Tunable Microwave and Sub-Terahertz Generation Based on Frequency Quadrupling Using a Single Polarization Modulator

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Abstract—Frequency quadrupling for tunable microwave and sub-terahertz generation using a single polarization modulator (PolM) in a Sagnac loop without using an optical filter or a wideband microwave phase shifter is proposed and experimentally demonstrated. In the proposed system, a linearly polarized continuous wave from a tunable laser source (TLS) is split into two orthogonally polarized optical waves by a polarization beam splitter (PBS) and sent to the Sagnac loop traveling along the clockwise and counter-clockwise directions. A PolM to which a reference microwave signal is applied is incorporated in the loop. The PolM is a traveling-wave modulator, due to the velocity mismatch only the clockwise light wave is effectively modulated by the reference microwave signal, and the counter-clockwise light wave is not modulated. This is the key point that ensures the cancelation of the optical carrier without the need of an optical filter. Along the clockwise direction, the joint operation of the PolM, a polarization controller (PC), and a polarizer corresponds to a Mach-Zehnder modulator (MZM) with the bias point controlled to suppress the odd-order sidebands. The optical carrier is then suppressed by the counter-clockwise light wave at the polarizer. As a result, only two ± 2 nd-order sidebands are generated, which are applied to a photodetector (PD) to generate a microwave signal with a frequency that is four times that of the reference microwave signal. A theoretical analysis is developed, which is validated by an experiment. A frequency-quadrupled electrical signal with a large tunable range from 2.04 to 100 GHz is generated. The performance of the proposed system in terms of stability and phase noise is also evaluated.

Index Terms—Carrier suppression, microwave photonics, microwave technology, optical interference, polarization modulation, photonic generation of microwave signals.

I. INTRODUCTION

ICROWAVE generation in the optical domain has been a topic of interest, and numerous approaches have been proposed in the last few years [1]. The key advantages of generating microwave signals by photonic means are the low phase

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noise, high frequency, and wide frequency tunability. In addition, photonic generation of microwave signals is inherently compatible with radio-over-fiber (RoF) systems, which offer several advantages over radio-over-coaxial systems, namely, wide bandwidth, low loss, and immunity to electromagnetic interference (EMI). To generate a microwave signal in the optical domain, optical heterodyning has been widely used [1]-[15], in which two optical waves of different wavelengths beat at a photodetector (PD). A microwave signal with a frequency corresponding to the wavelength spacing of the two optical waves is then generated at the output of the PD. The critical factor here is to produce two phase-correlated optical waves to ensure a low phase noise of the generated microwave. To obtain two phase-correlated wavelengths, the following techniques have been among those proposed: optical injection locking (OIL) [2], [3]; optical mode locking [4], [5]; optical phase-locked loop (OPLL) [6], [7]; using a dual-wavelength laser source [8]–[10]; and external modulation [11]–[19]. Among these techniques, external modulation has been considered an attractive solution due to its simplicity and stability, in addition to the large frequency tunability, and high spectral purity of the generated microwave signal [11].

In 1992, O'Reilly *et al.* first proposed and demonstrated a frequency doubling system using an external Mach–Zehnder modulator (MZM). A frequency-doubled electrical signal was optically generated by biasing the MZM at the minimum transmission point (MITP) to suppress the optical carrier and the even-order optical sidebands. If the modulation signal is small, then the \pm 2nd- and higher order sidebands are small and are ignored. Therefore, only the two \pm 1st-order sidebands are present at the output of the MZM. The beating of the two sidebands at a PD would generate a microwave signal with a frequency that is twice the frequency of the sinusoidal drive signal [12].

To achieve a higher frequency multiplication factor, several other approaches have been proposed [13]–[19]. These approaches can be classified into two categories. In the first category, the microwave frequency multiplication is achieved with the assistance of an optical filter [13]–[16]. In [13], O'Reilly *et al.* demonstrated an approach to achieving frequency quadrupling using an MZM biased at the maximum transmission point (MATP) to suppress the odd-order sidebands. By selecting the \pm 2nd-order sidebands with a Mach–Zehnder interferometer (MZI) based optical filter, a frequency-quadrupled microwave signal was generated. To avoid the limitation of frequency tunability due to the fixed free spectral range (FSR) of the MZI, Qi *et al.* proposed to use a narrow-bandwidth fiber Bragg grating (FBG) to remove the optical carrier [15]. Since the FBG has a narrow bandwidth and its central wavelength is not required to be tunable, the system provides good frequency tunability. To further increase the multiplication factor, Li and Yao proposed a system with frequency twelvetupling by a joint use of a polarization modulator (PolM) for frequency quadrupling and a semiconductor optical amplifier (SOA) for frequency tripling [16]. The PolM, in conjunction with two polarization controllers (PCs) and a polarization analyzer, operates equivalently as an MZM that is biased at the MATP. A fixed FBG functions as an optical notch filter to remove the optical carrier, the two ± 2 nd-order sidebands with a wavelength spacing corresponding to four times the microwave drive frequency are obtained. The introduction of the ± 2 nd-order sidebands as two pump waves to the SOA generates two idle waves due to the four wave mixing (FWM) in the SOA. The two pump waves are then filtered out by a stimulated Brillouin scattering (SBS) assisted filter, which is wavelength-independent. In all these techniques, an optical filter is always used, which limits the frequency tuning speed and tunable range.

