2\pi}{\Omega} \omega \right). \qquad (3)$$

Let $S_k(\omega) = A(\omega)\exp[jk(2\pi/\Omega) f(\omega)]$; Eq. (3) can be rewritten as

$$S(\omega) = \sum_k F_k S_k(\omega) \exp\left(jk \frac{2\pi}{\Omega} \omega \right). \qquad (4)$$

Applying the inverse Fourier transform to Eq. (4) by considering the time-domain shifting property, the time-domain representation of the nonuniformly spaced sampling function is

$$s(t) = \sum_k F_k s_k(t + kT), \qquad (5)$$

where $s_k(t)$ is the inverse Fourier transform of $S_k(\omega)$ and $T = (2\pi/\Omega)$. Note that the operation of the nonuniformly spaced sampling in the frequency domain can be considered as frequency-domain filtering. Therefore, $s(t)$ in Eq. (5) can be regarded as the impulse response of the filter. It is clearly seen that the impulse response has multiple pulses with a temporal separation between two adjacent pulses of T. If an ultrashort pulse is input into the pulse shaping system, the output would be a convolution between the input ultrashort pulse with $s_k(t+kT)$, and multiple pulses located at 0, $\pm T$, $\pm 2T$, $\pm 3T, \ldots$, would be generated as shown in Fig. 1. If the desired waveform is the mth pulse located at $-mT$, then $s_m(t)$ should be equal to the desired impulse response, $h(t)$, which means that the mth-order Fourier component of $S(\omega)$ in Eq. (3), $S_k(\omega)$, should be equal to the desired frequency response, $H(\omega)$:

$$A(\omega) \exp\left[jk \frac{2\pi}{\Omega} f(\omega) \right] = |H(\omega)| \exp[j\varphi(\omega)]. \qquad (6)$$

Then we have

$$A(\omega) = |H(\omega)|, \qquad (7a)$$

$$f(\omega) = \frac{\varphi(\omega)}{2\pi} \frac{\Omega}{m}. \qquad (7b)$$

With the above frequency-domain implementation described by Eqs. (2) and (7), the desired waveform is then generated at $t = -mT$. It should be noted that not only the amplitude profile but also the phase profile can be controlled. Since two adjacent pulses at the output are spaced by T, as indicated in Eq. (5), the pulses at the output of the shaping system will be well separated if T is large enough (corresponding to a small Ω), and the desired waveform can then be "filtered" by applying a window function with a proper open time (a maximum open time is T) in the time domain.

An example showing the generation of a rectangle pulse is demonstrated. Assume that the input ultrashort pulse is a transform-limited Gaussian pulse with a full width at half-maximum (FWHM) of 1 ps, and we are required to generate a rectangle pulse that is chirp free with a pulse width of $T_W = 20$ ps. Figure 2(a) shows the spectra of the input and the output pulses. Based on this requirement, the frequency response of the LTI filter to generate the required pulse can be calculated $H(\omega) = E_O(\omega)/E_I(\omega)$, which is usually complex valued. The frequency response of the filter is shown in Fig. 2(b). Note that $H(\omega)$ is cut off at $|\omega/2\pi| = 1125$ GHz, because in practice the implementation in the frequency domain is generally band limited. It is clearly seen from Fig. 2(b) a phase modulation with phase changes between 0 and π is required in $H(\omega)$, which cannot be realized based on intensity-only modulation.

However, the desired pulse can be generated by intensity-only modulation with nonuniformly spaced sampling in the frequency domain. Based on Eqs. (7) and (2), we calculate the sampling function $S(\omega)$, which is also shown in Fig. 2(b). In our design, the periodic sampling function $S_0(\omega)$ is a square wave with a period of $\Omega = 5$ GHz and a duty cycle of 0.5. As we will show later, a smaller m would lead to a higher pulse output. Therefore, in this example, $m = 1$ is selected. It is clearly seen the sampling function $S(\omega)$ is all positive, which can be realized by intensity-only modulation. A zoom-in display of $S(\omega)$ is shown in Fig. 2(c). It can be seen that the sampling

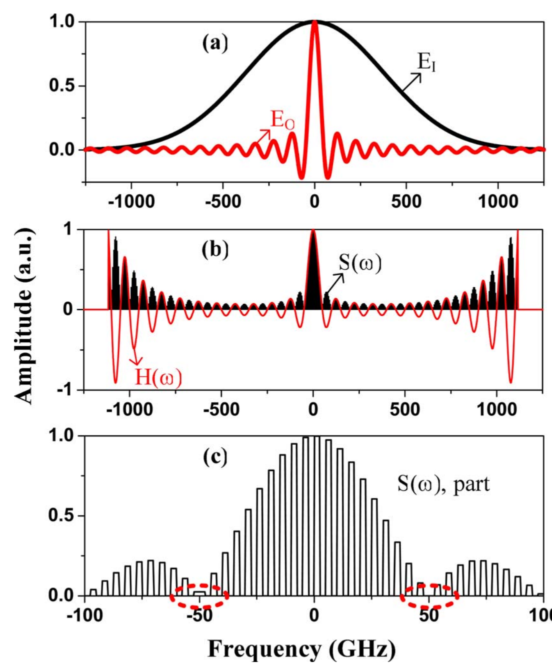

Fig. 2. (Color online) Frequency-domain illustration. (a) Spectra of the input and desired output waveforms. (b) Designed frequency response of a regular [$H(\omega)$] and the proposed [$S(\omega)$] filters. (c) Zoom-in display of $S(\omega)$.

periods at the frequencies marked in the circles in Fig. 2(c) are changed, which corresponds to an equivalent π phase shift in the frequency domain.

