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# A proof of wavelength conversion not improving Lagrangian bounds of the sliding scheduled RWA problem

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ABSTRACT

Extensive previous studies confirmed that wavelength conversion may only marginally improve the solutions to the static Routing and Wavelength Assignment (RWA) problem. This means that, for the static RWA problem, certain RWA schemes that do not use wavelength conversion can achieve a performance almost as good as the one from the best RWA scheme. Previous research work on sliding scheduled RWA problems, where a given set of lightpath demands are allowed to slide within their time windows, has also indicated in limited simulation results that the benefit of using wavelength conversion is marginal. However, the observation cannot be conclusive without the solid mathematical proof. We are thus motivated to investigate whether schedule sliding really requires wavelength conversion to achieve a better performance. In this paper, we prove that wavelength conversion does not improve the Lagrangian bound of the sliding scheduled RWA problem. In most test cases, this bound is very close to the best achieved objective function value. Our proof implies that, for those cases, the improvements achieved by making use of wavelength conversion are very marginal.



## 1. Introduction

Wavelength continuity is one of the key issues for the research and operation of wavelength–routed wavelength division multiplexing (WDM) networks. Without wavelength conversion, a lightpath must use a chain of wavelength channels with the same wavelength on all the fibre links along its route. Using wavelength converters at intermediate switching nodes, a lightpath may consist of wavelength channels with different wavelengths at different fibre links. However, in general, wavelength converters are expensive, and thus should only be used if the operating efficiency of a WDM network can be significantly improved.

The benefit of using wavelength conversion has been extensively studied for various network operating scenarios. For example, when all lightpath demands are given in advance, which forms the static RWA problem, previous studies concluded that wavelength conversion only slightly improves the network operating efficiency [1–12]. In Ref. [13], we proved that Lagrangian bounds do not change for the static RWA problem, even if abundant wavelength converters are installed on all switching nodes.

However, only limited simulation results were published in the literature about the benefit of using wavelength conversion in the *sliding scheduled RWA problem*, in which the source, destination and duration of all lightpath demands are given in advance, but all lightpaths are allowed to slide within their respective earliest starting and latest ending time slots. Resource provisioning for survivable WDM networks under a sliding scheduled traffic model was studied in [14]. Please note that we used the term ‘sliding’ to indicate that our formulation allows the starting time of the lightpath to slide with a certain penalty. However, the regular ‘accept-or-not’ scheduling is just a special case of this formulation by setting the sliding penalty to infinitely large and the sliding window size to be 1. Simulation results of heuristic algorithms showed that wavelength conversion only slightly reduced the required number of wavelength channels in the entire network. In [15], computation results from integer linear programing models for survivable traffic grooming of scheduled demands showed that resource utilization at the logical level was only slightly improved (sometimes too little to be observed) by allowing wavelength conversion. It was observed that wavelength conversion enabled lightpaths to use shorter routes. This indicates that by re-arranging the RWA schemes and scheduling of lightpaths (since all lightpath demands are known in advance), the benefit of using wavelength conversion to improve wavelength channel utilization is reduced. In [16], provisioning sub-wavelength multicast sessions with flexible scheduling over WDM networks was studied. Its simulation results of heuristic algorithms showed that the benefit of using wavelength conversion to reduce traffic blocking is much smaller

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for scheduled unicast traffic than for scheduled multicast traffic. In our opinion, the reason is that re-arranging the RWA schemes and scheduling for unicast traffic is much easier to achieve than that for multicast traffic, when wavelength conversion is unavailable. As the computation results shown in Table 1 ([17]), the wavelength conversion only slightly improved the solutions for the scheduled RWA problem. Note that the parameter $F$ denotes the number of wavelength converters installed on each node. Specifically, $F = 0$ means no converter at all, while $F = 4$ means abundant of wavelength converters. The other parameters $W$, $y_{sdn}$, $r_{sdn}$, and $d_{ij}$, represent, respectively, the number of wavelengths, the earliness penalty coefficient, the lateness penalty coefficient and the cost of a wavelength channel. The readers are referred to [17] for the detailed explanation for these parameters. We observed that no matter how we change the values of the other parameters, the Lagrangian (LR) bounds of the achieved optimization objectives remained almost unchanged regardless of using wavelength conversion or not, while the objective values have only marginal improvements. Please note that the improvements are so small that they are normally buried under the 'noise' of the randomness from the heuristic algorithm, i.e., we cannot derive any conclusion about the real impact on the primal problem.

In Ref. [18], lightpath rerouting strategies in WDM networks under scheduled traffic was studied. The authors made an assumption that no wavelength conversion was allowed, based on their expectation that efficient rerouting schemes might compensate the absence of wavelength converters so that lightpath blocking would not increase much. Unfortunately, neither numerical results nor formal proofs were provided to support such "expectation". So far, it largely remains a suspicion or a speculation that wavelength conversion only marginally improves the solutions to the scheduled RWA problem.

Therefore, we are motivated to further investigate whether schedule sliding requires wavelength conversion to achieve a better performance. Comparing with the static RWA problem, the scheduled RWA problem has an extra time dimension. Essentially, the scheduled RWA problem combines the static RWA problem and a scheduling problem, and thus is much more complicated in formulation and mathematical solution. However, we are able to identify similarities to the static RWA problem after proper mathematical manipulations. In this paper, we study the scheduled RWA problem by comparing two cases: the model with abundant wavelength converters, and the model without any wavelength converter. We will prove that the Lagrangian bound cannot be improved by adding abundant wavelength converters.