In the second category, microwave frequency multiplication is achieved without using an optical filter [17]-[19]. In [17], Zhang et al. proposed to use an optical carrier suppression modulation scheme in two cascaded MZMs by introduce a $\pi/2$ phase shift between two microwave driving signals; a frequency-quadrupled microwave signal is generated without the need of an optical or electrical filter to remove the optical carrier. A 4 to 40 GHz frequency quadrupler is experimentally demonstrated with an optical sideband suppression ratio (OSSR) of 20 dB [17]. To further simplify the system and increase the OSSR, Lin et al. proposed to use an integrated dual-parallel Mach-Zehnder modulator (DP-MZM) [18], which consists of two sub-MZMs and a phase modulator in an MZI structure with bias control. By introducing a $\pi/2$ phase shift between two input radio frequency (RF) signals, and setting the bias of the two parallel sub-MZMs at the MATP, and applying a DC bias to the phase modulator to introduce a π phase shift between the two parallel arms, ± 2 nd-order sidebands are obtained while the optical carrier and the odd-order sidebands are suppressed. A frequency-quadrupled microwave signal is generated by beating the ± 2 nd-order sidebands at a PD. The system is experimentally demonstrated to have an OSSR of more than 38 and 36 dB at 40 and 72 GHz, respectively. Shi et al. also proposed a frequency sextupling scheme by adjusting the phase shift and the power difference between the two microwave driving signals and controlling the bias voltages to make the two sub-MZMs at the minimum transmission point (MITP) in the DP-MZM, and applying a DC bias to the phase modulator to introduce a π phase shift between the two parallel arms [19]. In these techniques, although an optical filter is not used, a wideband microwave phase shifter is always needed, which again limits the frequency tunable range.

In this paper, we propose a novel approach to generating a frequency-quadrupled microwave signal using a single PolM without the need of an optical filter or a wideband microwave phase shifter. Since no optical filter or microwave phase shifter is needed, the proposed technique can be used to generate a frequency-tunable microwave signal with a large frequency tunable range. The proposed system is experimentally demonstrated. A microwave signal with a tunable frequency from



Fig. 1. Schematic of the proposed microwave frequency quadrupler. TLS: tunable laser source; PC: polarization controller; PBS: polarization beam splitter; RF: radio frequency; PolM: polarization modulator; Pol: polarizer, PD: photodetector.

2.04 to 100 GHz is generated. The phase noise performance of the generated signal is also evaluated.

The paper is organized as follows. In Section II, the principle of the proposed approach is presented, with an emphasis on the suppression of the optical sideband and carrier. A mathematical model for a non-reciprocal PC is also provided. In Section III, an experimental demonstration of the proposed technique for microwave generation is presented. A microwave signal with a tunable frequency from 2.04 to 100 GHz is generated. The phase noise of the generated microwave signal is measured and compared with that of the reference microwave signal. The system stability and frequency tunability is also discussed. A conclusion is drawn in Section IV.

II. PRINCIPLE

Fig. 1 shows the proposed system. A linearly polarized continuous-wave (CW) light wave from a tunable laser source (TLS) is split into two orthogonally polarized light waves by a polarization beam splitter (PBS) and then sent to a Sagnac loop, in which a PolM is incorporated. The two light waves will travel along the clockwise and counter-clockwise directions through the PolM and reach the PBS again. Along the clockwise direction, the light wave and the reference microwave signal are traveling in the same direction. Since the PolM is a traveling-wave modulator which is designed to match the velocities of the light wave and the microwave signal, strong modulation is resulted. Along the counter-clockwise direction, however, the light wave and the microwave are traveling in an opposite direction, since the light wave and the microwave only meet at a very short time, very weak modulation will be produced. Therefore, only the clockwise light wave is effectively modulated by the reference microwave signal at the PolM, and the counter-clockwise light wave is not modulated. As a result, a modulated and a non-modulated optical signal with opposite propagation directions are obtained. By introducing a phase shift to the non-modulated optical signal by controlling the PC before the polarizer (PC4), the two orthogonally polarized optical carriers will cancel at the polarizer. By controlling the PC (PC2) before the PolM, the odd-order sidebands of the modulated signal can be suppressed when the modulated optical signals interfere at the PBS. Thus, only two ± 2 nd-order



Fig. 2. Sidebands suppression with polarization modulation.

sidebands are obtained at the output of the polarizer, which are applied to a PD to generate a microwave signal with a frequency that is four times that of the reference microwave signal.

