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TRAFFIC GROOMING OPTIMIZATION IN MESH WDM NETWORKS WITHOUT WAVELENGTH CONVERTER

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Abstract. This is a tutorial paper on the traffic grooming optimization in mesh Wavelength Division Multiplexing (WDM) networks for static traffic patterns. The traffic grooming technique provides a two-layer traffic engineering capability by aggregating and routing low-bandwidth traffic flows from the upper layer over re-configurable high-bandwidth connections in the lower layer. This paper demonstrates how to use the Lagrangian Relaxation and Subgradient Methods (LRSM) to solve the optimization problem over a mesh WDM network where no wavelength converters are used, i.e., a lightpath must use the same wavelength from its source to destination. The framework makes use of a constrained Integer Linear Programming (ILP) formulation. The outputs of our optimization include the selection of profitable traffic flows, the routing schemes for the selected traffic flows over lightpaths, the selection of lightpaths, and the Routing and Wavelength Assignment (RWA) schemes for the selected lightpaths. Some interesting observations are noted.

Keywords. Network optimization, lagrangian relaxation and subgradient methods, traffic grooming, WDM networks, wavelength continuity constraint.

AMS (MOS) subject classification: 90B18

1 Introduction

We study the static traffic grooming problem in mesh WDM networks. It is a joint two-layer traffic engineering problem: the selection and routing of lightpaths over fibres, and the selection and routing of traffic flows over lightpaths. General surveys of the optimization models and methods can be found in [1 - 4]. Static traffic grooming in mesh networks is challenging and is reported in only a few publications [5 - 13]. The lightpath RWA problem was neglected in [7, 8, 10]. Heuristic algorithms were proposed in [6, 11 - 13]. In [5], a decomposition method was given with the objective of minimizing the total number of transponders, but it is unclear how the decomposition method applies to other optimization objectives, or how the solutions of the sub-problems should be coordinated.

We assume no wavelength conversion is used, so a lightpath must take the same wavelength from its source to destination without wavelength conversion. This constraint is known as the wavelength continuity constraint. Some existing approaches do not consider the lightpath RWA problem by assuming the virtual topology is given [7, 8, 10]. In the approaches that consider the lightpath RWA, so far only heuristic algorithms [6, 11, 12] can handle the additional complexity that arises from the wavelength continuity constraint. Others relax this constraint by assuming full wavelength conversion is available at all nodes [5, 9].

The selection of profitable traffic flows is untouched for the static traffic grooming problem. Assuming that all the traffic flows should be accepted, most previous studies aim at minimizing costs [1, 5, 7, 8, 10, 13]. A few previous studies deal with throughput or revenue maximization [6, 15]. In the throughput maximization, the traffic flows are prioritized based on their efficiency of resource utilization. The revenue maximization is a variation of the throughput maximization, where the revenue was modelled as a weighted throughput in [6]. In the throughput or revenue maximization, it is unknown whether providing service to a traffic flow is profitable or not, because the cost is not modelled. The throughput maximization and the resource minimization are modelled as two competing objectives, and use a multi-objective optimization technique [9] or heuristic algorithms [11, 12] to solve the problem. However, the relationship of these two objectives to the profit was not revealed. Different from the previous studies, the cost minimization and throughput maximization are incorporated into our grooming optimization.

Among many mathematical approaches that can be applied to the traffic grooming optimization problem, Lagrangian Relaxation (LR) has advantages over the existing three types of methods: exact ILP solution, heuristic algorithms, and approximation by bounds. LR is a computationally efficient approach with substantial proven mathematical background [19], and can be applied to problems with various constraints. Moreover, unlike most of the other methods, if there is only a small change in the network configuration or the traffic demand, the re-computation time of LR can be dramatically shortened by reusing the previously obtained Lagrange multipliers [10]. The Dual Problem (DP) is formed after the properly selected constraints are relaxed, and the optimal solution to DP constitutes a bound to the original problem. The performance of the final solution to the original problem can thus be evaluated by comparing with this bound. The DP is sovled by the subgradient method, which iteratively makes the bound closer to the optimum. This method has been formally proven to converge geometrically and it has been widely used in practical applications with a satisfactory convergence speed [19].

In our previous research, we successfully applied LR and subgradient framework on the Routing and Wavelength Assignment (RWA) problem [20].

The RWA problem was relaxed to a DP. Then, the DP was separated into subproblems, each corresponding to one lightpath. The subproblems can easily be solved optimally using the shortest path algorithm on a wavelength graph [20]. The computational complexity of the overall algorithm is compared with other solution methods. Superior performance of optimality and speed was observed.

The traffic grooming problem can also be formulated as a constrained ILP problem [5 - 13]. The exact ILP solution performs exhaustive search among all possible traffic selection combinations and all possible RWA schemes. Although it provides an optimal solution, it suffers from very high computational complexity. Even for a ring network, the static traffic grooming problem is Non-Polynomial (NP) complete [16]. The traffic grooming problem in a mesh network is NP-complete, since the RWA problem in a mesh network is NP-complete [6]. For practical scale networks, an ILP solver software (such as CPLEX) cannot solve this problem. For example, in [6], even for a small network consisting of 6 nodes and 8 links, CPLEX could not compute the optimal solution within a reasonable time, and the computation was terminated before reaching the optimal solution. Although the approximation by bounds is effective for some special topologies such as ring, no bound is known for arbitrary topologies. So far, for practical scale mesh networks, only heuristics were used to solve the ILP problem [5, 6, 11, 12, 13]. Heuristic algorithms yield reasonable solutions by just performing intelligent local search. However, their performance varies depending on network properties and traffic patterns. Their effectiveness is difficult to quantify, because the real optimal solution is unknown. In [17], the CPLEX computation was terminated after 48 hours, and the approximate results are used as a benchmark to compare with the results of heuristic algorithms. By applying the LR and subgradient methods to this problem, we can achieve a feasible solution for practical scale mesh networks (e.g., 13 nodes, 19 links, 32 wavelengths, and 1104 traffic flows). In addition, our approach obtains a performance bound to evaluate the optimality of a solution. In this paper, we will give an example of the application of our LR and subgradient methods in a mesh network without wavelength conversion.