Since the solution to the original problem has exponential complexity and it is thus computationally impossible to obtain the exact optima even for a medium-size network [17], we need to apply some near-optimal solution with a reasonable computation complexity. At the same time, generating the lower bound would be an additional advantage. For this purpose, the Lagrangian relaxation method is employed to generate the dual problem (DP) by relaxing the original scheduled RWA problem (i.e., the primal problem). A Lagrangian bound, which yields a lower bound to the primal problem, is obtained by solving the DP optimally [19]. Compared with the primal problem, the solution to the DP is straightforward and has polynomial complexity. Since the DP has relaxed constraints (from the primal problem), its solution is normally infeasible for the primal problem. We usually apply a simple heuristic algorithm with polynomial complexity to 'map' the dual solution back to the primal problem to obtain a near-optimal feasible solution. The subgradient method is often used to maximize the DP iteratively to obtain a better solution in the dual space, while the feasible solution is generated in each iteration by applying the heuristic mapping algorithm. The users are referred to [17] for the detailed description of the LR optimization framework. In this paper, we are focusing only on the Lagrangian bound, since we need to prove it to be constant with respect to the availability of wavelength converters. Please note that our objective function is non-linear as opposed to most of the linear formulations in the area.

Please note that the computation results and the observations have already been described in Ref. [17], while in this paper, we aim at providing the mathematical proof to support our observation in Ref. [17].

This paper is organized as follows: In Section 2, we provide a formulation of the scheduled RWA problem. In Section 3, a bound of the scheduled RWA problem is derived from its Lagrangian dual problem. In Section 4, we describe two cases: in one case, wavelength conversion is used in a special way; while in the other case, no wavelength conversion is used. We prove that the Lagrangian bound for the first case is not better than that for the second case in Section 5. In Section 6, we explain duality gaps. Then, in Section 7, the implications on resource requirements and acceptance of lightpath demands are exploited. We conclude this paper in Section 8.

**Table 1**
Using wavelength conversion to improve the achieved design objective and LR bounds [17].

| Parameters | | | | | Results | |
|---|---|---|---|---|---|---|
| $W$ | $y_{sdn}$ | $r_{sdn}$ | $d_{ij}$ | $F$ | LR bound | Objective value |
| 12 | 49 | 24 | 4 | 4 | 9122 | 9256 |
| | | | | 0 | 9146 | 9414 |
| 8 | 49 | 24 | 4 | 4 | 10165 | 11178 |
| | | | | 0 | 10174 | 11762 |
| 10 | 49 | 24 | 4 | 4 | 9263 | 10005 |
| | | | | 0 | 9287 | 10330 |
| 10 | 49 | 24 | 8 | 4 | 17820 | 17862 |
| | | | | 0 | 17829 | 17922 |
| 12 | 20 | 20 | 4 | 4 | 9278 | 9781 |
| | | | | 0 | 9288 | 10145 |
| 12 | 20 | 20 | 8 | 4 | 17723 | 17882 |
| | | | | 0 | 17726 | 17942 |

## 2. A formulation of the scheduled RWA problem

### 2.1. Notations

Our network model consists of $N$ nodes interconnected by $E$ fibres in an arbitrary mesh topology. Each fibre has $W$ wavelength channels. The fibre between nodes $i$ and $j$ is denoted by $e_{ij}$. The $c$th wavelength channel on $e_{ij}$ is denoted by $w_{ijc}$ ($0 \leqslant c < W$). The set $E$ represents all links in the network. Each link has a pair of fibres, one for each direction. The set $V$ represents all the WDM switch nodes in the network. $s_h$ is used to denote the $h$th lightpath demand. The set $S$ represents all lightpath demands. Note that $N = |V|$. We consider a total number of $Z$ time slots. $T_h$ denotes a constant holding time of $s_h$. We assume using wavelength channel $w_{ijc}$ for one time slot costs one unit, and then set the relative cost of using a wavelength converter at any node for one time slot to a constant $R$.

### 2.2. Design variables

We list the input parameters in Table 2 and design variables (i.e., decision variables) of our formulated problem in Table 3.

### 2.3. Design objective

We use the same penalty-based objective function (i.e., primal function) as [17], which can efficiently formulate the design

**Table 2**
Input parameters of our formulated problem.

| | |
|---|---|
| $d_{ij}$ | The cost of using a wavelength channel on link between node i and j for one time slot |
| $o_i$ | The cost of using a wavelength converter on node i for one time slot |
| $W$ | The number of wavelengths used in the network |
| $s_h$ | The hth lightpath demand |
| $S$ | All lightpath demands, i.e., $S = \{s_h\}$ |
| $T_h$ | The holding time of lightpath $s_h$ |
| $(b_h, b'_h)$ | The desired window of starting time for $s_h$, $0 \leqslant b_h \leqslant b'_h < Z$ |
| $F$ | The number of wavelength converters installed on each node |
| $N$ | The number of nodes in the network |
| $P_h$ | The penalty coefficient for rejecting $s_h$ |
| $Z$ | Total number of time slots of our scheduling problem |

