Photonic Network Communications, 10:2, 155–178, 2005

A Lagrangian-Relaxation Based Network Profit Optimization For Mesh SONET-Over-WDM Networks

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Received June 17, 2004; Revised March 10, 2005; Accepted March 31, 2005

Abstract. In Wavelength Division Multiplexing (WDM) networks, the huge capacity of wavelength channels is generally much larger than the bandwidth requirement of individual traffic streams from network users. Traffic grooming techniques aggregate low-bandwidth traffic streams onto high-bandwidth wavelength channels. In this paper, we study the optimization problem of grooming the static traffic in mesh Synchronous Optical Network (SONET) over WDM networks. The problem is formulated as a constrained integer linear programming problem and an innovative optimization objective is developed as network profit optimization. The routing cost in the SONET and WDM layers as well as the revenue generated by accepting SONET traffic demands are modelled. Through the optimization process, SONET traffic demands will be selectively accepted based on the profit (i.e., the excess of revenue over network cost) they generate. Considering the complexity of the network optimization problem, a decomposition approach using Lagrangian relaxation is proposed. The overall relaxed dual problem is decomposed into routing and wavelength assignment and SONET traffic routing sub-problems. The subgradient approach is used to optimize the derived dual function by updating the Lagrange multipliers. To generate a feasible network routing scheme, a heuristic algorithm is proposed based on the dual solution. A systematic approach to obtain theoretical performance bounds is presented for an arbitrary topology mesh network. This is the first time that such theoretical performance bounds are obtained for SONET traffic grooming in mesh topology networks. The optimization results of sample networks indicate that the proposed algorithm achieves good sub-optimal solutions. Finally, the influence of various network parameters is studied.

Keywords: WDM networks, SONET-over-WDM networks, traffic grooming, traffic engineering, routing and wavelength assignment, network profit optimization, Lagrangian relaxation

1 Introduction

The design of wavelength-routed Wavelength Division Multiplexing (WDM) networks employs a two-step approach [1]. In the first step, a virtual topology is designed for traffic that is directly carried over lightpaths [2, 3] and each traffic flow is routed onto the designed virtual topology. In this paper, we study the SONET-over-WDM architecture, so the electrical domain traffic is Time Division Multiplexing (TDM) based. A lightpath is a dedicated optical connection using non-overlapping frequency or Wavelength Channels (WCs). The SONET frames are carried transparently from the source to the destination of a lightpath. The electrical processing of SONET frames only happens at the source and destination of a lightpath. In the second step, point-to-point virtual links obtained from the first step are routed as lightpaths onto physical fibres and each lightpath is assigned to WCs. If wavelength converters are used, a lightpath

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may change wavelength along its route. The second step is called Routing and Wavelength Assignment (RWA) in the literature. Such two-step approaches were effective for early-deployed WDM networks, because at that time RWA was independent of the virtual topology design. Although two-step approaches reduce the complexity of design and optimization, they do not yield the best network resource utilization.

Integrated design approaches are required to improve the network resource utilization. This is of particular interest to the new generation WDM networks, e.g., WDM networks consisting of network elements (nodes) capable of manipulating SONET traffic. Such new nodes integrate switching and add-drop multiplexing in the optical domain with SONET traffic multiplexing, de-multiplexing, and grooming in the electrical domain [4,5]. Grooming means the aggregation of low bandwidth SONET connections onto high bandwidth lightpaths.

As an integrated design approach, SONET traffic grooming in WDM networks has received considerable attention recently [4–12]. Traffic grooming consists of four sub-problems, which are not necessarily independent: (1) determining the virtual topology of lightpaths over a fibre network; (2) routing SONET traffic onto the virtual topology; (3) routing the lightpaths over the fibre network; and (4) performing wavelength assignments to the lightpaths. Traffic grooming is studied for two scenarios, i.e., static network planning and dynamic network operation. Static traffic patterns are used in network planning, which means all SONET traffic requests are given in advance. The design and optimization method proposed in this paper is for network planning. In the network operational stage, SONET traffic requests arrive one after another. The best effort has to be made to provide service to as many traffic requests as possible with given network resources [4,13–18], but solutions must also be provided for various issues, such as the capacity fairness issue [19] and reconfiguring wavelength assignments for new traffic patterns [20].

The majority of research on traffic grooming in SONET-over-WDM networks is based on the ring topology because of its relevance to the established SONET networks and its simplicity in analysis [9,21–28]. Different traffic models (e.g., uniform or arbitrary distribution, all-to-all or all-to-one, distance dependent, etc.), network architectures (e.g., uni-directional or bi-directional rings, no hub, single or multiple hubs, etc.) and switching capability (i.e., whether traffic can be switched over from one WC to another) have been extensively studied. A number of heuristic algorithms were proposed and theoretical bounds on performance were discovered for ring networks [8,29,30]. Theoretical bounds were also discovered for other regular topologies such as the star or tree topology [31,32]. However, emerging SONET-over-WDM networks are increasingly deployed using the mesh topology due to its unprecedented efficiency [33]. Traffic grooming for the mesh topology is very challenging and only a few recent publications reported such research [10,16,34–42]. Although heuristic algorithms have been proposed and the efficiency of the heuristics has been demonstrated for a practical scale network [34], it is still an open issue as to how close the results obtained from the heuristics are to the optimal results, i.e., what is the theoretical bound of the performance metrics. So far no theoretical performance bound is available yet [43]. Thus, we have conducted our research on the mesh topology, and one of our contributions in this paper is the systematic approach to obtain the theoretical performance bounds for any mesh topology and algorithms will be proposed to achieve this performance objective.

The traffic grooming problem is NP-complete, and the numbers of variables and equations increase drastically with the number of nodes and links in a network. Thus, the traffic grooming problem requires dramatic amounts of time to solve for large networks. For practical scale networks, the traffic grooming problem formulated as the Integer Linear Programming (ILP) cannot be directly solved by ILP solver package software such as CPLEX [44]. Even for a small sample network of six nodes and eight links, CPLEX cannot compute the optimal solution due to high computational complexity, and the result presented in [34] is actually a sub-optimal solution by terminating the optimization process after a period of computational time. The optimization method we will propose in this paper can achieve better results for the same network within reasonable computational time. Such results will be presented in Section 6 in this paper and compared with the reported results obtained with CPLEX. Our method has the potential for solving large-scale traffic grooming problems.