We start our analysis from the sideband suppression by using a PolM jointly with a PC and a polarizer. Then, a mathematical model for a fiber-optic PC used in the proposed system is developed. Finally, an analysis on the suppression of odd-sidebands and the optical carrier is provided.

A. Sideband Suppression

The PolM used in our proposed system is a special phase modulator that supports both TE and TM modes with opposite phase modulation indexes [20]. The joint operation of a PolM, a PC and a polarizer is equivalent to an MZM with the bias point being controlled by adjusting the phase difference between the two orthogonal modes via tuning the PC.

Fig. 2 shows the operation of a PolM as an MZM. A linearly polarized light wave from a TLS is applied to the PolM with its state of polarization (SOP) aligned at an angle of 45° relative to one principal axis of the PolM, so the light wave is equally projected to the two orthogonal directions. Mathematically, the electrical field at the output of the PolM along its principal axes (x and y) can be expressed as

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = E_0 \begin{bmatrix} \frac{\sqrt{2}}{2} \exp(j\omega t + j\beta \sin \omega_m t) \\ \frac{\sqrt{2}}{2} \exp(j\omega t - j\beta \sin \omega_m t) \end{bmatrix}$$
(1)

where E_0 is the amplitude of the input optical field, ω is the angular frequency of the optical carrier, β is the phase modulation index, and ω_m is the angular frequency of the microwave signal applied to the PolM. At the output of the PolM, the two modulated light waves are sent to a polarizer via a PC. The principal axis of the polarizer is aligned at 45° relative to the x axis of the PolM. Thus, the electrical field at the output of the polarizer is given by

$$E_{\text{out}} = \frac{\sqrt{2}}{2} E_x + \frac{\sqrt{2}}{2} E_y \exp(j\varphi)$$
(2)

where φ is a static phase term introduced by the PC placed before the polarizer. Thus, by substituting (1) into (2), we have

$$E_{\text{out}} = \frac{1}{2} E_0 \exp(j\omega t) [\exp(j\beta \sin \omega_m t) + \exp(-j\beta \sin \omega_m t + j\varphi)] \quad (3)$$

As can be seen (3) is exactly an expression for a standard MZM with a 1:1 splitting ratio for the two Y junctions. This

confirms that a PolM in conjunction with a PC and a polarizer is equivalent to an MZM with the bias controlled by the PC. Then, (3) is expanded in terms of Bessel functions of the first kind, which is given in (4),

$$\begin{aligned} \int_{\text{put}} &= \frac{1}{2} E_0 \exp(j\omega t) [\exp(j\beta \sin \omega_m t) \\ &+ \exp(-j\beta \sin \omega_m t + j\varphi)] \\ &= \frac{1}{2} E_0 \exp(j\omega t) \left\{ J_0(\beta) + \sum_{k=1}^{\infty} J_k(\beta) \exp(jk\omega_m) \\ &+ \sum_{k=1}^{\infty} (-1)^k J_k(\beta) \exp(-jk\omega_m) \\ &+ \exp(j\varphi) \left[J_0(\beta) + \sum_{k=1}^{\infty} J_k(\beta) \exp(-jk\omega_m) \\ &+ \sum_{k=1}^{\infty} (-1)^k J_k(\beta) \exp(jk\omega_m) \right] \right\} \\ &= \frac{1}{2} E_0 \exp(j\omega t) \left\{ [1 + \exp(j\varphi)] J_0(\beta) \\ &+ \sum_{k=1}^{\infty} [(-1)^k + \exp(j\varphi)] J_k(\beta) [(-1)^k \\ &\times \exp(jk\omega_m) + \exp(-jk\omega_m)] \right\} \end{aligned}$$

$$(4)$$

where $J_n(x)$ represents the *n* th-order Bessel functions of the first kind. As can be seen, the odd-order sidebands are suppressed when $\varphi = 2l\pi, l = 0, 1, 2, 3, \ldots$, and the even-order sidebands are suppressed when $\varphi = (2l + 1)\pi, l =$ $0, 1, 2, 3, \ldots$ Theoretically, odd- or even-order sidebands can be suppressed with an infinite OSSR by changing the phase difference, φ , between the TE and the TM modes.

There are two ways to introduce a tunable phase difference between the TE and the TM modes. One is to tune the PC before the polarizer; the other is to apply a tunable DC voltage to the bias of the PolM. Since the static phase term, φ , can be introduced before or after the PolM, the PC shown in Fig. 2 can also be placed before the PolM to achieve the same sidebands suppression. In our proposed system in Fig. 1, instead of using a DC bias, PC2 is tuned to introduce a tunable phase difference between the two orthogonal modes.