In this tutorial paper, we shall discuss the use of the LRSM to the traffic grooming optimization problem, which is more complicated than the RWA problem. We explain how to apply the LRSM to solve the grooming problem. In particular, we extend our previous study on the traffic grooming problem for SONET-over-WDM networks [14] by forbidding the use of wavelength converters in the mesh WDM networks and then providing a modified optimization model where profit is modelled as the difference between the revenue generated by accepted traffic flows and the cost of used resources. In this way, our model can identify the profitable traffic flows and examine the impact of cost parameters on the grooming objective. We can achieve a feasible solution for practical scale mesh networks such as the example of 13 nodes, 19 links, 32 wavelengths, and 1104 traffic flows. In summary, our paper demonstrates:

- The LRSM approach to formulate the grooming optimization problem in mesh WDM networks;
- The different behaviours of traffic flows with various bandwidths as the grooming cost in the electrical layer increases;
- The effectiveness of the LRSM through an example.

Our tutorial paper is organized as follows: in Section 2, the grooming optimization problem is formulated. A solution to the model is proposed based on the LR and subgradient methods in Section 3. The numeric results are presented in Section 4, followed by the conclusions in Section 5. For the remainder of the paper, the following notations and variables pertain.

A	the total number of potential lightpaths to be set up;
C	a lightpath's bandwidth;
d_{ijc}	the cost of w_{ijc};
D_{sdn}	the routing cost for lightpath s_{sdn};
E_{pqz}	the electrical domain traffic grooming cost for χ_{pqz};
G_{pqz}	the bandwidth requirement of χ_{pqz};
n_{ij}	the number of Wavelength Channels(WCs) on the link between node pair (i, j);
N	the number of nodes;
N_{sd}	the maximum number of lightpaths between node pair (s, d);
P_{pqz}	the revenue coefficient for accepting the traffic flow χ_{pqz}, measured by the service charge per bandwidth unit;
q	the dual function;
q^U	an approximation to the optimal dual value;
$q^{(h)}$	the value of the dual function q at the h^{th} iteration;
r_d	the cost of a receiver at node d ;
R_d	the number of receivers at destination node d;
s_{sdn}	the n^{th} lightpath between the node pair (s, d);
t_s	the cost of a transmitter at node s;
T_s	the number of transmitters at source node s;
V_{pqz}	the coefficient for the traffic grooming cost;
w_{ijc}	the c^{th} WC between node pair (i, j);
W	the WC count on a fibre;
Z	the number of traffic flows;
Z_{pq}	the maximum number of traffic flows between the node pair (p, q);

α	the lightpath setup status, where $\alpha = (\alpha_{sdn})$ for all lightpaths s_{sdn};
α_{sdn}	the setup status of s_{sdn}, which equals one when s_{sdn} is set up; otherwise zero;
$\beta^{(h)}$	the step size for the $(h+1)^{\text{th}}$ iteration;
γ_{pqz}	the admission status of χ_{pqz}, which equals one if χ_{pqz} is determined to be profitable and therefore accepted; otherwise zero;
γ	the traffic flow admission status, where $\gamma = (\gamma_{pqz})$ for all traffic flows χ_{pqz};
δ	the lightpath RWA scheme, where $\delta = \left(\delta_{ijc}^{sdn}\right)$ for all lightpaths s_{sdn} and all WCs w_{ijc};
δ_{ijc}^{sdn}	the usage of the WC w_{ijc} by s_{sdn}, which equals one when s_{sdn} travels through w_{ijc}; otherwise zero;
η	the Lagrange multipliers corresponding to the transmitter quantity constraint;
θ	the Lagrange multipliers corresponding to the lightpath bandwidth constraint;
υ	the traffic flow routing scheme, where $\upsilon = (\upsilon_{sdn}^{pqz})$ for all traffic flows χ_{pqz} and all lightpaths s_{sdn};
υ_{sdn}^{pqz}	the routing status of χ_{pqz} over lightpath s_{sdn}, which equals one when χ_{pqz} is routed through the lightpath s_{sdn}; otherwise zero;
ξ	the Lagrange multipliers corresponding to the WC exclusive usage constraint;
$(\xi, \eta, \theta)^{(h)}$	the value of vectors (ξ, η, θ) obtained in the h^{th} iteration;
χ_{pqz}	the z^{th} traffic flow between the node pair (p, q).

2 The Grooming Optimization Problem

2.1 Network model and assumptions

We consider a general WDM mesh network of N nodes interconnected by E fibers. Each fiber has W non-interfering Wavelength Channels (WCs). Two nodes can be logically connected through a lightpath (optical channels) defined to be of a concatenated sequence of WCs of the same wavelength color. Since there could be multiple lightpath demands between any source-destination pair, we allow more than one lightpath being set up between them. For simplicity, we use multiples of the equivalent bandwidth (51.84 Mbps) of a Synchronous Transport Signal of the level-1 (STS-1) channel as the bandwidth unit.

We make the following assumptions:

1. Bi-directional WCs. Every WC can carry optical signals travelling in both directions at the same time;

2. The same bandwidth of all WCs. Every WC has the same bandwidth, denoted as C;
3. Single fibre on a link. There is only one fibre connecting a pair of neighbour nodes;
4. The same number of WCs in all fibres. Every fibre has the same number of WCs, denoted as W;
5. Tunable transmitters and receivers. Transmitters and receivers can be tuned to any WC;
6. Unlimited electrical grooming capacity. If a node is equipped with a traffic grooming fabric, its capacity is unlimited;
7. Non-blocking electrical grooming. An input electrical signal can be groomed and switched to any output port of a traffic grooming fabric, regardless of the usage of other input and output ports;
8. Non-blocking wavelength switching. An input lightpath can be switched to any output port, regardless of the usage of other input and output ports;
9. No wavelength conversion;
10. Uni-directional traffic demands. A traffic demand is uni-directional from its source to destination;
11. Asymmetric traffic pattern. The traffic demand from a given node to another is not necessarily identical for the reverse direction;
12. Non-bifurcation traffic. A traffic demand must be handled as its entirety, and cannot be split.