Most previous studies were conducted based on the assumption that all traffic requests should be accepted and tried to directly or indirectly minimize the network cost [23,24,45]. Simply accepting all traffic requests is not a practical approach. Network cost optimization needs to be related to the revenue generated by accepting traffic demands. Although the optimization objective used in [34] modelled the revenue as the weighted network throughput, the cost for accepting traffic demands is not considered. As a result, maximizing the weighted network throughput does not mean the real profit is maximized, because the profit is the excess of revenue over cost. Multi-objective optimization was applied to the traffic grooming problem in [46], where maximizing traffic throughput and minimizing the number of transceivers or lightpaths are modelled as two competing objectives. However, their relation to the network profit are not revealed. A new optimization objective will be proposed in this paper, which contains terms including both the revenue and cost, thus the potential profit generated by accommodating traffic demands can be optimized. The new optimization objective will be elaborated in Section 2.

Lagrangian Relaxation (LR) and subgradient methods have been successfully employed to solve the RWA problem [47–49]. Compared to the exponential complexity of CPLEX, LR and subgradient methods have polynomial complexity as long as the solution to all the sub-problems of the relaxed dual problem is polynomial, thus can be used to solve the RWA problem for fairly large networks. Theoretical bounds are generated, and at the same time, feasible solutions satisfying constraints are obtained [50]. A Lagrangian-based heuristic algorithm was proposed for traffic grooming in WDM networks [51]. However, the physical topology was not considered, so the constraints from the physical configuration were neglected. Essentially, the problem has been simplified to a virtual topology design problem, which as discussed early on, does not necessarily result in the best network utilization when separately solved with the RWA problem.

In this paper, we study the design and optimization problem of traffic grooming in mesh SONET-over-WDM networks. An innovative optimization objective is proposed to consider both the revenue generated by accepting traffic demands and the actual network resource cost for providing services to the accepted traffic demands. The profit is maximized as opposed to either minimizing network cost or maximizing network throughput in other studies. An LR-based optimization method is presented to solve the network profit maximization problem for SONET-over-WDM networks. Theoretical bounds are derived for the optimization problem. A systematic approach is developed to obtain theoretical bounds as well as feasible solutions, which provide acceptable sub-optimal results. Such optimization approach is generic and may be applied to arbitrary topologies. This paper is organized as follows. In Section 2, the network profit optimization objective is presented for SONET traffic grooming in mesh wavelength-routed WDM networks. In Section 3, the network model and assumptions are introduced. Then, the problem is formally formulated in Section 4. A solution based on LR and subgradient methods is proposed in Section 5. The numerical results are presented with comparisons to published results in Section 6. Section 7 contains the conclusions.

2 Design and Optimization Objective

Two types of optimization problems have been studied for traffic grooming under static traffic patterns. The first optimization type is to minimize network cost [21,28]. In minimizing network cost, a given static traffic matrix is completely satisfied and the optimization objective is to use minimal network resources to provide services to all traffic demands. Fundamentally, minimizing network cost is to allocate network resources to match a given set of traffic demands. Various cost functions are adopted, e.g., [7,28]. For example, the total number of lightpaths is used as a cost function [7], which represents the total cost of transmitters and receivers, as well as wavelength switching fabric. Another cost function is the total capacity of SONET switches [7], which represents the cost of electrical multiplexing, de-multiplexing and switching, and indirectly represents the cost of transmitters and receivers. Similarly, the maximum number of lightpaths terminating/originating at a node serves as another cost function, which reflects the cost of transmitters and receivers, as well as the cost of SONET switches.

The second optimization type is to maximize network throughput [34]. In maximizing network throughput, a given network has such limited resources that not all given traffic demands can be accepted, and therefore the optimization objective is to maximize the total traffic that is carried by the network. Essentially, to maximize network throughput is to prioritize traffic demands based on how efficient the traffic demands can be provisioned and only the traffic demands that most efficiently use network resources are accepted. Maximizing the total network throughput implicitly evaluates the efficiency of traffic flow provisioning. A variation of maximizing network throughput is network revenue optimization, where the network revenue is modelled as a weighted network throughput [34], or based on service differentiation for lightpath protection [52]. Since for a given network, the total network cost is fixed, maximal network revenue will lead to maximal network profit. However, it is difficult to study the impact of network cost factors to the overall network profit, because the network cost factors are not explicitly modelled. In maximizing network throughput, it is still unknown whether providing service to a certain traffic demand is profitable or not. The reason is that an arbitrarily selected network configuration will unlikely match a given traffic matrix and such miss-matching has two negative impacts. In some cases, non-profitable traffic demands are accepted because the given network resources are abundant. It is also possible that potential profitable traffic demands cannot be accepted because the given network resources are less than necessary. By adding small amounts of network resources with relatively low cost, the potential profitable traffic can be actually accepted to generate profit. Unfortunately, the network throughput optimization does not provide clues for improving network configurations, i.e., no guide is provided for adding network resources.

The ultimate goal for minimizing network cost and maximizing network throughput is to maximize network profit. Optimizing either network cost or network throughput separately cannot achieve the goal. For example, in the network cost minimization, the assumption that all traffic demands should be satisfied is questionable with respect to the profit objective. Some traffic demands may be costly and no efficient provisioning plan exists for them. To provide service to them is an excessive burden to the whole network, and the revenue generated from them cannot recover the provisioning cost. Such non-profitable traffic demands should be rejected instead of being accepted regardless of the provisioning cost. On the other hand, network throughput optimization may not achieve the profit objective, either, because the implicit evaluation for the network resource utilization of traffic demands may not reflect the cost factor. So far, no study has been reported to link the minimizing network cost and maximizing network throughput so that network profit can be maximized.