B. The PC Model

 E_{c}

Since PCs are playing an important role in the proposed system, the mathematical model for a PC is developed. An in-line fiber-optic PC consists of three fiber birefringence components; the first and the last are quarter-wave plates and the central one is a half-wave plate. With this combination, any input SOP can be transformed to any other output SOP [21]. Thus, a PC could be written as a fully tunable wave plate (see Appendix)

$$P_F(\theta, \Delta) = \begin{bmatrix} \cos\theta & -\sin\theta\\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \exp\left(j\frac{\Delta}{2}\right) & 0\\ 0 & \exp\left(-j\frac{\Delta}{2}\right) \end{bmatrix}$$
(5)



Fig. 3. Polarization status changes of both the clockwise and counter-clockwise propagation waves, where φ_{δ} is the accumulated static phase difference between the two orthogonal components.

where θ is the rotation angle, and Δ is the phase difference between the two orthogonal components introduced by the PC. If the same light is incident from the other side, the backward transfer function is given by

$$P_B(\theta, \Delta) = P_F^T(\theta, \Delta) = \begin{bmatrix} \exp\left(j\frac{\Delta}{2}\right) & 0\\ 0 & \exp\left(-j\frac{\Delta}{2}\right) \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta\\ -\sin\theta & \cos\theta \end{bmatrix}$$
(6)

This model will be applied to the analysis of the proposed system.

C. Frequency Quadrupling

As discussed in Subsections A and B, simultaneous suppression of the first-order sidebands and the optical carrier is achievable using the proposed system shown in Fig. 1. Assuming that the SOP of the incident light from the TLS is aligned at an angle of α relative to one principal axis of the PBS, we have the clockwise and counter-clockwise electrical fields given by

$$\begin{bmatrix} E_{cw} \\ E_{ccw} \end{bmatrix} = E_{in} \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix}$$
(7)

where E_{cw} and E_{ccw} are the electrical fields of the clockwise and counter-clockwise light waves, respectively, and $E_{in} = E_0 \exp(j\omega t)$ is the electrical field of the light wave from the TLS. To assist the analysis of the SOP change in the proposed system, the principal axes of the polarization devices, the PCs, PBS, and PolM, are shown in Fig. 3 with the SOP of light propagating along the loop in the same Cartesian coordinate system. As can be seen from Fig. 3, the principal axes of the PCs and the PolM match with each other without angle rotation, and one of these principal axes pairs is set to be the reference Cartesian coordinates. The principal axes of the PBS are assumed aligned at 45° relative to the reference Cartesian coordinates, which can be achieved in the experiment by tuning PC2 and PC3.

For the clockwise light wave, it is modulated by the reference microwave signal applied to the PolM when it passes through the PolM. According to the discussion in Subsection A, the clockwise light wave at the output of the PolM can be written as

$$E_{PolM,out} = \begin{bmatrix} E_{PolM,x} \\ E_{PolM,y} \end{bmatrix}$$
$$= \begin{bmatrix} \exp(j\beta\sin\omega_m t) & 0 \\ 0 & \exp(-j\beta\sin\omega_m t) \end{bmatrix}$$
$$\times P_{F2}(0,\Delta_2) \begin{bmatrix} E_{cw,x} \\ E_{cw,y} \end{bmatrix}$$
$$= E_c \begin{bmatrix} \exp(j\beta\sin\omega_m t + j\frac{\Delta_2}{2}) \\ \exp(-j\beta\sin\omega_m t - j\frac{\Delta_2}{2}) \end{bmatrix}$$
(8)

where $E_{PolM,out}$ is the clockwise light wave at the output of the PolM, and $P_{F2}(0, \Delta_2)$ is the forward transfer function of PC2, $E_{cw,x} = E_{cw,y} = \sqrt{2}/2|E_{cw}|$, and Δ_2 is the phase difference between the two orthogonal components induced by PC2. If (8) is expanded in terms of Bessel functions of the first kind, ignoring the third- and higher order sidebands considering small signal modulation, (8) can be written as

$$E_{PolM,x} = E_{cw,x} \exp\left(j\frac{\Delta_2}{2}\right) \left[J_0(\beta) + J_1(\beta)\exp(j\omega_m t) - J_1(\beta)\exp(-j\omega_m t) + J_2(\beta)\exp(j2\omega_m t) + J_2(\beta)\exp(-j2\omega_m t)\right]$$
(9.a)

$$E_{PolM,y} = E_{cw,y} \exp\left(-j\frac{\Delta_2}{2}\right) [J_0(\beta) - J_1(\beta) \exp(j\omega_m t) + J_1(\beta) \exp(-j\omega_m t) + J_2(\beta) \exp(j2\omega_m t) + J_2(\beta) \exp(-j2\omega_m t)]$$
(9.b)