objectives. Thus, we want to minimize the objective function $J$ over all feasible solutions of the primal problem, that is to find $\min_{v\in V}\{J\}$, where

$$J(V) = \sum_{s_h \in S}[(1-\alpha_h)P_h + \alpha_h(C_h + E_h)] \quad (1)$$

For each lightpath demand $s_h$, either the penalty of rejecting it ($P_h$), or the penalty of using resources ($C_h$) to set up the lightpath and its timing violation ($E_h$) is added to the objective function ($J$), depending on $s_h$'s admission status $\alpha_h$. If the scheduling of lightpath $s_h$ respects its timing requirement, $E_h = 0$; otherwise, we assign $E_h$ with a certain penalty value. Specifically,

$$E_h = \begin{cases} EP_h(\beta_h) & \text{if} \quad \beta_h < b_h \\ 0 & \text{if} \quad b_h \leqslant \beta_h \leqslant b'_h \\ TP_h(\beta_h) & \text{if} \quad \beta_h > b'_h \end{cases},$$

where $EP_h(\beta_h)$ stands for earliness penalty and $TP_h(\beta_h)$ stands for tardiness penalty, both of which can be either constants or functions with increasing values as $\beta_h$ deviates from the window $[b_h, b'_h]$.

The total cost of using wavelength channels and wavelength converters is the cost of $s_h$, denoted by $C_h$:

$$C_h = \sum_{\beta_h \leqslant t < (\beta_h + T_h)} \left( \sum_{e_{ij}\in E} \sum_{0 \leqslant c < W} \delta^h_{ijct} + R \times \sum_{i\in V} \phi^h_{it} \right), \quad s_h \in S \quad (2)$$

### 2.4. Design constraints

The above problem must conform to the following constraints.

(a) **Lightpath conservation constraints**: A lightpath must be continuous along a path from its source to its destination.

$$\sum_{j\in V}\sum_{0\leqslant c<W} \delta^h_{ijct} - \sum_{j\in V}\sum_{0\leqslant c<W} \delta^h_{jict} = \begin{cases} \alpha_h & \text{if} \quad \beta_h \leqslant t < (\beta_h + T_h), \; i \text{ is the source node of } s_h \\ -\alpha_h & \text{if} \quad \beta_h \leqslant t < (\beta_h + T_h), \; i \text{ is the destination node of } s_h \\ 0 & \text{otherwise} \end{cases},$$
$$s_h \in S, \; 0 \leqslant t < Z \quad (3)$$

(b) **Wavelength channel exclusive usage constraints**: A wavelength channel can only be used by no more than one lightpath.

$$\sum_{s_h \in S} \delta^h_{ijct} \leqslant 1, \quad e_{ij} \in E, \quad 0 \leqslant c < W, \quad 0 \leqslant t < Z \quad (4)$$

(c) **Wavelength converter quantity constraints**: The number of used converters at a node must be no more than the number of installed converters at the node.

$$\sum_{s_h \in S} \phi^h_{it} \leqslant F, \quad i \in V, \quad 0 \leqslant t < Z \quad (5)$$

(d) **Wavelength conversion constraints**: For any time slot, a wavelength converter at an intermediate node $j$ is used, when different wavelengths are assigned to $s_h$ for the incoming and outgoing portions of the lightpath at this node.

$$\phi^h_{jt} = \begin{cases} 1 & \text{if } \exists m, \quad k \in V \quad \text{and} \quad b \neq a, \quad \delta^h_{mjat} = \delta^h_{jkbt} = 1 \\ 0 & \text{otherwise} \end{cases},$$
$$j \in V, 0 \leqslant t < Z \quad (6)$$

(e) **Lightpath persistency constraints**: During the lifespan of $s_h$, its RWA scheme must remain the same for all time slots.

$$\delta^h_{ijcx} = \delta^h_{ijcy}, \quad s_h \in S, \quad e_{ij} \in E, \quad \beta_h \leqslant x < (\beta_h + T_h), \quad \beta_h \leqslant y < (\beta_h + T_h) \quad (7)$$

Note that together with constraints (b), the allocation of wavelength converters $\phi^h_{jt}$ remains the same, too.

## 3. Lagrangian dual problem and bounds of the scheduled RWA problem

### 3.1. Lagrangian dual problem (DP)

We use the Lagrangian relaxation framework to derive a Lagrangian DP from the primal problem $\min_{v\in V}\{J\}$. We relax the primal problem's constraints that represent resource limitations