In this paper, we propose a model for network profit, which bridges the gap between the minimizing network cost and maximizing network throughput. We assume two factors are defined first: the predicted traffic matrix and the expected revenue for each traffic demand. The predicted traffic matrix is defined in the form of traffic demands between node pairs. The expected revenue for a traffic demand is the estimated charge for providing service to the traffic demand. Using static network planning, every traffic demand can be identified whether it is profitable or not. Profitable traffic demands will be accepted and an optimized provisioning plan will be arranged.

The following are some notes on our design and optimization objective:

1. Representation of the revenue generated by a traffic demand: In order to incorporate the revenue factor into the network cost function, we will use the revenue penalty for rejecting a traffic demand. The amount of revenue penalty is the same as the amount of revenue the traffic demand will generate

if it is accepted. With this modification, the actual optimization objective will be to minimize the summation of the revenue penalty and the total network cost. This objective is in fact minimizing the profit loss, which is equivalent to maximizing profit. For the total network cost, a direct measurement of network cost needs to be used as opposed to any of the previously used indirect cost functions. This is because the network cost has to be measured by a monetary value and will be added to the revenue penalty. This paper adopts a virtual unit for cost, revenue and profit, which is proportional to the dollar unit.

2. Handling non-profitable traffic demand: If the profit situation for a traffic demand is marginal, i.e., close to break-even, it may be re-considered. This can be done by adjusting the estimated charge after the first round of network planning, when the overall situation of the network becomes clearer than at the beginning. Hence the charge can be estimated more accurately.
3. Limiting network capacity: Because of the extremely high complexity associated with the integrated design of SONET-over-WDM networks, we simplify the problem by fixing the network capacity. The network capacity is set to a limited level such that not all traffic demands can be accommodated. After the LR-based optimization, a subset of traffic demands will be identified as high profitable traffic. A series of trials will then be made by adjusting the network capacity, i.e., by adjusting the number of transmitters, receivers and wavelength converters at some selected nodes, and the number of WCs in each fibre. The results will be presented in Section 6.
4. Difference between identifying the profit situation of a traffic demand in the static network planning and in the dynamic network operational stage: The philosophy we use to create the new design and optimization objective is that no such traffic demand should be accepted when it is known to be non-profitable before the network is built. However, after the network is built and all network resources are deployed, the dynamic network operational stage should adopt a different strategy for accepting or rejecting traffic demands. This is because the objective is changed to generate as much revenue as possible with existing network resources. How to utilize spare network resource is beyond the scope of this paper.

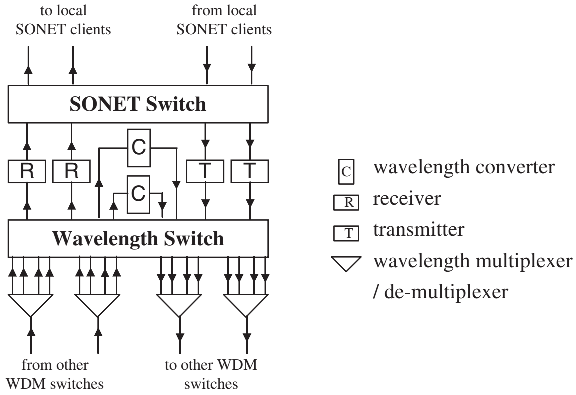

Fig. 1. An integrated SONET-over-WDM node.

3 Network Model and Assumptions

The network model used in this paper consists of integrated SONET-over-WDM nodes as shown in Fig. 1. In the optical domain, WCs in the input fibres are de-multiplexed and then switched to either receivers for optical-to-electrical conversion, or wavelength converters, or output fibres. All the transmitters and receivers are tuneable to any wavelength used in the network. If there is any wavelength converter (some nodes may not have any such wavelength converter at all), it would operate at the full wavelength range, i.e., it can convert any input wavelength to any output wavelength. The formulation of the wavelength converter structure can be found in [47]. The transmitters, receivers and wavelength converters are assumed to be set up in a share-per-node structure, so that the least number of such components is required to achieve the same performance [53].

In addition, we make the following assumptions:

1. SONET-over-WDM nodes are interconnected in a mesh topology.

2. In the SONET layer, the switching fabric has unlimited multiplexing, de-multiplexing and time slot exchanging capability. The granularity of traffic requests can be any of the Synchronous Transport Signal (STS) or the Optical Carrier (OC) levels of 1, 3, 12 or 48, i.e., having a bit rate of 51.84, 155.52, 622.08 or 2488.32 Mbps. A traffic demand may be groomed with other traffic and rounded up to any granularity of OC-3, OC-12 or OC-48.
3. The capacity of SONET switching fabric is unlimited.
4. One fibre connects a pair of neighbour nodes. Every WC on fibres can carry optical signals travelling in both directions, i.e., all WCs are bi-directional.
5. Full-range wavelength conversion is used. Therefore, each input wavelength can be converted into any possible output wavelength if a wavelength converter is available.
6. A traffic demand must be handled as its entirety, and cannot be split.
7. All nodes are equipped with wavelength switching capability. However, the SONET switching capability is optional. When SONET switching is not available at a node, we set the number of transmitters and receivers at that node to zero.
8. We assume all fibres use the same number of wavelengths and denoted as W.

4 Problem Formulation

4.1 Notations

The following notations are used in the rest of the paper. For better understanding the notations, we organize them into three categories:

Optical transmission domain:

- N node count of the network;
- i, j two end nodes of a fibre;
- e_{ij} fibre between nodes i and j;
- n_{ij} WC count on the fibre e_{ij};
- w_{ijc} the cth WC on the fibre e_{ij}. Without losing generality, the WCs on all fibres with the same index c are assumed to carry the same wavelength, and this wavelength is referred to as the *wavelength c*;
- d_{ijc} cost of the WC w_{ijc};
- c_j cost of a wavelength converter at node j;
- r_j cost of a receiver at node j;
- t_j cost of a transmitter at node j;
- $(\mathcal{V}, \mathcal{E})$ bi-directional graph representing the physical WDM network, with $\mathcal{V}$ standing for nodes and $\mathcal{E}$ standing for fibres.