To obtain a maximum power reception at the PBS for the return light waves, a third PC, PC3, is placed before the PBS, which is used to rotate the incoming light wave by 90° as shown in Fig. 3. According to (5), we have,

$$E_{F} = \begin{bmatrix} E_{F,x} \\ E_{F,y} \end{bmatrix} = P_{F3} \left(\frac{\pi}{2}, \Delta_{3}\right) \begin{bmatrix} E_{PolM,x} \\ E_{PolM,y} \end{bmatrix}$$
$$= \begin{bmatrix} -E_{PolM,y} \exp\left(-j\frac{\Delta_{3}}{2}\right) \\ E_{PolM,x} \exp\left(j\frac{\Delta_{3}}{2}\right) \end{bmatrix}$$
(10)

where E_F is the electrical filed at the output of PC3 for the clockwise light wave, and $P_{F3}(\pi/2, \Delta_3)$ is the forward transfer function of PC3. Thus, the two orthogonal components of the clockwise light wave would interfere at port 2 of the PBS, and its output is given by

$$E_{cw,out} = \frac{\sqrt{2}}{2} (E_{F,x} - E_{F,y})$$
 (11)

Substituting (7), (9.a), (9.b), and (10) into (11), we have

$$E_{cw,out} = -\frac{1}{2} |E_{cw}| \exp\left(j\frac{\Delta_2 + \Delta_3}{2}\right)$$

$$\times \left\{ [1 + \exp(-j\Delta_2 - j\Delta_3)] J_0(\beta) + [1 - \exp(-j\Delta_2 - j\Delta_3)] J_1(\beta) \exp(j\omega_m t) - [1 - \exp(-j\Delta_2 - j\Delta_3)] J_1(\beta) \exp(-j\omega_m t) + [1 + \exp(-j\Delta_2 - j\Delta_3)] J_2(\beta) \exp(j2\omega_m t) + [1 + \exp(-j\Delta_2 - j\Delta_3)] J_2(\beta) \exp(-j2\omega_m t) \right\}$$

$$(12)$$

To suppress the first order sidebands, we must have

$$\Delta_2 + \Delta_3 = 2l\pi \quad (l = 0, 1, 2, 3...) \tag{13}$$

Thus, (12) can be written as

$$E_{cw,out} = -|E_{cw}|e^{jl\pi} \left\{ J_0(\beta) + J_2(\beta) \exp(j2\omega_m t) + J_2(\beta) \exp(-j2\omega_m t) \right\}$$
(14)

Since the reference microwave signal is not modulated on the counter-clockwise light wave due to the velocity mismatch at the PolM, we have the counter-clockwise light wave at the output of PC2, given by

$$E_{PC} = \begin{bmatrix} E_{PC,x} \\ E_{PC,y} \end{bmatrix} = P_{B2}(0, \Delta_2) P_{B3} \left(\frac{\pi}{2}, \Delta_3\right)$$
$$\times \begin{bmatrix} E_{ccw,x} \\ E_{ccw,y} \end{bmatrix}$$
$$= \begin{bmatrix} E_{ccw,y} \exp\left(j\frac{\Delta_2 + \Delta_3}{2}\right) \\ -E_{ccw,x} \exp\left(-j\frac{\Delta_2 + \Delta_3}{2}\right) \end{bmatrix}$$
(15)

where $E_{ccw,x} = -E_{ccw,y} = \sqrt{2}/2|E_{ccw}|$. Therefore, the electrical filed of the counter-clockwise light wave at the output of port 1 of the PBS is given as

$$E_{ccw,out} = \frac{\sqrt{2}}{2} (E_{PC,x} + E_{PC,y})$$

= $-\frac{1}{2} |E_{ccw}| \exp\left(j\frac{\Delta_2 + \Delta_3}{2}\right)$
× $[1 + \exp(-j\Delta_2 - j\Delta_3)]$
= $-|E_{ccw}|e^{jl\pi}$ (16)

Since the clockwise and counter-clockwise light waves, $E_{cw,out}$ and $E_{ccw,out}$, are combined at the PBS, and $E_{cw,out}$ and $E_{ccw,out}$ are orthogonal at the output of the PBS. By connecting a fourth PC, PC4, and a polarizer after the PBS, and the principal axis of the polarizer is aligned at an angle of ρ to one principal axis of the PBS, we have the light wave at the output of the polarizer, written as

$$E_{\rm out} = E_{cw,\rm out} \cos \rho + E_{ccw,\rm out} \sin \rho \exp(-j\varphi) \qquad (17)$$

where φ is the phase difference between $E_{cw,out}$ and $E_{ccw,out}$ introduced by PC4. By tuning PC4, the two orthogonal propagation waves can have an arbitrary phase difference. Substituting (7), (14) and (16) into (17), we have

$$E_{\text{out}} = -E_{\text{in}}e^{jt\pi} \{ [\cos\alpha\cos\rho J_0(\beta) + \sin\alpha\sin\rho\exp(-j\varphi)] + \cos\alpha\cos\rho [J_2(\beta)\exp(j2\omega_m t) + J_2(\beta)\exp(-j2\omega_m t)] \}$$
(18)