**Table 3**
Design variables of our formulated problem.

| | |
|---|---|
| $\alpha_h$ | Lightpath demand $s_h$'s admission status. If $s_h$ is admitted, $\alpha_h = 1$; if $s_h$ is rejected, $\alpha_h = 0$ |
| $A$ | Admission status of all lightpath demands, i.e., $A = (\alpha_h)$ |
| $\delta^h_{ijct}$ | Lightpath $s_h$'s usage of wavelength channel $w_{ijc}$ at time slot t. If $s_h$ uses $w_{ijc}$ at time slot t, $\delta^h_{ijct} = 1$; otherwise, $\delta^h_{ijct} = 0$ |
| $\Delta_h$ | The RWA scheme of $s_h$, i.e., all the $\delta^h_{ijct}$ for a given h |
| $\Delta$ | The RWA schemes for all lightpaths, i.e., $\Delta = (\Delta_h)$ |
| $\beta_h$ | The starting time slot of lightpath $s_h$ |
| $B$ | The scheduling for all lightpaths, i.e., $B = (\beta_h)$ |
| $\phi^h_{it}$ | Lightpath $s_h$'s usage of a wavelength converter at node i at time slot t. If $s_h$ uses a wavelength converter at node i at time slot t, $\phi^h_{it} = 1$; otherwise, $\phi^h_{it} = 0$ |
| $\phi_h$ | The assignment of wavelength converters at all nodes to $s_h$, i.e., all the $\phi^h_i$ for given h |
| $\phi$ | The assignment of wavelength converters to all lightpaths, i.e., $\Phi = (\Phi_h)$ |
| $V$ | The set composed of all the feasible solutions to the primal problem, i.e., the set of solutions fulfilling constraints a, b, c, d, and e |
| $v$ | A feasible solution to the primal problem |
| $Vr$ | The set composed of all the feasible solutions to the relaxed problem, i.e., the set of solutions fulfilling constraints a, d, and e |
| $v^*$ | Optimal solutions to the scheduled RWA problem, i.e., $v^* = (A^*, B^*, \Delta^*, \Phi^*)$. Note that we denote optimal solutions with the star superscript and feasible solutions without the star superscript |

(i.e., the wavelength channel exclusive usage constraints (b), and the wavelength converter quantity constraints (c)) and then add items corresponding to the relaxed constraints into the objective function via Lagrange multipliers. We introduce Lagrangian multipliers $\xi_{ijct}$ ($e_{ij} \in E,\ 0 \leqslant c < W,\ 0 \leqslant t < Z$) in association with the wavelength channel exclusive usage constraints (b), and Lagrangian multipliers $\lambda_{it}$ ($i \in V,\ 0 \leqslant t < Z$) in association with the wavelength converter quantity constraints (c). The vectors of Lagrange multipliers ($\xi_{ijct}$) and ($\lambda_{it}$) are denoted as $\xi$ and $\lambda$, respectively. Thus, the objective function of the primal problem is transformed into the Lagrangian function $L$ [19]:

$$L(V,\xi,\lambda) = J(V) + \sum_{e_{ij}\in E}\sum_{0\leqslant c<W}\sum_{0\leqslant t<Z}\xi_{ijct}\left(\sum_{s_h\in S}\delta^h_{ijct} - 1\right) + \sum_{i\in V}\sum_{0\leqslant t<Z}\lambda_{it}\left(\sum_{s_h\in S}\phi^h_{it} - F\right) \quad (8)$$

The Lagrangian relaxation of the primal problem is defined as $\min_{v\in V}\{L(v,m)\}$ with parameters of the Lagrange multipliers $m = (\xi,\lambda)$, subject to constraints (a), (d) and (e).

For $m = (\xi,\lambda) \geqslant 0$, the Lagrangian relaxation of the primal problem $\min_{v\in V}\{L(v,m)\}$ provides a lower bound to the primal problem $\min_{v\in V}\{J\}$. This fact may be explained as follows: (1) a feasible solution to the primal problem is a feasible solution to the Lagrangian relaxation of the primal problem; (2) for any feasible solution $v$ to the primal problem and with any Lagrange multipliers $m \geqslant 0$, we have $L(v,m) \leqslant J$, and thus $\min_{v\in V}\{L(v,m)\} \leqslant \min_{v\in V}\{J\}$, for all $m \geqslant 0$.

Among the lower bounds provided by the Lagrangian relaxation of the primal problem, we are interested in finding the best lower bound for the primal problem, which can be formulated as the Lagrangian DP:

$$\max_{m\geqslant 0}\{q(m)\}, \quad (9)$$

subject to the constraints in (a), (d) and (e). The Lagrangian DP (9) is relative to the constraints (b) and (c). The dual function is defined as $q(m)$.

There are a large but finite number of feasible solutions to the primal problem, over which when the Lagrangian function is minimized, we obtain the dual function $q(m)$ defined as the infimum of the Lagrangian function $L$:

$$q(m) = \min_{v\in Vr}\{L(v,m)\}, \quad m \geqslant 0 \quad (10)$$

The optimal value of the Lagrangian DP (denoted by $q^*$) is the best lower bound of the primal problem [19]. The corresponding optimal Lagrange multiplier values are denoted by $m^* = (\xi^*,\lambda^*)$. We thus have

$$q^*(m^*) = \min_{v\in Vr}\{L(v,m^*)\} \leqslant \min_{v\in V}\{L(v,m^*)\} \leqslant \min_{v\in V}\{J\} \quad (11)$$

The Lagrangian DP has the following properties:

- The solution space of the Lagrangian DP is dimensioned by the Lagrangian multipliers corresponding to the relaxed constraints in the primal problem together with the original variables of the primal problems;
- The dual function is concave over its solution space, which is convex [19, proposition 5.1.2], resulting in the applicability of more efficient solution methods, such as sub-gradient based methods.