Optical switching domain:

- s, d source and destination nodes of a lightpath;
- s_{sdn} the nth lightpath between a node pair (s, d);
- M lightpath set, i.e., $\{s_{sdn}\}$;
- N_{sd} maximum number of lightpaths between node pair (s, d);
- C bandwidth of a lightpath, measured by the number of OC-1 or STS-1 channels. In this paper, the bandwidth of a lightpath is OC-48, so $C = 48$;
- D_{sdn} total network resource cost of s_{sdn}, which is defined as $\alpha_{sdn}\left(r_d + t_s + \sum_{(i,j)} \sum_{0<c\le n_{ij}} d_{ijc}\delta_{ijc}^{sdn} + \sum_j c_j \phi_j^{sdn}\right)$. In this paper, only the cost of transmitters, receivers, wavelength converters and wavelength channels is modelled;
- F_i number of wavelength converters at node i;
- T_i number of transmitters at node i;
- R_i number of receivers on node i;
- Q_{sdn} dual routing cost for the lightpath s_{sdn};
- $(\mathcal{V}_v, \mathcal{E}_v)$ unidirectional graph representing the virtual WDM network topology, with $\mathcal{V}_v$ standing for the electrical switches at nodes and E_v standing for all the unidirectional lightpaths that could be set up;
- α_{sdn} admission status of the lightpath s_{sdn}, which equals to one, if the lightpath s_{sdn} is set up; zero otherwise;
- δ_{ijc}^{sdn} usage of the WC w_{ijc} by the lightpath s_{sdn}, which equals to one, if the lightpath s_{sdn} goes through the WC w_{ijc}; zero otherwise;

ϕ_j^{sdn} usage of wavelength converters, which equals to one, if a wavelength converter is used by the lightpath s_{sdn} at node j; zero otherwise;

A set of the admission status of all lightpaths, i.e., $\{\alpha_{sdn}\}$;

Δ set of the wavelength assignment for all lightpaths, i.e., $\left\{\delta_{ijc}^{sdn}\right\}$;

Φ set of the wavelength converter assignment for all lightpaths, i.e., $\left\{\phi_j^{sdn}\right\}$.

Electrical switching domain:

x_{pqz} the zth traffic demand between node pair (p, q);

p, q endpoint nodes of a traffic;

E_{pqz} total electrical routing cost of the traffic x_{pqz}, which is defined as $\sum_{(s,d)} \sum_{0<n\le N_{sd}} V_{pqz} v_{sdn}^{pqz}$;

V_{pqz} coefficient for the electrical routing cost of the traffic x_{pqz};

v_{sdn}^{pqz} usage of the lightpath s_{sdn} by the traffic x_{pqz}, which equals to one, if the traffic x_{pqz} goes through s_{sdn}; zero otherwise;

G_{pqz} bandwidth of the traffic x_{pqz}, which is measured by the equivalent number of OC-1 / STS-1 channels, i.e., $G_{pqz} \in \{1, 3, 12, 48\}$;

P_{pqz} penalty coefficient for rejecting the traffic demand x_{pqz};

R_{pqz} dual routing cost of the traffic x_{pqz};

γ_{pqz} admission status of the traffllic x_{pqz}, which equals to zero, if the traffic x_{pqz} is rejected; one otherwise;

Y set of use of lightpaths by all traffic, i.e., $\{v_{sdn}^{pqz}\}$;

Y_{pqz} set $\{v_{sdn}^{pqz}\}_{pqz}$, representing the routing of the traffic x_{pqz};

Γ set of admission status of all traffic, i.e., $\{\gamma_{pqz}\}$;

Z_{pq} maximum number of traffic demands between node pair (p, q). Note that Z_{pq} not necessarily equals Z_{qp}, i.e., traffic demands may be asymmetric;

4.2 Formulation of the Optimization Objective and Constraints

As discussed before, our objective function is to minimize the profit loss, which is equivalent to maximize the profit. We can see objective function contains composed of three parts: the first summation is the penalty of rejecting traffic demands; the second summation is the cost associated with the electrical domain for routing traffic demands; and the third summation is the cost associated with the optical domain for setting up lightpaths.

Objective Function:

$$\min_{A,\Delta,\Phi,\Gamma,Y} \{J\}, \text{ with}$$

$$J \equiv \sum_{(p,q)} \sum_{0<z\le Z_{pq}} P_{pqz} G_{pqz}(1-\gamma_{pqz}) + \sum_{(p,q)} \sum_{0<z\le Z_{sd}} E_{pqz} + \sum_{(s,d)} \sum_{0<n\le N_{sd}} D_{sdn} \tag{1}$$

Constraints:

The constraints are organized into two parts.

(A) Relation between lightpaths and the optical transmission network

There are five constraints to be considered:

- Lightpath flow continuity constraint: Lightpath continuity means every lightpath has to be continuous from source to destination. Thus, the balance of entering and exiting lightpaths at all intermediate nodes does not change. Only at the source and destination nodes, the lightpath adds the number of entering and exiting lightpaths by one, respectively. So, we have

$$\sum_j \sum_{0<c\le n_{ij}} \delta_{ijc}^{sdn} - \sum_j \sum_{0<c\le n_{ij}} \delta_{jic}^{sdn} = \begin{cases} \alpha_{sdn} & \text{if } i=s, \\ -\alpha_{sdn} & \text{if } i=d, \quad \forall (s,d),\ 0<n\le N_{sd} \\ 0 & \text{otherwise.} \end{cases} \tag{1a}$$

- Wavelength conversion constraint: A wavelength converter at an intermediate node j is used only when different wavelengths are assigned to the lightpath s_{sdn} for the incoming and outgoing WCs at node j.

$$\phi_j^{sdn} = \begin{cases} 1, & \text{if } \exists\ m>0, k\le N, b\ne c, \\ & \quad \delta_{mjb}^{sdn} = \delta_{jkc}^{sdn} = 1 \\ 0, & \text{otherwise} \end{cases} \tag{1b}$$