2.2 Formulation

We formulate the objective function as $\max(f)$, where

$$f = \sum_{(p,q)} \sum_{0<z\leq Z_{pq}} \gamma_{pqz} P_{pqz} G_{pqz} - \sum_{(p,q)} \sum_{0<z\leq Z_{pq}} E_{pqz} - \sum_{(s,d)} \sum_{0<n\leq N_{sd}} D_{sdn} \tag{1}$$

The grooming objective is composed of three parts: the revenue generated by accepted traffic flows, the traffic grooming cost, and the lightpath cost. We adopt a virtual unit for cost, revenue and profit, which is proportional to the dollar unit. The expected revenue for each traffic flow is given. In this paper, as an example, we assume the expected revenue being the bandwidth requirement of a traffic flow. However, our model and solution method can investigate various revenue functions, as long as the revenue is deterministic.

The traffic grooming cost E_{pqz} is modelled as the total cost of the electrical domain grooming cost for χ_{pqz} before χ_{pqz} is carried by a lightpath.

$$E_{pqz} = \sum_{(s,d)} \sum_{0<n\leq N_{sd}} \upsilon_{sdn}^{pqz} V_{pqz}, \text{ for all traffic flows } \chi_{pqz} \tag{2}$$

We model the lightpath cost as the sum of the transmitter cost, the receiver cost, and the total cost of all the WCs that the lightpath uses.

$$D_{sdn} = \alpha_{sdn}\left(t_s + r_d\right) + \sum_{(i,j)} \sum_{0<c\le n_{ij}} \delta_{ijc}^{sdn} d_{ijc}, \text{ for all lightpaths } s_{sdn} \tag{3}$$

The constraints are:

a) Lightpath bandwidth constraint.

$$\sum_{(p,q)} \sum_{0<z\le Z_{pq}} \upsilon_{sdn}^{pqz} G_{pqz} \le C\alpha_{sdn}, \text{ for all lightpaths } s_{sdn} \tag{4}$$

C is a constant, measured in the units of multiples of the equivalent bandwidth of an STS-1 channel.

b) Traffic flow continuity constraint.

$$\sum_{d} \sum_{0<n\le N_{sd}} \upsilon_{sdn}^{pqz} - \sum_{d} \sum_{0<n\le N_{ds}} \upsilon_{dsn}^{pqz} = \begin{cases} \gamma_{pqz} & \text{if } s = p \\ -\gamma_{pqz} & \text{if } s = q \\ 0 & \text{otherwise} \end{cases}, \quad \text{for all traffic flows } \chi_{pqz} \tag{5}$$

c) Wavelength continuity constraint.

$$\sum_{i} \delta_{ikc}^{sdn} = \sum_{j} \delta_{kjc}^{sdn}, \text{ if } k \ne s \text{ or } d, \text{ for all lightpaths } s_{sdn}, \text{ all wavelengths } c \tag{6}$$

$$\sum_{i} \delta_{isc}^{sdn} = 0, \text{ for all lightpaths } s_{sdn}, \text{ all wavelengths } c \tag{7}$$

$$\sum_{j} \delta_{djc}^{sdn} = 0, \text{ for all lightpaths } s_{sdn}, \text{ all wavelengths } c \tag{8}$$

$$\sum_{j} \sum_{0<c\le n_{sj}} \delta_{sjc}^{sdn} = \alpha_{sdn}, \text{ for all lightpaths } s_{sdn} \tag{9}$$

$$\sum_{i} \sum_{0<c\le n_{id}} \delta_{idc}^{sdn} = \alpha_{sdn}, \text{ for all lightpaths } s_{sdn} \tag{10}$$

d) WC exclusive usage constraint.

$$\sum_{(s,d)} \sum_{0<n\le N_{sd}} \delta_{ijc}^{sdn} \le 1, \text{ for all WCs } w_{ijc} \tag{11}$$

e) Transmitter quantity constraint.

$$\sum_{d} \sum_{0<n\le N_{sd}} \alpha_{sdn} \le T_s, \text{ for all source nodes } s \tag{12}$$

f) Receiver quantity constraint.

$$\sum_{s} \sum_{0<n\le N_{sd}} \alpha_{sdn} \le R_d, \text{ for all destination nodes } d \tag{13}$$

Using the above notations, the design variables in our model are:

- Traffic flow admission status γ;

- Traffic flow routing scheme υ;
- Lightpath setup status α;
- Lightpath RWA scheme δ.

The objective function $\max\limits_{\gamma,\upsilon,\alpha,\delta}\left[f\left(\gamma,\upsilon,\alpha,\delta\right)\right]$ is to maximize a weighted summation of all design variables. All the design variables are integers of value either 1 or 0. Unlike the previous problems, .e.g., [14, 20], we reflect the absence of wavelength converters through constraints (6)-(10).

3 Optimization Method and Analysis of Traffic Grooming

By relaxing selected constraints, and transforming the relaxed constraints into soft "price" terms, we derive a Lagrangian function [18], in which Lagrange multipliers (soft "prices") are used to reflect the relaxed hard constraints. By relaxing the lightpath bandwidth constraint (i.e., the constraint (4)), we can transform it into a soft "price" term

$$\sum_{(s,d)}\sum_{0<n\leq N_{sd}}\theta_{sdn}\left[\sum_{(p,q)}\sum_{0<z\leq Z_{pq}}\upsilon_{sdn}^{pqz}G_{pqz}-C\alpha_{sdn}\right].$$

Likewise other constraints (11) and (12) are converted into soft term from which we can derive a Lagrangian function, as will be shown in the next section. They are then solved in the subsequent section 3.2 and 3.3.