### *3.2. Decomposition of the Lagrangian DP*

The Lagrangian DP can be decomposed into sub-problems, where each sub-problem corresponds to the scheduled RWA problem of one lightpath demand.

$$q(m) = \sum_{s_h\in S}\left\{\min_{\alpha_h}[(1-\alpha_h)P_h + \alpha_h D_h]\right\} - \sum_{e_{ij}\in E}\sum_{0\leqslant c<W}\sum_{0\leqslant t<Z}\xi_{ijct} - \sum_{i\in V}\sum_{0\leqslant t<Z}\lambda_{it}F, \quad (12)$$

where

$$D_h = \min_{\beta_h}\left\{E_h + \min_{\Delta_h,\Phi_h}\left[\sum_{e_{ij}\in E}\sum_{0\leqslant c<W}\sum_{\beta_h\leqslant t<(\beta_h+T_h)}\delta^h_{ijct}(\xi_{ijct}+1) + \sum_{i\in V}\sum_{\beta_h\leqslant t<(\beta_h+T_h)}\phi^h_{it}(\lambda_{it}+R)\right]\right\},$$

subject to the constraints in (a), (d) and (e), which contains the exact same form as the Shortest Path Algorithm for Wavelength Graph (SPAWG) proposed in [22,23]. We can thus simply apply SPAWG to obtain the optimum for $\min_{\Delta_h,\Phi_h}[\cdot]$ and then obtain the minimal $\beta_h$ among all possible time slots. Please note that SPAWG is essentially a shortest path algorithm on a wavelength graph (WG) [22,23].

Now, we define two important parameters of using a resource. Both parameters are functions of the starting time slot of using the resource (denoted by $\beta$) and the duration of using the resource (denoted by $T$).

The *accumulative dual cost* $\widehat{\xi}^{\beta,T}_{ijc}$ of using wavelength channel $w_{ijc}$ ($e_{ij} \in E,\ 0 \leqslant c < W$) is defined as:

$$\widehat{\xi}^{\beta,T}_{ijc} = \sum_{\beta\leqslant t<(\beta+T)}(\xi_{ijct}+1), \quad e_{ij}\in E, \quad 0 \leqslant c < W. \quad (14)$$

The *accumulative dual cost* $\widehat{\lambda}^{\beta,T}_{i}$ of using a wavelength converter on node $i$ ($i \in V$) is defined as:

$$\widehat{\lambda}^{\beta,T}_{i} = \sum_{\beta\leqslant t<(\beta+T)}(\lambda_{it}+R), \quad i \in V \quad (15)$$

We can thus simplify the representation of $D_h$ as

$$\min_{\beta_h}\left\{E_h + \min_{\Delta_h,\Phi_h}\left[\sum_{e_{ij}\in E}\sum_{0\leqslant c<W}\delta^h_{ijc,\beta_h}\widehat{\xi}^{\beta_h,T_h}_{ijc} + \sum_{i\in V}\phi^h_{j,\beta_h}\widehat{\lambda}^{\beta_h,T_h}_{i}\right]\right\} \quad (16)$$

Note that due to the lightpath persistency constraints (e), the design variables $\delta^h_{ijct}$ and $\phi^h_{jt}$ do not change over the lifespan of lightpath $s_h$. So we use their values at the starting time slot to represent their values, i.e., $\delta^h_{ijct} = \delta^h_{ijc,\beta_h}$ and $\phi^h_{jt} = \phi^h_{j,\beta_h},\ \beta_h \leqslant t < (\beta_h + T_h)$.

## 4. Lagrangian bounds of the scheduled RWA problems with and without wavelength conversion

To study the impact of wavelength conversion, we will derive Lagrangian bounds of the scheduled RWA problems with and without wavelength conversion. Then in the next section, we will prove that the Lagrangian bound cannot be improved by wavelength converters for the same network and lightpath demands.

We derive the Lagrangian bound of the scheduled RWA problem with wavelength conversion for a benchmark case that has three properties:

- Wavelength conversion is available at all nodes. The number of wavelength converters at a node is set to such a large constant number that there are more than enough for all lightpaths at any node to use one. We denote this large constant number as $F$;
- Wavelength converters are installed in a share-per-node manner [20], in which any input lightpath may use any available wavelength converter before entering any output port. This is the best sharing structure of wavelength converters;

- The cost of using any wavelength converter is set to zero (i.e., $R = 0$).

For this benchmark case, the Lagrangian dual problem of the scheduled RWA problem with wavelength conversion is denoted as $DP_1$.

Without wavelength conversion, the Lagrangian bound of the scheduled RWA problem may be derived from a baseline case, in which wavelength conversion is unavailable at all nodes. The Lagrangian dual problem of the scheduled RWA problem without wavelength conversion is denoted as $DP_2$. Note that in $DP_2$, $\phi_{it}^h = 0$, $s_h \in S$, $i \in V$, $0 \leqslant t < Z$.

When we prove that wavelength conversion does not improve the Lagrangian bound even for the benchmark case, our conclusions are able to be extended to other configurations and cost values of wavelength converters. This can be explained by the non-negative requirements of Lagrange multipliers and $D_h$ in Eq. (13), and the formulation of the Lagrangian DP in Eq. (12). The Lagrangian bound for the case other than the benchmark case cannot be less than that for the benchmark case.