- WC capacity constraint: This constraints stipulate that no more than one lightpath may be routed through a WC in either direction.

$$\sum_{(s,d)} \sum_{0<n \le N_{sd}} \delta_{ijc}^{sdn} \le 1, \quad \forall (i,j), \quad 0 < c \le n_{ij} \tag{1c}$$

- Wavelength converter capacity constraint: The number of converters used in one node cannot exceed the number of available converters, because of the share-per-node wavelength converter configuration (Fig. 1).

$$\sum_{(s,d)} \sum_{0<n \le N_{sd}} \phi_j^{sdn} \le F_j, \quad \forall j \tag{1d}$$

- Transmitter and receiver capacity constraint: The total number of transmitters and receivers used on a node cannot exceed the corresponding number of transmitters and receivers installed on this node.

$$\sum_j \sum_{0<c \le n_{sj}} \sum_d \sum_{0<n \le N_{sd}} \delta_{sjc}^{sdn} \le T_s, \quad \forall s \tag{1e}$$

$$\sum_i \sum_{0<c \le n_{id}} \sum_s \sum_{0<n \le N_{sd}} \delta_{idc}^{sdn} \le R_d, \quad \forall d \tag{1f}$$

The parameters T_i, R_i represent, respectively, the number of transmitters and receivers on switch i. Constraints (1e) and (1f) confine that at most $N_{sd} \le \min(T_s, R_d)$ lightpaths can be set up between source–destination pair (s,d). However, in the optimal solution, it is very unlikely that N_{sd} will go very close to $\min(T_s, R_d)$, because if the lightpaths between (s,d) occupy too many of the transceivers and other resources, the lightpaths between s–d, and other switches will likely be blocked and eventually limit the number of the traffic demands being routed through the network.

From our computation experience, $N_{sd} \le \frac{1}{2} \min(T_s, R_d)$ is adequate enough for most of the network examples to obtain very good near-optimal result. Although not required, we usually set T_i and R_i to the same value.

(B) Relation between traffic flows and lightpaths

There are two such constraints:

- Traffic flow continuity constraint: The explanation of these constraints is similar to the lightpath flow continuity constraints (1a).

$$\sum_d \sum_{0<n \le N_{sd}} \upsilon_{sdn}^{pqz} - \sum_d \sum_{0<n \le N_{ds}} \upsilon_{dsn}^{pqz} = \begin{cases} \gamma_{pqz} & \text{if } s = p, \\ -\gamma_{pqz} & \text{if } s = q, \\ 0 & \text{otherwise.} \end{cases} \quad \forall (p,q), \ 0 < z \le Z_{pq} \tag{1g}$$

- Lightpath capacity constraint: The total traffic being routed through a lightpath should be less than the capacity of the lightpath.

$$\sum_{(p,q)} \sum_{0<z \le Z_{sd}} \upsilon_{sdn}^{pqz} G_{pqz} \le C\alpha_{sdn}, \quad \forall (s,d), \quad 0 < n \le N_{sd} \tag{1h}$$

5 Solution Method and Analysis

5.1 Overview of the Solution Framework

The traffic grooming problem is comprised of the RWA (routing and wavelength assignment) and the electrical traffic routing sub-problems, which are very complicated by themselves and closely coupled to each other. It is thus very difficult to obtain a good solution considering both sub-problems at the same time. We shall apply the LR method to decouple and solve the two sub-problems independently. Similar to the pricing concept in the marketplace, the LR method can replace “hard” coupling constraints by “soft” prices for the use of resources [50].

In the traffic grooming problem, lightpath capacity is the primary constraint, because it should be less than the capacity of a WC and on the other hand, more than the aggregated traffic onto it. The Lagrange multipliers in the LR method will serve as the “prices” for lightpaths to use optical transmission resources, and the “prices” for traffic to use the lightpaths. The multipliers are updated iteratively to reflect eventually the criticality of the resources. The integration of the RWA and electrical traffic routing sub-problems will be realized by adjusting the multipliers at the outer loop, and the two problems are solved independently as inner problems. This solution framework greatly simplifies the solution method without losing the consideration of the interaction of the two sub-problems. Note that our approach is different from other approaches which, while separating the RWA and electrical traffic routing sub-problems for simplicity, lose the interaction between the two sub-problems. In our solution method, although the two sub-problems are separated in solving the inner problem, they are always associated in the outer loop.

In our LR framework, some constraints of the original optimization problem (1) (i.e., the primal problem) are relaxed and a dual problem is created by using Lagrange multipliers. The derived dual problem is then decomposed into independent RWA and electrical traffic routing sub-problems, and solved independently by the Revised Minimum Cost Lightpath (RMC-SLP) and the Revised Shortest Path Algorithm (RSPA). The optimal solution to the dual problem is a lower bound for the primal problem [50]. Since in transforming the original optimization problem into the dual problem, some constraints are relaxed, the solution to the dual problem is generally an infeasible solution to the primal problem, which mean some constraints are violated. We therefore propose a heuristic algorithm to obtain a feasible solution based on the dual solution. The subgradient method [50] will be used in updating Lagrange multipliers. Solving the sub-problems becomes the inner process with regard to updating the multipliers, which can be thought of as an outer loop process. The overall structure of the algorithm will be given in Section 5.6.