Our method can solve the optimization problem for practical scale networks. CPLEX is able to obtain the optimal solution of this problem for very small networks, but cannot solve this problem for practical networks. Existing heuristic algorithms cannot obtain a quantitative performance bound. Thus, the performance of heuristics is never checked for any practical network. Our method obtains a performance bound for the grooming optimization problem for practical scale networks. It should be noted that our algorithm does not favour any particular traffic pattern or network topological property. Therefore, our algorithm's performance hardly depends on these factors.

In the following, we shall illustrate how to map the grooming problem into the LRSM framework.

3.1 Decomposition of the grooming optimization problem

We relax selected constraints to derive a Lagrangian function, and decompose the grooming optimization problem into sub-problems. The constraints that we choose to relax are the lightpath bandwidth constraint (4), the WC exclusive usage constraint (11), and the transmitter quantity constraint (12). Corresponding to each constraint, a Lagrange multiplier is introduced and a term is added into the Lagrangian function. The Lagrange multipliers are denoted as θ_{sdn}, ξ_{ijc} and η_s for constraints (4), (11) and (12), respectively. We propose a novel decomposition that removes the dependence between sub-problems in the Lagrangian function and constraints. The relaxation of the selected constraints leads to the following Lagrangian function $L(\gamma, \upsilon, \alpha, \delta, \xi, \eta, \theta)$:

$$\begin{aligned}
&L(\gamma, \upsilon, \alpha, \delta, \xi, \eta, \theta) = \\
&\sum_{(p,q)} \sum_{0<z\leq Z_{pq}} \gamma_{pqz} P_{pqz} G_{pqz} - \sum_{(p,q)} \sum_{0<z\leq Z_{pq}} E_{pqz} - \sum_{(s,d)} \sum_{0<n\leq N_{sd}} D_{sdn} + \\
&\sum_{(s,d)} \sum_{0<n\leq N_{sd}} \theta_{sdn} \left[\sum_{(p,q)} \sum_{0<z\leq Z_{pq}} \upsilon_{sdn}^{pqz} G_{pqz} - C\alpha_{sdn} \right] + \\
&\quad \sum_{(i,j)} \sum_{0<c\leq n_{ij}} \xi_{ijc} \left[\sum_{(s,d)} \sum_{0<n\leq N_{sd}} \delta_{ijc}^{sdn} - 1 \right] + \sum_{s} \eta_s \left[\sum_{d} \sum_{0<n\leq N_{sd}} \alpha_{sdn} - T_s \right]
\end{aligned} \tag{14}$$

We define the dual function $q(\zeta, \eta, \theta)$ as the supremum of the Lagrangian function (14). The design variables of the dual problem are the Lagrange multipliers θ, ξ and η.

$$\min_{\xi,\eta,\theta\leq 0} (q) \geq \max_{\gamma,\upsilon,\alpha,\delta} L(\gamma, \upsilon, \alpha, \delta, \xi, \eta, \theta) \tag{15}$$

After manipulation and re-grouping of the Lagrangian function, the maximization of the Lagrangian function is transformed to:

$$\begin{aligned}
&\sum_{(s,d)} \sum_{0<n\leq N_{sd}} \max_{\alpha_{sdn}} \left\{ \alpha_{sdn} \left[H_{sdn} - \min_{\delta} (Q_{sdn}) \right] \right\} + \\
&\sum_{(p,q)} \sum_{0<z\leq Z_{pq}} \max_{\gamma_{pqz}} \left\{ \gamma_{pqz} \left[P_{pqz} G_{pqz} - \min_{\upsilon} (R_{pqz}) \right] \right\} \quad - \\
&\sum_{(i,j)} \sum_{0<c\leq n_{ij}} \xi_{ijc} - \sum_{s} \eta_s T_s
\end{aligned} \tag{16}$$

where

$$H_{sdn} = -C\theta_{sdn} + \eta_s - t_s - r_d \tag{17}$$
$$Q_{sdn} = \sum_{(i,j)} \sum_{0<c\leq n_{ij}} \delta_{ijc}^{sdn} (d_{ijc} - \xi_{ijc}) \tag{18}$$
$$R_{pqz} = \sum_{(s,d)} \sum_{0<n\leq N_{sd}} \upsilon_{sdn}^{pqz} (V_{pqz} - \theta_{sdn} G_{pqz}) \tag{19}$$

Since the re-grouped Lagrangian function is decomposable, the maximization of the Lagrangian function can be decomposed into sub-problems. The first group of (16) represents the design of the lightpath setup status α, and the lightpath RWA scheme δ. We call it the RWA sub-problem. The second group of (16) represents the design of the traffic flow admission statusγ, and the traffic flow routing schemeυ. We call it the traffic flow routing sub-problem. The last two groups of (16) are independent of any design variables.

3.2 Solving the sub-problems

The RWA sub-problem is described as:

$$\max_{\alpha_{sdn}} \left\{ \alpha_{sdn} \left[H_{sdn} - \min_{\delta} \left(Q_{sdn} \right) \right] \right\}, \text{ for all lightpaths } s_{sdn}, \tag{20}$$

subject to the lightpath flow continuity constraints (6)-(10), and the receiver quantity constraint (13).

We develop a method to solve the RWA sub-problem. By assuming a virtual cost of w_{ijc} as $(d_{ijc} - \xi_{ijc})$, the meaning of Q_{sdn} can be understood as the total virtual cost of WCs that the lightpath s_{sdn} travels through. Because the WC exclusive usage constraint (11), and the transmitter quantity constraint (12) are relaxed, we are able to sequentially solve the RWA sub-problem for each lightpath, as opposed to considering all lightpaths together. The receiver quantity constraint (13) is ignored first and then we enforce the constraint by using the following policy. For each destination node d, the obtained lightpaths are sorted in the ascending order of their minimal dual cost. Lightpath s_{sdn} is only accepted, if its minimal dual cost is less than H_{sdn}, and a spare receiver is available at the destination node d. Otherwise, this lightpath should be rejected.