To suppress the optical carrier, we must have

$$\cos\alpha\cos\rho J_0(\beta) + \sin\alpha\sin\rho\exp(-j\varphi) = 0 \qquad (19)$$

Thus,

$$\alpha = -\tan^{-1} \left[\frac{J_0(\beta)}{\tan \rho} \exp(j\varphi) \right]$$
(20)

Therefore, the output at the polarizer is, (if l = 1)

$$E_{\text{out}} = E_{\text{in}} J_2(\beta) \cos \alpha \cos \rho [\exp(j2\omega_m t) + \exp(-j2\omega_m t)] \quad (21)$$

By applying E_{out} to a high-speed PD, a microwave signal with an angular frequency of $4\omega_m$ is obtained.



Fig. 4. Simulation results: microwave powers of the second-order harmonic and the fourth-order harmonic versus the modulation index.

D. Simulation

To have a better understanding of the tunability of the proposed system, a simulation is performed.

Since the two light waves along the clock-wise and counterclockwise travel along the same optical path, no phase difference between the two light waves is introduced by the optical path. Thus, the length of the optical path will not affect the cancellation of the optical carrier. Here we only evaluate the effects of the input microwave power level on the microwave powers of the second and the fourth order harmonics, because the odd-order harmonics are suppressed and the powers of the higher order harmonic are very low and negligible.

The following parameters are used in the simulation: the wavelength of the optical carrier is 1550 nm, the input microwave frequency is 20 GHz, and the modulation index is from 0 to 5. Fig. 4 shows the simulation results. For a given input RF power, the system is optimized to completely suppress the second order harmonic, then by increasing the input microwave power from 0 to 25 dBm, or equivalently the modulation index from 0 to 5, the power of the second order harmonic will increase. When the modulation index is 3.62 or 4.04 (corresponding to an input microwave power of 22.12 dBm or 23.06 dBm) the second order harmonic is completely suppressed, and this is the optimized state of operation of the system.

Since no optical filter is employed in the system, the frequency of the frequency-quadrupled signal can be tunable from zero to a maximum value that is limited by the bandwidth of the PD and the PolM. This is the key advantage of this technique. Note that during the frequency tuning, the input microwave power should be maintained constant to avoid the change of the optimal state of the system for the second-order harmonic suppression.

III. EXPERIMENT

An experiment based on the setup shown in Fig. 1 is implemented. A linearly polarized CW light wave at 1550.4 nm from the TLS (AQ2201, YOKOGAWA) is sent to the Sagnac loop through PC1 and the PBS. Two orthogonally polarized light waves travel along opposite directions in the Sagnac loop and pass through the PolM (Versawave 40-GHz Polarization Modulator). Due to the velocity mismatch, only the clockwise light wave is effectively modulated by the reference microwave



Fig. 5. Experimental results. The blue dashed curve shows the spectrum of the modulated optical signal without sideband suppression while the red solid curve shows the spectrum of the modulated optical signal with both carrier and first-order sideband suppressed. The modulation microwave signal has a frequency of 10 GHz.

signal applied to the PolM, and the counter-clockwise propagation light wave is not modulated. As a result, a modulated and a non-modulated optical signal with opposite propagation directions are obtained. By controlling PC2 and PC3, the oddorder sidebands of the modulated signal are suppressed at the PBS, while the non-modulated optical signal would have a π phase difference with the modulated optical carrier by controlling PC4, to cancel the optical carrier in the modulated optical signal. Two \pm 2nd-order sidebands are obtained at the polarizer, which are applied to a PD and a microwave signal with a frequency that is four times that of the reference microwave signal is generated.

A. Microwave Signal Generation

In the first experiment, a reference microwave signal of 10 GHz from a signal generator (Agilent E8254A) is applied to the PolM. The power of the input microwave is 22.6 dBm which is the maximum power available from the signal generator and is close to the optimal input power to achieve a complete suppression of the second order harmonic. In the experiment, the principal axis of the polarizer is aligned at 45° relative to one principal axis of the PBS, and PC4 is adjusted to introduce a π phase shift between the two orthogonal light waves. To cancel the optical carrier, the optical carriers in both directions from the two ports of the PBS must have the same amplitude when they return to the PBS. Thus, according to (20), the incident light must be aligned at an angle of α relative to one principal axis of the PBS, which is calculated to be

$$\alpha = -0.07\pi \tag{22}$$

By tuning PC1 and PC4 to achieve the optical carrier suppression, while the odd-order sidebands suppression are achieved by tuning PC2 and PC3, an optical signal with only the \pm 2nd-order is generated. The optical spectrum of the optical signal at the output of the polarizer is shown in Fig. 5. As can be seen, the optical carrier is suppressed by 42 dB, and the first-order sidebands are suppressed by more than 32 dB. By beating the two \pm 2nd-order sidebands at a PD (U2T XPDV2150R-VF-VA, 50 GHz), a microwave signal with a frequency that is four times



Fig. 6. Electrical spectrum of the generated microwave signal when the reference microwave signal is at 10 GHz.