5.2 Application of the LR (Lagrangian Relaxation) Method

According to the LR method, we should relax the WC capacity constraint (1c), converter capacity constraint (1d), transmitter capacity constraint (1e) and lightpath capacity constraint (1h), by introducing Lagrange multipliers $\zeta_{ijc}, \lambda_j, \eta_s$ and κ_{sdn}, respectively. This leads to the following Lagrangian dual problem, denoted as DP.

$$
\begin{aligned}
&\max_{\xi,\lambda,\eta,\theta\geq 0} q \\
&= \min_{\mathrm{A},\Delta,\Phi,\Gamma,Y} \Bigg\{ \sum_{(p,q)} \sum_{0<z\leq Z_{pq}} P_{pqz} G_{pqz} \left(1-\gamma_{pqz}\right) \\
&\quad + \sum_{(p,q)} \sum_{0<z\leq Z_{sd}} E_{pqz} + \sum_{(s,d)} \sum_{0<n\leq N_{sd}} D_{sdn} \\
&\quad + \sum_{(i,j)} \sum_{0<c\leq n_{ij}} \xi_{ijc} \left(\sum_{(s,d)} \sum_{0<n\leq N_{sd}} \delta_{ijc}^{sdn} - 1 \right) \\
&\quad + \sum_{j} \lambda_j \left(\sum_{(s,d)} \sum_{0<n\leq N_{sd}} \phi_j^{sdn} - F_j \right) \\
&\quad + \sum_{s} \eta_s \left(\sum_{j} \sum_{0<c\leq n_{sj}} \sum_{d} \sum_{0<n\leq N_{sd}} \delta_{sjc}^{sdn} - T_s \right) \\
&\quad + \sum_{(s,d)} \sum_{0<n\leq N_{sd}} \kappa_{sdn} \Bigg(\sum_{(p,q)} \sum_{0<z\leq Z_{sd}} \upsilon_{sdn}^{pqz} G_{pqz} \\
&\quad - C\alpha_{sdn} \Bigg) \Bigg\}
\end{aligned} \tag{2}
$$

subject to the constraints (1a), (1b), (1f) and (1g).

After regrouping the relevant terms, our DP can be written as:

$$
\begin{aligned}
\min_{\mathrm{A},\Delta,\Phi,\Gamma,Y} \Bigg\{ & \sum_{(s,d)} \sum_{0<n\leq N_{sd}} \Bigg[D_{sdn} + \sum_{(i,j)} \sum_{0<c\leq n_{ij}} \xi_{ijc} \delta_{ijc}^{sdn} \\
& + \sum_{j} \lambda_j \phi_j^{sdn} + \sum_{j} \sum_{0<c\leq n_{sj}} \eta_s \delta_{sjc}^{sdn} \\
& - C\kappa_{sdn}\alpha_{sdn} \Bigg] \\
& + \sum_{(p,q)} \sum_{0<z\leq Z_{sd}} \Bigg[\sum_{(s,d)} \sum_{0<n\leq N_{sd}} \kappa_{sdn} \upsilon_{sdn}^{pqz} G_{pqz} \\
& + E_{pqz} + P_{pqz} G_{pqz} \left(1-\gamma_{pqz}\right) \Bigg] \\
& - \sum_{(i,j)} \sum_{0<c\leq n_{ij}} \xi_{ijc} - \sum_{s} \eta_s T_s - \sum_{j} \lambda_j F_j \Bigg\}
\end{aligned} \tag{3}
$$

Because the last three terms are independent of the decision variables, the problem can be simplified as:

$$
\begin{aligned}
\min_{\mathrm{A},\Delta,\Phi,\Gamma,Y} \Bigg\{ & \sum_{(s,d)} \sum_{0<n\leq N_{sd}} \Bigg[D_{sdn} + \sum_{(i,j)} \sum_{0<c\leq n_{ij}} \xi_{ijc} \delta_{ijc}^{sdn} \\
& + \sum_{j} \lambda_j \phi_j^{sdn} + \sum_{j} \sum_{0<c\leq n_{sj}} \eta_s \delta_{sjc}^{sdn} - C\kappa_{sdn}\alpha_{sdn} \Bigg] \\
& + \sum_{(p,q)} \sum_{0<z\leq Z_{sd}} \Bigg[\sum_{(s,d)} \sum_{0<n\leq N_{sd}} \kappa_{sdn} \upsilon_{sdn}^{pqz} G_{pqz}
\end{aligned}
$$

$$+E_{pqz}+P_{pqz}G_{pqz}\left(1-\gamma_{pqz}\right)\Bigg]\Bigg]\Bigg\} \tag{4}$$

which is referred to as RP (the Relaxed Problem).

By using the fact that $\delta_{ijc}^{sdn}=\alpha_{sdn}\delta_{ijc}^{sdn}$, $\phi_j^{sdn}=\alpha_{sdn}\phi_j^{sdn}$, $\upsilon_{sdn}^{pqz}=\gamma_{pqz}\upsilon_{sdn}^{pqz}$, and using constraint (1a), we can rewrite the problem RP as:

$$\begin{aligned}
&\min_{A,\Delta,\Phi,\Gamma,Y}\left\{\sum_{(s,d)}\sum_{0<n\le N_{sd}}\left[\alpha_{sdn}\left(D_{sdn}+\sum_{(i,j)}\sum_{0<c\le n_{ij}}\xi_{ijc}\delta_{ijc}^{sdn}\right.\right.\right.\\
&\left.\left.+\sum_j\lambda_j\phi_j^{sdn}+\eta_s\right)-C\kappa_{sdn}\alpha_{sdn}\right]\\
&+\sum_{(p,q)}\sum_{0<z\le Z_{sd}}\left[\gamma_{pqz}\left(\sum_{(s,d)}\sum_{0<n\le N_{sd}}\kappa_{sdn}\upsilon_{sdn}^{pqz}G_{pqz}+E_{pqz}\right)\right.\\
&\left.\left.+\ \left(1-\gamma_{pqz}\right)P_{pqz}G_{pqz}\right]\right\}\\
&=\min_{A,\Delta,\Phi}\left\{\sum_{(s,d)}\sum_{0<n\le N_{sd}}\left[\alpha_{sdn}\left(\sum_{(i,j)}\sum_{0<c\le n_{ij}}\left(\xi_{ijc}+d_{ijc}\right)\delta_{ijc}^{sdn}\right.\right.\right.\\
&\left.\left.\left.+\sum_j\left(\lambda_j+c_j\right)\phi_j^{sdn}+\eta_s+t_s+r_d\right)-C\kappa_{sdn}\alpha_{sdn}\right]\right\}\\
&+\min_{\Gamma,Y}\left\{\sum_{(p,q)}\sum_{0<z\le Z_{sd}}\left[\gamma_{pqz}\left(\sum_{(s,d)}\sum_{0<n\le N_{sd}}\upsilon_{sdn}^{pqz}\left(\kappa_{sdn}G_{pqz}\right.\right.\right.\right.\\
&\left.\left.\left.\left.+V_{pqz}\right)\right)+\ \left(1-\gamma_{pqz}\right)P_{pqz}G_{pqz}\right]\right\}\\
&=\sum_{(s,d)}\sum_{0<n\le N_{sd}}\min_{\alpha_{sdn}}\left\{\alpha_{sdn}\left[\min_{\Delta_{sdn},\Phi_{sdn}}\left(Q_{sdn}\right)-C\kappa_{sdn}\right]\right\}\\
&+\sum_{(p,q)}\sum_{0<z\le Z_{sd}}\min_{\gamma_{pqz}}\left[\gamma_{pqz}\min_{Y_{pqz}}\left(R_{pqz}\right)+\left(1-\gamma_{pqz}\right)P_{pqz}G_{pqz}\right]
\end{aligned} \tag{5}$$