## 5. Wavelength conversion's impact on the Lagrangian bounds of the scheduled RWA problems

We now prove that the lower bounds of the two Lagrangian DPs presented above are the same, i.e., $(q_{DP_1})^* = (q_{DP_2})^*$, where $(q_{DP_1})^*$ and $(q_{DP_2})^*$ denote the optimal values of $DP_1$ and $DP_2$, respectively.

**Lemma 1.** *In any of the optimal solutions $(m^*, v^*)$ to $DP_1$,* $(\lambda_{it})^* = 0$, $i \in V$.

**Proof.** When we ignore the integer constraints for the design variables $v$, the Lagrange multiplier theory [19, Chapter 3, proposition 3.3.6, pp.327] proves that for the optimal solution $v^*$ of the Lagrangian function (8), there always exist Lagrange multipliers $m^*$ that satisfy:

$$L(m^*, v^*) = 0 \tag{17}$$

Then we have $(\lambda_{it})^* \left(\sum_{s_h \in S} (\phi_{it}^h)^* - F\right) = 0$ ($i \in V$, $0 \leqslant t < Z$). Because we assume the number of wavelength converters at any node is abundant, the term $\sum_{s_{sdn} \in S} (\phi_{it}^{sdn})^* - F$ must be strictly less than zero, $i \in V$, $0 \leqslant t < Z$. Thus, $(\lambda_{it})^* = 0$, $i \in V$, $0 \leqslant t < Z$. □

**Lemma 2.** *In any of the optimal solutions $(m^*, v^*)$ to $DP_1$,* $\left(\widehat{\lambda}_i^{\beta,T}\right)^* = 0$, $i \in V$.

**Proof.** Put Lemma 1 and the zero-cost assumption of wavelength converters in $DP_1$ into the definition of accumulative dual cost $\widehat{\lambda}_i^{\beta,T}$ in Eq. (15), we have $\left(\widehat{\lambda}_i^{\beta,T}\right)^* = 0$, $i \in V$. □

Lemma 2 implies that a change from the resource quantity does not impact the optimization objective if there is abundance of it.

**Lemma 3.** *In any of the optimal solutions $(m^*, v^*)$ to $DP_1$, for given $\beta$ and $T$,* $\left(\widehat{\xi}_{ij0}^{\beta,T}\right)^* = \left(\widehat{\xi}_{ij1}^{\beta,T}\right)^* = \left(\widehat{\xi}_{ij2}^{\beta,T}\right)^* = \ldots$, $e_{ij} \in E$, $T \geqslant 1$, $0 \leqslant \beta < (Z - T)$.

**Proof.** If $0 \leqslant \left(\widehat{\xi}_{ijl}^{\beta,T}\right)^* < \left(\widehat{\xi}_{ijk}^{\beta,T}\right)^*$, $e_{ij} \in E$, $T \geqslant 1$, $0 \leqslant \beta < (Z - T)$, $k \neq l$, $0 \leqslant k < W$, and $0 \leqslant l < W$, then, we can see from Eq. (16) that $s_h$'s least-cost paths $(\Delta_h)^*$ only include the wavelength channel with the lowest cost on any link $e_{ij} \in E$ that $s_h$ is routed through. Thus, $w_{ijl}$ is chosen, instead of $w_{ijk}$, i.e., $(\delta_{ijlx}^h)^* = 1$ and $(\delta_{ijkx}^h)^* = 0$. Note that according to the lightpath persistency constraints described in Eq. (7), $w_{ijl}$ is used by $s_h$ starting from time slot $\beta$ and for a duration of $T$ time slots. Since $(\delta_{ijkx}^h)^* = 0$, we are able to find one of the optimal solutions that satisfies $\left(\widehat{\xi}_{ijk}^{\beta,T}\right)^* = 0$, $k \neq l$, $0 \leqslant k < W$, and $0 \leqslant l < W$, which contradicts the assumption that $0 \leqslant \left(\widehat{\xi}_{ijl}^{\beta,T}\right)^* < \left(\widehat{\xi}_{ijk}^{\beta,T}\right)^*$. □

Lemma 3 means that for any given $\beta$ and $T$, the summation of Lagrange multipliers for all the wavelength channels on a given link must be the same in an optimal solution to $DP_1$. Otherwise, all lightpaths routed through the link would only use the least-cost wavelength channels.

**Lemma 4.** *At least one of the optimal solutions to $DP_1$ satisfies $\Phi^* = 0$.*

**Proof.** If $(\phi_{jx}^h)^* = 1$, due to the existence of $m$, $k \in V$ and $b \neq a$ that satisfies $\left(\delta_{mjax}^h\right)^* = \left(\delta_{jkbx}^h\right)^* = 1$, we can then obtain an optimal solution that satisfies $\left(\phi_{jx}^h\right)^* = 0$, by letting $s_h$ use $w_{jka}$, instead of $w_{jkb}$. In this way, the requirement of using a wavelength converter at node $j$ is eliminated. Lemma 3 ensures that this change satisfies all the constraints, but does not influence the optimal value $q^*$. So the obtained solution is also one of the optimal solutions to $DP_1$. □

Lemma 4 means that despite the fact that the zero-cost wavelength conversion is available, plus some optimal solution to $DP_1$ uses the wavelength conversion, we are able to find at least one optimal solution to $DP_1$ not using wavelength conversion. Specifically, an optimal solution of the scheduled RWA problem does not have to use wavelength conversion.