The traffic flow routing sub-problem is:

$$\max_{\gamma_{pqz}} \left\{ \gamma_{pqz} \left[P_{pqz} G_{pqz} - \min_{\upsilon} \left(R_{pqz} \right) \right] \right\}, \text{ for all traffic flows } \chi_{pqz}, \tag{21}$$

subject to the traffic flow continuity constraint (5).

This sub-problem requires the selection of traffic flows based on their profitability. We evaluate the profitability of a traffic flow based on the resources that the traffic flow requires in both the electrical and optical domains. By assuming a virtual cost of lightpath s_{sdn} as $(V_{pqz} - \theta_{sdn} G_{pqz})$, the meaning of R_{pqz} can be understood as the total virtual cost of lightpaths that the traffic flow χ_{pqz} travels through. We obtain the minimal dual cost by sequentially using Dijkstra's shortest path algorithm for each traffic flow. The minimal dual cost of χ_{pqz} is compared to the revenue $P_{pqz} G_{pqz}$ that χ_{pqz} generates if it is accepted. A routing scheme υ is obtained for each accepted traffic flow.

3.3 Solving the dual problem

We use a subgradient-based method to coordinate the solutions of the sub-problems. The sub-problems are linked together by the Lagrange multipliers, which are updated in iterations. We update (ξ, η, θ) in iterations towards the direction of its subgradient.

$$(\xi, \eta, \theta)^{(h+1)} = (\xi, \eta, \theta)^{(h)} + \beta^{(h)} g\left((\xi, \eta, \theta)^{(h)} \right) \tag{22}$$

The subgradient of q in Eq. (15) is denoted as$(g(\xi), g(\eta), g(\theta))$. We have the subgradient components:

$$g\left(\theta_{sdn}\right)=\sum_{(p,q)}\sum_{0<z\leq Z_{pq}}\upsilon_{sdn}^{pqz}G_{pqz}-C\alpha_{sdn},\text{ for all lightpaths }s_{sdn} \tag{23}$$

$$g\left(\xi_{ijc}\right)=\sum_{(s,d)}\sum_{0<n\leq N_{sd}}\delta_{ijc}^{sdn}-1,\text{ for all WCs }w_{ijc} \tag{24}$$

$$g\left(\eta_{s}\right)=\sum_{d}\sum_{0<n\leq N_{sd}}\alpha_{sdn}-T_{s},\text{ for all source nodes }s \tag{25}$$

The step size for the iterative updates is computed as

$$\beta^{(h)}=\mu\times\frac{q^{U}-q^{(h)}}{g^{T}\left((\xi,\eta,\theta)^{(h)}\right)g\left((\xi,\eta,\theta)^{(h)}\right)} \tag{26}$$

We set an initial estimation of q^U to the value of the objective function f for a feasible solution. The range of the parameter μ is $0<\mu<2$, which is adjusted adaptively as the algorithm converges. Specifically, if the value of $q^{(h)}$ remains unchanged in 3 consecutive iterations, the value of μ is decreased by a factor $p<1$, and if the value of $q^{(h)}$ increases in 5 consecutive iterations, the value of μ is increased by a factor $1/p$. From our simulation, fast convergence is achieved when $p=0.95$.

4 Performance Evaluation

We first provide an analysis on the computational complexity of our grooming optimization in order for readers to appreciate its efficiency. Then further capabilities are illustrated by an example.

4.1 Computational complexity

he computational complexities to solve the traffic flow routing and RWA subproblems are $O\left(A\left(N+W\right)N^2W\right)$ and $O\left(ZA^2N^2\right)$, respectively. Thus the computation complexity to solve the dual problem is $O\left(A\left(N+W\right)N^2W\right)+O\left(ZA^2N^2\right)$. The dual problem needs to be solved for many iterations in the overall framework. The good convergence of the subgradient-based iterations keeps the computational complexity of the overall optimization approach at an acceptable level. Our simulation on practical scale networks demonstrates the efficiency of the approach.

4.2 An example

We study the impact of cost parameters to the profit for a hypothetic network of 13 nodes and 19 links, whose topology is shown in Figure 1. We evaluate our algorithm for an arbitrary traffic pattern. We assume that C is 48 bandwidth unit. The randomly generated traffic pattern is shown in Tables 1-3. We fix other cost parameters and investigate the impact of a single cost parameter. Between each node pair, there are traffic flows with three different bandwidths. Our algorithm accepts or rejects traffic flows between

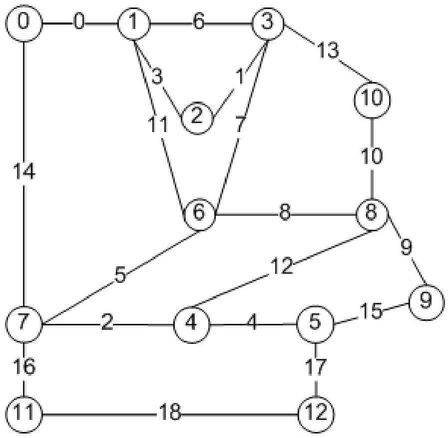

Figure 1: 13-node sample network topology.