where $Q_{sdn}=\sum_{(i,j)}\sum_{0<c\le n_{ij}}\left(\xi_{ijc}+d_{ijc}\right)\delta_{ijc}^{sdn}+\sum_j\left(\lambda_j+c_j\right)\phi_j^{sdn}+\eta_s+t_s+r_d$, $R_{pqz}=\sum_{(s,d)}\sum_{0<n\le N_{sd}}\upsilon_{sdn}^{pqz}\left(\kappa_{sdn}G_{pqz}+V_{pqz}\right)$.

In this step, we can see that Equation (5) consists of two independent parts. The first minimization is over the variables in the optical domain (A,Δ,Φ), while the second minimization is over the variables in the electrical domain (Y,Γ). This important feature leads to the separation of the sub-problems to be solved below.

5.3 Solving the Sub-Problems

5.3.1 The Decomposition of the Dual Problem

Since the first and second summations in Equation (5) are independent of each other, we can separate the problem into two independent sub-problem sets: a set of RWA sub-problems, one for each lightpath; a set of electrical traffic routing sub-problems, one for each traffic demand.

(A) RWA Sub-problems (RWAS):

Considering the optical domain subproblems in Equation (5), we have

$$\sum_{(s,d)}\sum_{0<n\le N_{sd}}\min_{\alpha_{sdn}}\left\{\alpha_{sdn}\left[\min_{\Delta_{sdn},\Phi_{sdn}}\left(Q_{sdn}\right)-C\kappa_{sdn}\right]\right\},$$

subject to the constraints (1a),(1b) and (1f), where

$$Q_{sdn}=\sum_{(i,j)}\sum_{0<c\le n_{ij}}\left(\xi_{ijc}+d_{ijc}\right)\delta_{ijc}^{sdn}+\sum_j\left(\lambda_j+c_j\right)\phi_j^{sdn}+\eta_s+t_s+r_d.$$

Without considering the constraint (1f), we can now decompose the RWAS to $|A|$ sub-problems $\min_{\alpha_{sdn}}\left\{\alpha_{sdn}\left[\min_{\Delta_{sdn},\Phi_{sdn}}\left(Q_{sdn}\right)-C\kappa_{sdn}\right]\right\}$ each of which corresponds to one lightpath. We shall use RWA_{sdn} to denote the sub-problem in RWAS corresponding to the lightpath s_{sdn}. Note that Q_{sdn} represents the minimum routing cost for the lightpath s_{sdn}.

(B) Electrical Traffic Routing Sub-problems (ETRS):

Considering the electrical domain subproblems in equation (5), we have

$$\sum_{(p,q)}\sum_{0<z\le Z_{sd}}\min_{\gamma_{pqz}}\left[\gamma_{pqz}\min_{Y_{pqz}}\left(R_{pqz}\right)+\left(1-\gamma_{pqz}\right)P_{pqz}G_{pqz}\right],$$

subject to constraints (1g), where $R_{pqz}=\sum_{(s,d)}\sum_{0<n\le N_{sd}}\upsilon_{sdn}^{pqz}\left(G_{pqz}\kappa_{sdn}+V_{pqz}\right)$.

By assuming the cost of the lightpath s_{sdn} is $G_{pqz}\kappa_{sdn}+V_{pqz}$, it is obvious that $\min_{Y_{pqz}}\left(R_{pqz}\right)$ represents the cost of the shortest path from nodes p to q for routing the traffic x_{pqz} over the virtual topology $(\mathcal{V}_\upsilon,\mathcal{E}_\upsilon)$, which is formed by the electrical switches and all the lightpath candidates. We decompose the ETRS into $|Y|$ subproblems $\min_{\gamma_{pqz}}\left[\gamma_{pqz}\min_{Y_{pqz}}\left(R_{pqz}\right)+\left(1-\gamma_{pqz}\right)P_{pqz}G_{pqz}\right]$, each of which corresponds to one traffic demand. We shall use $\mathrm{ETR}_{\mathrm{pqz}}$ to denote the sub-problem in ETRS corresponding to x_{pqz}.

5.3.2 The Solution to the Sub-Problems

The total number of sub-problems is $|A| + |Y|$, where $|\cdot|$ denotes the number of elements in the set. The optimal solutions for RWAS and ETRS may be obtained separately by applying RMCSLP and RSPA, respectively.

(A) RMCSLP:

Intuitively, the lightpath s_{sdn} can be thought of as a "broker" in the dual problem, which "negotiates" between cost and "offer". The cost of the lightpath s_{sdn} is $\min_{\Delta_{sdn},\Phi_{sdn}} \{Q_{sdn}\}$, which reflects the criticality of the physical resources that the lightpath uses. The more critical the resources that the lightpath uses are, the higher cost the lightpath has. The "offer" from the electrical traffic demands is $C\kappa_{sdn}$, which reflects the "intention" of the electrical traffic demands to use the lightpath. The acceptance decision of the lightpath is based on whether the lightpath is "profitable". In this way, the RWAS and ETRS sub-problems are linked to each other.