**Theorem 1.** $(q_{DP_1})^* \leqslant (q_{DP_2})^*$.

**Table 4**
Computation results using various topologies and traffic matrices [17].

| Network settings | | | | Total number of demands | $J$ | $q$ | Duality gap (%) |
|---|---|---|---|---|---|---|---|
| Network | $N$ | $E$ | $W$ | | | | |
| NSFNET | 14 | 21 | 12 | 231 | 11653 | 11344 | 2.72 |
| | 14 | 21 | 4 | 105 | 5379 | 5081 | 5.86 |
| | 14 | 21 | 7 | 165 | 8198 | 8035 | 2.03 |
| A random network | 22 | 35 | 10 | 352 | 18278 | 17641 | 3.61 |
| | 22 | 35 | 6 | 211 | 11015 | 10715 | 2.80 |
| | 22 | 35 | 8 | 263 | 13708 | 13346 | 2.72 |
| European network | 28 | 61 | 8 | 403 | 19421 | 19162 | 1.35 |
| | 28 | 61 | 10 | 605 | 28180 | 27222 | 3.52 |
| | 28 | 61 | 4 | 201 | 9673 | 9217 | 4.95 |
| | 28 | 61 | 12 | 807 | 37837 | 35434 | 5.43 |
| | Average duality gap (%) | | | | | | 3.50 |

**Proof.** From Lemmas 2 and 4, we can see that there always exists an optimal solution $((m_{DP_1})^*, (v_{DP_1})^*)$ to $DP_1$ where $(\lambda_{DP_1})^* = 0$ and $(\Phi_{DP_1})^* = 0$, which means $((m_{DP_1})^*, (v_{DP_1})^*)$ satisfies all the constraints of $DP_2$, and is thus a solution to $DP_2$. Thus, $(q_{DP_1})^* = q_{DP_2}((m_{DP_1})^*, (v_{DP_1})^*)$. Since the optimal solution to $DP_2$ is at least as good as the solution $((m_{DP_1})^*, (v_{DP_1})^*)$, we have $q_{DP_2}((m_{DP_1})^*, (v_{DP_1})^*) \leqslant (q_{DP_2})^*$. So we have $(q_{DP_1})^* \leqslant (q_{DP_2})^*$. □

Theorem 1 means that if an optimal solution to $DP_1$ does not use any wavelength conversion, this solution is a solution to $DP_2$ too. In another word, the optimal solution to $DP_2$ should generate a bound at least better (i.e., greater) than the bound from this solution.

Now we proceed to prove $(q_{DP_1})^* \geqslant (q_{DP_2})^*$.

**Lemma 5.** *For an optimal solution $(m^*, v^*)$ to $DP_2$, for given $\beta$ and $T$,* $\left(\widehat{\xi}_{ij0}^{\beta,T}\right)^* = \left(\widehat{\xi}_{ij1}^{\beta,T}\right)^* = \left(\widehat{\xi}_{ij2}^{\beta,T}\right)^* = \ldots, e_{ij} \in E, T \geqslant 1, 0 \leqslant \beta < (Z - T)$.

**Proof.** Conceptually, since there is no wavelength conversion available in $DP_2$, the WDM network can be considered as layered planes. Each layer (i.e., a wavelength plane) is a separate network and corresponds to exactly one wavelength. Every wavelength plane is independent of each other, but is identical. The Lagrangian multiplier of each wavelength channel on the same fibre should thus be identical too.Specifically, we can start with an assumption that in an optimal solution, consider a link (say link $(i,j)$) that has different Lagrangian multiplier values for its wavelength channels. For any wavelength channel $c$, if we consider its capacity (currently set to constant 1 in constraints (4)) as a continuous variable (note that it does not need to be integer), and we have

$$(\xi_{ijct})^* = -\frac{d(q_{DP_2})^*}{dC_{ijct}}$$

(see [21]). If we have another wavelength channel $d$ that has different Lagrangian multiplier value

$$(\xi_{ijdt})^* = -\frac{d(q_{DP_2})^*}{dC_{ijdt}}.$$

It means that by increasing the capacity of $C_{ijct}$ and $C_{ijdt}$ by $dC_{ijct}$ and $dC_{ijdt}$, we will result in different optimized values of $(q_{DP_2} + dq_{DP_2})^*$. This contradicts the fact that wavelength channels $c$ and $d$ are identical (i.e., cannot be distinguished). □

**Theorem 2.** $(q_{DP_1})^* \geqslant (q_{DP_2})^*$.

**Proof.** From Lemma 5, we can see that at least an optimal solution to $DP_2$ can construct a solution to $DP_1$ by assigning $((m_{DP_2})^*, (v_{DP_2})^*)$ to $(m_{DP_1}, v_{DP_1})$ while setting $(\lambda_{DP_1})^* = 0$ and $(\Phi_{DP_1})^* = 0$. Specifically, for the dual function $q_{DP_1}(m)$ (see Eq. (10)), Lemma 5 ensures that this assignment satisfies all the constraints of $DP_1$, since the Lagrangian multipliers of wavelength channels on the same link all have the same value. We can thus set $(\Phi_{DP_1})^* = 0$ with $\min_{v_{DP_1} \in V}\{J\}$ still holds without violating any constraint, since using the wavelength conversion to switch to any other wavelength channel on the same link would result in the same dual value (see Lemmas 3 and 4 for the detailed conditions of not using wavelength conversion). Similar to Theorem 1, we have $(q_{DP_1})^* \geqslant (q_{DP_2})^*$. □

**Theorem 3.** $(q_{DP_1})^* = (q_{DP_2})^*$.

**Proof.** This is a natural derivation from Theorems 1 and 2. □

With Theorem 3, the optimum of $DP_1$ is proven to be the same as $DP_2$. In other words, the optimum of the scheduled RWA problem cannot be improved by wavelength conversion.