Table 1: Traffic flows of 1 bandwidth units

0	5	4	12	6	2	5	0	1	0	3	1	0
0	0	1	1	1	9	2	2	2	1	4	0	2
4	1	0	4	0	2	1	4	11	4	2	1	1
4	2	8	0	11	2	1	4	1	1	2	1	2
2	4	0	1	0	3	0	1	0	4	0	2	2
2	1	2	4	1	0	2	4	4	9	0	5	10
1	1	4	2	10	4	0	0	4	2	1	0	4
4	10	1	4	6	3	2	0	0	4	1	0	4
4	0	12	4	4	4	2	0	0	1	2	7	9
0	0	0	2	1	0	1	0	2	0	2	0	0
3	0	0	10	0	4	0	5	0	4	0	4	6
1	4	2	2	4	1	2	1	10	1	1	0	2
2	0	11	1	0	2	1	0	9	0	1	2	0

Table 2: Traffic flows of 3 bandwidth units

0	2	1	2	4	2	0	1	1	0	4	1	0
4	0	0	1	1	1	2	2	2	1	4	0	2
2	1	0	2	0	1	1	4	2	1	4	6	7
7	2	0	0	2	4	1	2	1	1	2	1	2
2	1	8	1	0	2	0	1	0	7	0	6	1
5	3	2	2	1	0	2	2	4	3	0	2	2
1	5	8	6	2	1	0	0	4	2	1	0	0
1	4	1	2	4	3	5	0	0	2	1	0	8
2	0	5	5	1	2	1	0	0	1	2	4	2
0	0	7	2	1	0	1	0	4	0	1	0	0
2	3	8	4	0	0	4	4	0	2	0	2	2
1	4	1	5	7	8	6	1	1	1	1	0	2
0	0	2	1	1	4	3	0	2	0	4	5	0

Table 3: Traffic flows of 12 bandwidth units

0	1	4	5	4	3	1	2	1	4	1	2	4
3	0	3	2	3	5	1	4	5	3	5	5	2
0	2	0	0	0	1	5	0	4	5	1	2	0
2	2	0	0	5	2	2	0	1	3	3	5	2
5	0	0	1	0	5	4	1	3	1	4	3	0
2	5	1	0	2	0	1	0	0	2	0	2	4
2	3	0	3	0	0	0	0	3	5	2	1	1
0	5	2	0	4	0	3	0	0	0	5	0	3
0	3	2	1	5	4	2	3	0	1	2	4	3
4	2	0	2	2	3	0	0	2	0	2	0	0
2	5	5	0	3	5	2	1	0	0	0	0	2
2	0	1	1	4	2	0	5	5	5	1	0	5
0	4	1	5	3	2	1	0	1	2	2	2	0

the same node pair independently based on the grooming objective. The accepted traffic flows are allowed to use different traffic routing schemes.

A heuristic algorithm is developed to construct a feasible solution for the primal problem. Because some constraints for the primal problem are relaxed and transformed into the price terms in the dual problem, the solution to the dual problem is generally infeasible, i.e., some constraints are violated. Our heuristic algorithm repeatedly detours the least profitable traffic flows and the traffic flows on the least profitable lightpaths to other existing lightpaths, until all conflicts are resolved.

We study the impact of the WC cost. With an increase of the WC cost, the total number of the established lightpaths decreases (shown in Figure 2). When the WC cost increases from 0 to 9, the total number of the established lightpaths decrease by (110-76)/110=31%. The reason is that more lightpaths become non-profitable as the WC cost increases. The remaining profitable traffic flows tend to be packed into a lesser number of lightpaths. An increase of the WC cost dramatically reduces the lightpaths that use multiple fibre hops. In Figure 3, we show that the percentage of 3-fibre hop lightpaths reduces from 30% to 16%, as the WC cost increases from 0 to 9. More lightpaths of single fibre hop are established when the WC cost is high. The lightpaths of single fibre hop increase from 24% to 51%, as the WC cost increases from 0 to 9. The final primal values and the bounds are shown in Figure 4. The bound shows the upper limit of the profit with the given network configuration and traffic.

We study the impact of the traffic grooming cost under the fixed optical layer cost. The traffic grooming cost is modelled by the coefficient V_{pqz}. We set V_{pqz} to a fraction of the bandwidth requirement G_{pqz} of χ_{pqz}. Figures 5-7 show the number of the accepted traffic flows with different bandwidths. When the traffic grooming cost V_{pqz} increases from 0 to $0.6G_{pqz}$, the traffic flows with different bandwidths exhibit dramatically different behaviours: 1) for large bandwidth (i.e., OC-12) traffic flows, the number of traffic flows using multiple hop lightpaths decreases significantly, by (26-6)/26=77%; the number of traffic flows using single hop lightpaths slightly decreases, by (240-

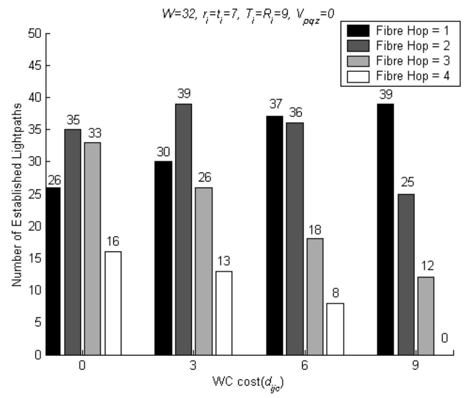

Figure 2: Number of the established lightpaths of different fibre hops w.r.t. the WC cost.

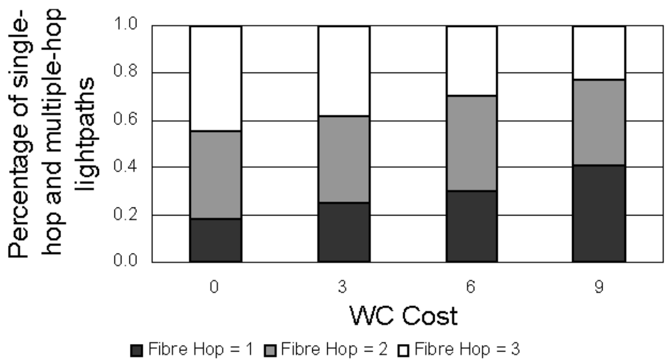

Figure 3: Percentage of single-hop and multiple-hop lightpaths in all established lightpaths w.r.t. the WC cost.