Fig. 7. Optical spectra of the generated frequency-quadrupled microwave signals at different frequencies of 5 GHz, 10 GHz, 15 GHz and 25 GHz of the reference microwave signal.

that of the reference microwave signal is generated. The spectrum of the generated microwave signal is measured by an electrical spectrum analyzer (ESA, E4448A, Agilent), as shown in Fig. 6. As can be seen, the power difference between the frequency-quadrupled signal and the highest unwanted harmonic is 17.8 dB.

B. Stability and Tunability

Another important feature of the proposed system is the excellent short-term stability. Since the two counter propagation light waves are traveling within the same optical loop, the two light waves are experiencing the same perturbations and can be cancelled out at the PD, which ensures an excellent short-term stability. This conclusion is justified by the experiment. At room temperature in a laboratory environment, the system is allowed to operate for more than five hours with the spectrum of the generated microwave signal monitored by the ESA. Negligible amplitude variations are observed. The long term stability may be affected due to the use of the multiple PCs. For real applications, the system can be implemented using polarization maintaining components and the PCs are not needed.

To further evaluate the system performance of the proposed technique, frequency tunability is investigated. One significant feature of the proposed technique is the frequency independent tuning due to the elimination of an optical filter and a microwave phase shifter. To evaluate the frequency tuning independence, we tune the frequency of the reference microwave signal from 5 to 25 GHz while keeping the microwave power unchanged (22.6 dBm). The optical spectra are shown in Fig. 7. As can be seen, when the frequency of the reference microwave signal is tuned, the carrier and the first-order sidebands are maintained suppressed with no need to fine tune the system. It can be seen that the system can achieve frequency quadrupling with a reference microwave signal at different frequencies while the other parameters (the setting of PC1, PC2, PC3, and PC4, and input microwave power) remain unchanged. This important feature makes the proposed system greatly simplified for the generation of a widely frequency-tunable microwave signal.



Fig. 8. The spectra of the generated microwave signals for a reference microwave signal at different frequencies of (a) 510 MHz, (b) 800 MHz, (c) 2 GHz, (d) 17 GHz, and (e) 25 GHz.

Fig. 8 shows the frequency tuning with a wider tunable range from 2.04 to 100 GHz when the frequency of the reference microwave signal is tuned from 510 MHz to 25 GHz with an identical microwave power of 22.6 dBm applied to the PolM. Again, a frequency-quadrupled signal is generated for the reference microwave signal at different frequencies. Note that the 100 GHz signal is detected by a different PD (U2T XPDV4120R, 100 GHz) with a wider bandwidth. Again, since no optical filter or microwave phase shifter is used, the tunable range is only limited by the bandwidths of the PolM and the PD.

C. Phase Noise

The power spectrum of the phase noise in a frequency-multiplied microwave signal can be expressed as [22]

$$\begin{aligned} &10 \log_{10}[S(f)] \\ &= 10 \log_{10}[S_{\rm res}(f)] + 10 \log_{10}[m^2 \cdot S_e(f)] \\ &= 10 \log_{10}[S_{\rm res}(f)] + 10 \log_{10}[S_e(f)] + 12 \end{aligned}$$
(23)

where m is the frequency multiplication factor, and $S_{res}(f)$ and $S_e(f)$ are the power spectra of the residual phase noise of the system and of the microwave drive signal, respectively. In our proposed system, m = 4.

To evaluate the phase noise performance, the phase noises of a reference microwave signal at 10 GHz and that of a frequency-quadrupled microwave signal at 40 GHz are measured by a signal source analyzer (Agilent E5052B), which are shown



Fig. 9. Measured phase noises at driving signal of 10 GHz.



Fig. 10. Measured phase noises at different frequencies.

in Fig. 9. It can be seen that the phase noise of the frequencyquadrupled microwave signal is deteriorated by about 12.8 dB compared to that of the reference microwave signal, which indicates that the phase noise of the frequency- quadrupled signal is mainly from the reference microwave signal in addition to the 12 dB deterioration due to the frequency quadrupling operation. The system induced phase noise, $S_{res}(f)$, is small and can be neglected. The phase noise of the frequency-quadrupled microwave signal at other frequencies is also measured, as shown in Fig. 10. As can be seen the phase noise at other frequencies is also mainly from the reference microwave signal and the frequency quadrupling operation.