## 6. Duality gaps

A gap may exist between the optimal value of the Lagrangian DP (denoted by $q^*$) and the optimal value of the primal problem. This means that, although the optimal value of the Lagrangian DP is the best lower bound of the primal problem, the optimal values of the Lagrangian DP and the primal problem may not be identical. Such a gap is called a duality gap.

In practice, only near-optimal solutions are obtained for both the Lagrangian DP and the primal problem, due to limitations on the computational time and efforts. As long as the duality gap between a near-optimal value of the Lagrangian DP and the best achieved value of the primal problem is controlled below a threshold (e.g., 5–10% of the best achieved value of the primal problem), we are satisfied that a feasible solution to the primal problem is obtained with a reasonably good quality. The quality of the obtained solution is readily evaluated by the duality gap between a near-optimal value of the Lagrangian DP and the best achieved value of the primal problem.

Relatively small duality gaps between a near-optimal value of the Lagrangian DP and the best achieved value of the primal problem were reported in the Figs. 9, 11 and 14 in [17]. The results for various topologies and traffic matrices are also shown in Table 4 [17], which indicates that in most cases, the duality gap is less than 5% (sometimes <1%). The meaning of the tight duality gap is twofold: (1) the near-optimum solutions are within a very close range from the LR bounds, which does not change by adding wavelength conversion. The improvement (by adding wavelength conversion) on these near-optimum solutions can thus be only marginal; (2) since the optimal solution lies in-between the obtained objective value and the LR bound and since the LR bound is proven not to be affected by adding wavelength conversion, its impact on the optimum solution is also limited by the tight duality gap. Intuitively, the duality gap can be considered as a 'noise' added on top of the LR bound and since the LR bound is constant, the impact from the wavelength conversion on the optimal solution cannot be higher than the 'noise-level'. Please note that in our computation examples, we noticed that the duality gap is quite independent of the network topology or the network traffic pattern and we do not observe any obvious trend as the topology/traffic pattern changes.

## 7. Implications on resource requirements and acceptance of lightpath demands

The implications of our conclusion may be viewed from two complementary perspectives: resource requirements to accommodate a given number of lightpath demands; and acceptance/rejection of lightpath demands under a given amount and configuration of resources.

With or without wavelength conversion, similar amount of resources are required to accommodate a given number of lightpath demands. Our conclusion is consistent with the previous work in [14,15]. In Fig. 7 of [15], from an integer linear programing formulation, the maximum amount of resources used during any time slot (in terms of the weighted hop count at the logical level) were computed and shown to be almost identical in wavelength-continuous (i.e., without wavelength conversion) and wavelength-convertible (i.e., with wavelength conversion) networks. Results for different traffic models (low/medium/high demand overlap, or holding time unaware) followed a similar pattern, regardless of the network size. In Figs. 8–10 of [14], using a heuristic algorithm, the total required number of wavelength channels on all links to accommodate all lightpath demands was shown similar for the cases with or without wavelength conversion. Larger discrepancies between the cases with or without wavelength conversion were observed in the results from heuristic algorithms than from integer linear programing, because wavelength conversion tends to offer more flexibility in most heuristic algorithms.

With or without wavelength conversion, lightpath demands are accepted at a similar ratio, under a given amount and configuration of resources. Our conclusion is in agreement with existing results in [16,17]. In Fig. 13 of [16], the rejection ratios to lightpath demands were similar for the cases "Opaque_Unicast" (equivalent to lightpaths with wavelength conversion) and "AllOptical_Unicast" (equivalent to lightpaths without wavelength conversion). In Table 7 of [17], the numbers of rejected scheduled sliding lightpath demands (SSLDs) were similar in each pair of compared cases. Note that considering the total number of SSLDs being 286 as shown in Table 6 of [17], the differences of rejected numbers of SSLDs were quite small for the cases with or without wavelength conversion, and such differences may be partially attributed to the randomness of the heuristic algorithm.

## 8. Conclusions

We proved that wavelength conversion does not improve the scheduled RWA problem's Lagrangian bound, which is obtained using the Lagrangian relaxation method on the scheduled RWA problem. Although it was not a direct proof that wavelength conversion does not improve the quality of the solutions to the scheduled RWA problem, the result implied that in solving the scheduled RWA problem, the contribution of wavelength conversion is very marginal, since the bound is very close to the achieved objective function value in most cases. Please note that similar results have been obtained for various traffic patterns and for other network topologies as shown in Table 8 of [17].

The practical implications of our conclusion may be explained from two aspects: with or without wavelength conversion, similar amount of resources are required to accommodate a given number of scheduled lightpath demands; and scheduled lightpath demands are accepted at a similar ratio, under a given amount and configuration of resources.

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