219)/240=9%. It is more efficient to carry them over single hop lightpaths; 2) for medium bandwidth (i.e., OC-3) traffic flows, the number of traffic flows using multiple hop lightpaths decreases by (80-67)/80=16%; the number of traffic flows using single hop lightpaths increases slightly, by (225-213)/225=5%. More lightpaths should be set up so that more such traffic flows can be carried over single hop lightpaths. However, the revenue generated by such traffic flows remains at the same level; 3) for small bandwidth (i.e., OC-1) traffic flows, the number of traffic flows using multiple hop lightpaths increases significantly, by (136-104)/136=24%; the number of traffic flows using single hop lightpaths increases slightly, by (279-255)/279=8.6%. More such traffic flows are accepted by using multiple hop lightpaths (particularly two hop lightpaths). The reason is that such traffic flows use the segregated bandwidth that cannot be used by medium and large bandwidth

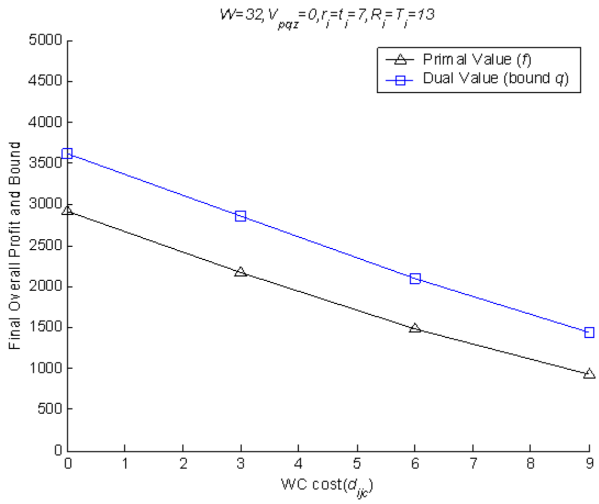

Figure 4: The final overall profit and bound w.r.t. the WC cost.

traffic flows. Since we assume that the traffic grooming cost is proportional to the bandwidth requirement, the traffic grooming cost for small bandwidth traffic flows is not a major cost compared to other costs such as the optical layer cost.

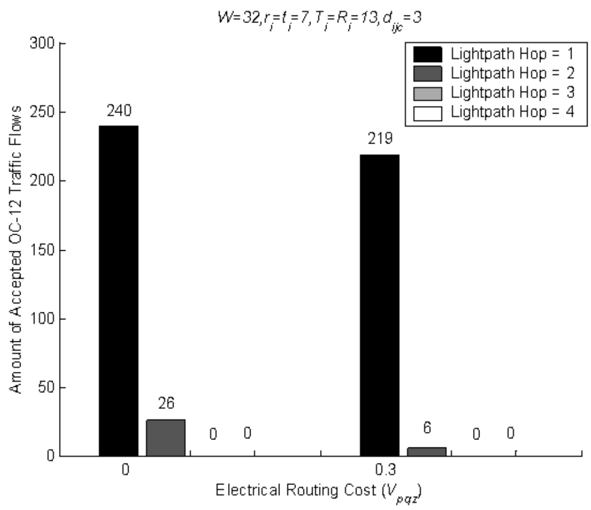

Figure 5: Total number of the accepted OC-12 traffic flows (there are 334 offered OC-12 traffic flows with the traffic pattern shown in Table 3).

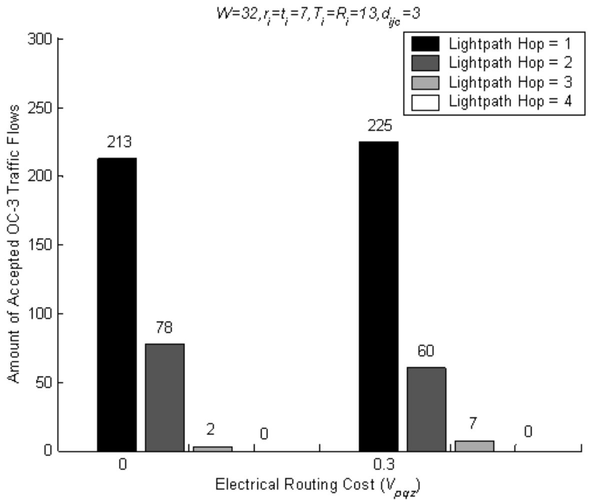

Figure 6: Total number of the accepted OC-3 traffic flows (there are 344 offered OC-3 traffic flows with the traffic pattern shown in Table 2).

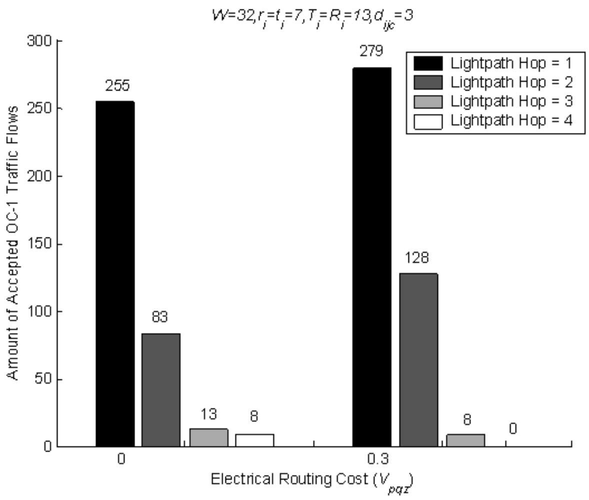

Figure 7: Total number of the accepted OC-1 traffic flows (there are 426 offered OC-1 traffic flows with the traffic pattern shown in Table 1).

4.3 Disccusion

We have also conducted extensive simulations of different networks (e.g., 13 nodes and 19 links; 14 nodes and 21 links; 22 nodes and 35 links). These simulations demonstrated similar trends, although the results were not presented. Our solution method can obtain fair results for a 13-node network within 30 minutes of computation time, for a 14-node network within 1 hour, and for a 22-node network within 4 hours on a PC configured with an Intel® 2.4GHz CPU and 1GB of RAM.