IV. CONCLUSION

We have proposed and experimentally demonstrated a new approach to achieving microwave frequency quadrupling using only one PolM without using an optical filter or a microwave phase shifter. The functionality is achieved due to the bidirectional use of the PolM which can perform effective modulation for a light wave along the clock-wise direction while no modulation is impressed on a light wave along the counter-clockwise direction due to the velocity mismatch. The key significance of

$$E_{Fout} = P_F \cdot E_{in}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \exp\left(j\frac{\Delta}{2}\right) & 0 \\ 0 & \exp\left(-j\frac{\Delta}{2}\right) \end{bmatrix} \begin{bmatrix} A_x \exp\left(j\frac{\delta}{2}\right) \\ A_y \exp\left(-j\frac{\delta}{2}\right) \end{bmatrix}$$

$$= \begin{bmatrix} A_x \cos\theta \exp\left[j\left(\frac{\delta}{2} + \frac{\Delta}{2}\right)\right] - A_y \sin\theta \exp\left[-j\left(\frac{\delta}{2} + \frac{\Delta}{2}\right)\right] \\ A_x \sin\theta \exp\left[j\left(\frac{\delta}{2} + \frac{\Delta}{2}\right)\right] + A_y \cos\theta \exp\left[-j\left(\frac{\delta}{2} + \frac{\Delta}{2}\right)\right] \end{bmatrix}$$
(A.4)

$$E_{Bout} = P_B \cdot E_{in}$$

$$= \begin{bmatrix} \exp\left(j\frac{\Delta}{2}\right) & 0\\ 0 & \exp\left(-j\frac{\Delta}{2}\right) \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta\\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} A_x \exp\left(j\frac{\delta}{2}\right)\\ A_y \exp\left(-j\frac{\delta}{2}\right) \end{bmatrix}$$

$$= \begin{bmatrix} A_x \cos\theta \exp\left[j\left(\frac{\delta}{2} + \frac{\Delta}{2}\right)\right] + A_y \sin\theta \exp\left[-j\left(\frac{\delta}{2} - \frac{\Delta}{2}\right)\right]\\ -A_x \sin\theta \exp\left[j\left(\frac{\delta}{2} - \frac{\Delta}{2}\right)\right] + A_y \cos\theta \exp\left[-j\left(\frac{\delta}{2} + \frac{\Delta}{2}\right)\right] \end{bmatrix}$$
(A.5)



Fig. 11 An in-line fiber optic PC consisting of three wave plates. QWP: quarter wave plate; HWP: half wave plate.

the proposed system is the frequency independent tuning due to the elimination of both an optical filter and a microwave phase shifter. Thus, the proposed system can be used to generate a microwave signal with a large tunable range. In addition, because of the use of a Sagnac loop, where the two counter traveling light waves are traveling within the same fiber loop, high system stability is ensured. The proposed system was theoretically analyzed and experimentally demonstrated. A frequency-quadrupled microwave signal with a frequency tunable from 2.04 to 100 GHz was demonstrated.

APPENDIX

A fiber-optic PC, shown in Fig. 11, consists of a quarter-wave, a half-wave, and a quarter-wave plates [20]. Any input SOP can be transformed to any other output SOP by jointly adjusting the $\lambda/4$, $\lambda/2$, and $\lambda/4$ plates. The rotation of a classical halfwave or a quarter-wave plate with respect to the incident light is functionally equivalent to the rotation of the fiber loop, which rotates the principle axes of the birefringent fiber sections with respect to the input polarization state.

The transfer function of a PC for a forward incident light could is given [23]

$$P_F = \begin{bmatrix} \cos\theta & -\sin\theta\\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \exp\left(j\frac{\Delta}{2}\right) & 0\\ 0 & \exp\left(-j\frac{\Delta}{2}\right) \end{bmatrix}$$
(A.1)

where θ is the rotation angle, and Δ is the phase difference between the two orthogonal components introduced by the birefringence in a PC. For a backward incident light [24], we have

$$P_B = P_F^{*}$$

$$= \begin{bmatrix} \exp\left(j\frac{\Delta}{2}\right) & 0\\ 0 & \exp\left(-j\frac{\Delta}{2}\right) \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta\\ -\sin\theta & \cos\theta \end{bmatrix}$$
(A.2)

where P_F^T is the transpose of P_F . Generally, a polarized light could be expressed in a Jones vector:

$$E_{\rm in} = \begin{bmatrix} A_x \exp\left(j\frac{\delta}{2}\right) \\ A_y \exp\left(-j\frac{\delta}{2}\right) \end{bmatrix}$$
(A.3)

where A_x and A_y are the amplitude of the orthogonal components, δ is the phase difference between the two orthogonal components. If the light is forwarded into the PC, we have (A.4), shown at the top of the page. If the light is back-warded into the PC with the same status, we have (A.5), shown at the top of the page.

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