Figure 3(b) shows the output pulses when one input ultrashort Gaussian pulse is sampled by $S(\omega)$ in the frequency domain. Multiple pulses are generated as indicated by Eq. (5). The desired waveform, based on the above theory, is located at $t \approx -T$. Since all the pulses in Fig. 3(b) are well separated, the desired waveform could be filtered by a time window. The desired waveform and its phase distribution are plotted in Fig. 3(c). Clearly the obtained waveform has the same profile as that obtained by the regular filter shown in Fig. 3(a). Since the waveform has a constant phase over the pulse width, it is chirp free.

The whole process can be described using the flow chart shown in Fig. 4. With a given ultrashort input pulse and the desired output pulse, the frequency response of a regular LTI filter can then be calculated using $H(\omega)=E_O(\omega)/E_I(\omega)$. Using Eqs. (7) and (2), we then design the intensity-only modulation function with a given m and Ω. If a time window with an open time at $t=-mT$ is used, the desired waveform is then selected.

Some issues should be discussed when using the proposed technique. First, the nonuniformly spaced sampling will lead to equivalent phase shifts in $S_k(\omega)$ except for $k=0$, as can be seen from Eq. (3). Second, compared with the pulse shaping using a regular LTI filter $H(\omega)$, there would be a loss, described by F_m in the proposed system, which could also be seen from the simulation results in Fig. 3(b) where the desired waveform has a lower amplitude. This is the cost paid for this technique. Since F_m generally decreases

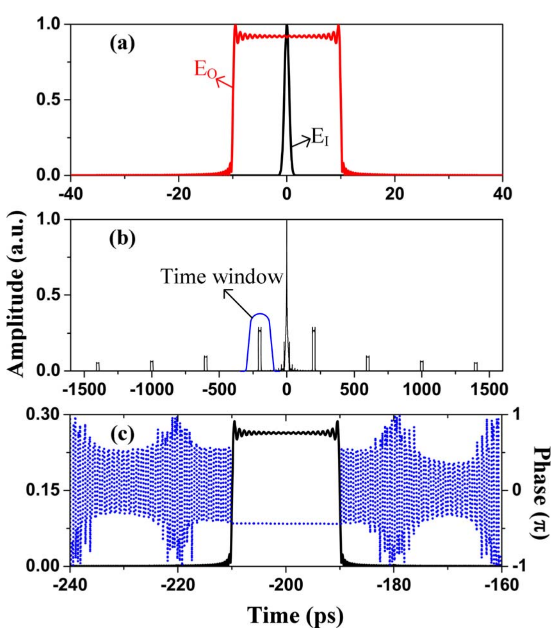

Fig. 3. (Color online) Time-domain illustration. (a) Waveform of the input pulse and the generated pulse by the regular filter $H(\omega)$. (b) Generated multiple pulses by the proposed filter $S(\omega)$. (c) Generated rectangle waveform selected by a time window and its phase distribution.

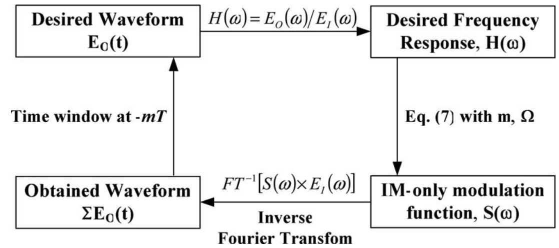

Fig. 4. Flow chart for the design of arbitrary pulse shaping with intensity-only modulation in the frequency domain. IM, intensity modulation.

with the increase of m, $m=\pm 1$ is usually recommended. In addition, if a square wave with a duty cycle of 0.5 is used, as in the above simulation, $F_m=0$ if m is even, which can be seen from Fig. 3(b).

In the proposed approach, a time window is required to select the desired waveform. Based on Eq. (5) the temporal spacing between two adjacent pulses at the output of the system is T, which is determined by the sampling period, Ω, in the frequency domain. Generally, the sampling resolution in the frequency domain can be as high as 5 GHz (corresponding to $T=200$ ps), which can be realized using an SLM. A much higher resolution can be obtained by using a fixed mask or by using a dispersive device with a large dispersion in the TPS. When the output pulses are well separated, the required pulse can be easily selected by using an amplitude modulator or an optical switch. Actually, if the sampling in the frequency domain is implemented by an SLM, the input pulse will not only be time-domain pulse shaped but also be diffracted by the spatial mask. It has been demonstrated in [6,7] that such a diffraction will spatially separate the multiple pulses with different k; i.e., the desired time window could be realized by a spatial filter.

In summary, we have proposed a technique to implement arbitrary waveform generation using intensity-only modulation in the frequency domain. The required phase modulation can be achieved by nonuniformly spaced sampling in the frequency domain. An example showing the generation of a chirp-free rectangle waveform was numerically studied.

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