5 Conclusions

In this tutorial paper, we have demonstrated the use of the LRSM to solve the grooming optimization problem of a mesh WDM network without using wavelength converters. The grooming optimization objective incorporates both cost minimization and throughput maximization objectives. In contrast to using CPLEX or heuristic algorithms to solve ILP problems, the LRSM can solve mesh networks of practical scales, and meanwhile can obtain an upper bound to evaluate the optimality of the feasible solution. Through the numerical example, we made an interesting observation on the different behaviours of various bandwidth traffic flows and their contributions to the grooming objective, when the relative cost of electrical and optical layers changes.

6 References

[1] R. Dutta and G.N. Rouskas, "Traffic grooming in WDM Networks: Past and Future", IEEE Network, Vol. 16, No. 6, pp. 46-56, Nov./Dec. 2002.

[2] I. Cerutti and A. Fumagalli. "Traffic Grooming in Static Wavelength Division Multiplexing Networks" IEEE Communications Magazine. Vol. 43, No. 1. Jan. 2005. pp. 101 – 107.

[3] S. Balasubramanian and A.K. Somani. "On Traffic Grooming Choices for IP over WDM networks". Proceedings of the 3^{rd} International Conference on Broadband Communications, Networks, and Systems (BROADNETS 2006). October, 2006. San José, California, USA.

[4] R. Barr, M.S. Kingsley, and R. Patterson. "Grooming Telecommunications Networks: Optimization Models and Methods". in M.G.C. Resende and P. M. Pardalos, eds., *Handbook of Optimization in Telecommunications*. Springer, 2006. pp. 837-862.

[5] J.Q. Hu and B. Leida, "Traffic Grooming, Routing, and Wavelength Assignment in Optical WDM Mesh Networks", Proceedings of the INFOCOM 2004. March, 2004. Hong Kong, China. pp. 495-501.

[6] K. Zhu and B. Mukherjee, "Traffic grooming in an Optical WDM Mesh Network", IEEE Journal on Selected Areas in Communications, Vol. 20, No. 1, pp. 122-133, Jan. 2002.

[7] V.R. Konda and T.Y. Chow, "Algorithm for traffic grooming in optical networks to minimize the number of transceivers", Proceedings of the IEEE Workshop on High Performance Switching and Routing (HPSR 2001). May 2001. Dallas, Texas, USA. pp. 218-221.

[8] Z. Patrocinio Jr. and G.R. Mateus, "A Lagrangian-based Heuristic for Traffic grooming in WDM Optical Networks", Proceedings of the IEEE 2003 Global Communications Conference (GLOBECOM 2003). Dec. 2003. San Francisco, CA, USA. pp. 2767-2771.

[9] P. Prathombutr, J. Stach and E.K. Park "An Algorithm for Traffic grooming in WDM Optical Mesh Networks with Multiple Objectives". Telecommunication Systems. Vol. 28, No.3-4. 2005 pp. 369 – 386.

[10] A. Jaekel, A. Bari, Y. Chen and S. Bandyopadhyay. "New Techniques for Efficient Traffic Grooming in WDM Mesh Networks". Proceedings of 16^{th} International Conference on Computer Communications and Networks (ICCCN 2007). Aug. 2007. Honolulu, Hawaii, USA. pp. 303 – 308.

[11] K. Lee and M.A. Shayman. "Optical Network Design with Optical Constraints in IP/WDM Networks". IEICE Transactions on Communications. Vol. E88-B, No. 5. May 2005, pp. 1898-905.

[12] K. Lee and M.A. Shayman. "Rollout Algorithms for Logical Topology Design and Traffic Grooming in Multihop WDM Networks". Proceedings of the 2005 IEEE Global Telecommunications Conference (GLOBECOM 2005). Vol. 4. Nov. 2005.

[13] J. Fang and A.K. Somani. "IP Traffic Grooming Over WDM Optical Networks". Proceedings of the 9^{th} Conference on Optical Network Design and Modeling (ONDM 2005). Feb. 2005. Milan, Italy. pp. 393 – 402.

[14] Y. Zhang, J. Wu, O. Yang and M. Savoie. "A Lagrangian-Relaxation Based Network Profit Optimization For Mesh SONET-Over-WDM Networks", Photonic Network Communications, Vol. 10, No. 2, pp. 155-178, Sept. 2005.

[15] M. Sridharan, A.K. Somani and M.V. Salapaka. "Approaches for Capacity and Revenue Optimization in Survivable WDM Networks," Journal of High Speed Networks, Vol. 10, No. 2. 2001. pp. 109-125.

[16] A. Chiu and E. Modiano. "Traffic Grooming Algorithms for Reducing Electronic Multiplexing Costs in WDM Ring Networks". IEEE/OSA Journal of Lightwave Technology. Vol. 18, No. 1. Jan. 2000. pp. 2–12.

[17] S. Xiao, G. Xiao and Y.W. Leung. "A Network Flow Approach for Static and Dynamic Traffic Grooming in WDM Networks". Computer Networks. Vol. 50, No. 17. Dec. 2006. pp. 3400-3415.

[18] D. Palomar, and M. Chiang, "A Tutorial on Decomposition Methods for Network Utility Maximization", IEEE Journal on Selected Areas in Communications, Vol. 24, No. 8, pp. 1439-1451, Aug. 2006.

[19] D. P. Bertsekas, Nonlinear Programming, Athena Scientific, Athena Scientific, 1999.

[20] Y. Zhang, O. Yang and H. Liu, "A Lagrangean Relaxation And Subgradient Approach For The Routing And Wavelength Assignment Problem In WDM Networks", IEEE Journal On Selected Areas In Communications, Vol. 22, No. 9, pp. 1752-1765, Nov. 2004

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