We can see from the expression of Q_{sdn} that the first two summations assume the same form as the well-known problem that has been solved optimally by the MCSLP [54], which is essentially a shortest path algorithm. Therefore, we can first obtain $\min_{\Delta_{sdn},\Phi_{sdn}} \{Q_{sdn}\}$ by applying MCSLP and then add the constants $\eta_s + t_s + r_d$. Afterwards each node i, we do the sorting of the s_{sin}'s to satisfy constraints (1f). This is captured in our *Revised Minimum Cost Semi-lightpath* (RMCSLP) algorithm below in order to solve RWA_{sdn}:

(A.1) Employ the MCSLP to solve $\min_{\Delta_{sdn},\Phi_{sdn}} \left[\sum_{(i,j)} \sum_{0<c\le n_{ij}} \left(\xi_{ijc} + d_{ijc}\right) \delta_{ijc}^{sdn} + \sum_j \left(\lambda_j + c_j\right) \phi_j^{sdn}\right]$, subject to constraints (1a), (1b). By assuming the cost of using WC w_{ijc} to be $d_{ijc} + \xi_{ijc}$, and the cost of using one wavelength converter on node j to be $\lambda_j + c_j$, add $\eta_s + t_s + r_d$ to obtain $\min_{\Delta_{sdn},\Phi_{sdn}} (Q_{sdn})$. The solution of the MCSLP algorithm can be found in [54].

(A.2) For any node i, among all the $\mathrm{RWA}_{\mathrm{sin}}$'s,

(A2.1) Compute $\min_{\Delta_{sin},\Phi_{sin}} (Q_{sin}) - C\kappa_{sin}$ for all $\mathrm{RWA}_{\mathrm{sin}}$'s, and select the R_i $\mathrm{RWA}_{\mathrm{sin}}$'s with the lowest $\min_{\Delta_{sin},\Phi_{sin}} (Q_{sin}) - C\kappa_{sin}$ values (the ties are broken arbitrarily);

if $\min_{\Delta_{sin},\Phi_{sin}} (Q_{sin}) - C\kappa_{sin} > 0$, set $\alpha_{sin} = 0$;

if $\min_{\Delta_{sin},\Phi_{sin}} (Q_{sin}) - C\kappa_{sin} < 0$, set $\alpha_{sin} = 1$. The tie is broken arbitrarily.

(A2.2) If the number of $\mathrm{RWA}_{\mathrm{sin}}$'s is more than R_i, then set $\alpha_{sin} = 0$ for the rest of the $\mathrm{RWA}_{\mathrm{sin}}$'s.

The computational complexity of step A.1 in the RMCSLP is the same as the MCSLP provided in [54], i.e., $O\left((N+W)\,N^2 W\right)$. The worst-case computational complexity for the sorting operation in step A.2 is $\sum_i O\left(G_i \log\left(G_i\right)\right)$, where G_i represents the number of lightpaths that are destined for node i. The value of G_i is generally much smaller than NW, so the computational complexity is generally dominated by step a. There are altogether $|A|$ sub-problems in the sub-problem RWAS, so the computational complexity to solve RWAS is $O\left(|A|\,(N+W)\,N^2 W\right)$.

(B) RSPA:

In part A, we have solved RWAS optimally, and now we present the following *Revised Shortest Path Algorithm* (RSPA) to solve $\mathbf{ETR}_{\mathrm{pqz}}$.

(B.1) Solve $\min_{\Upsilon_{pqz}}\{R_{pqz}\}$, subject to constraints (1 g), after applying SPA [55], and assuming the cost of using lightpath s_{sdn} is $G_{pqz}\kappa_{sdn} + V_{pqz}$.

(B.2) If $\min_{\Upsilon_{pqz}}\{R_{pqz}\} > P_{pqz}G_{pqz}$, set $\gamma_{pqz} = 0$; If $\min_{\Upsilon_{pqz}}\{R_{pqz}\} < P_{pqz}G_{pqz}$, set $\gamma_{pqz} = 1$. The tie is broken arbitrarily.

The computational complexity of the RSPA algorithm is dominated by step B.1. Since there are altogether $|\Gamma|$ sub-problems in the sub-problem ETRS and the computational complexity for the SPA is $O\left(|M|^2 N^2\right)$. so the computational complexity to solve the sub-problem TRS is $O\left(|\Gamma|\,|M|^2 N^2\right)$.

5.4 Solving the Dual Problem

Because of the integer variables involved in the formulation, the subgradient method [50] can be used to solve the dual problem.

Let ζ, λ, η, κ be the vectors of Lagrange multipliers $\{\zeta_{ijc}\}$, $\{\lambda_i\}$, $\{\eta_s\}$ and $\{\kappa_{sdn}\}$, respectively.

The multiplier vector $z=(\zeta,\lambda,\eta,\kappa)$ are updated using the following formula:

$$z^{(h+1)}=z^{(h)}+\alpha^{(h)}g\left(z^{(h)}\right), \qquad (6)$$

where $z^{(h)}$ denotes the value of vectors z obtained at the hth iteration, and $\alpha^{(h)}$ denotes the step size at the hth iteration. The notation $g(z)$ is the subgradient of q with respect to z, i.e. $g(z)=\{g(\zeta),g(\lambda),g(\eta),g(\kappa)\}$. The vectors $g(\zeta)$, $g(\lambda)$, $g(\eta)$ and $g(\kappa)$ are composed of $g_{ijc}(\zeta)$, $g_j(\lambda)$, $g_i(\eta)$ and $g_{sdn}(\kappa)$, respectively, where

$$g_{ijc}(\xi)=\sum_{(s,d)}\sum_{0<n\le N_{sd}}\delta_{jic}^{sdn}-1, \qquad (7)$$

$$g_j(\lambda)=\sum_{(s,d)}\sum_{0<n\le N_{sd}}\phi_j^{sdn}-F_j, \qquad (8)$$

$$g_i(\eta)=\sum_j\sum_{(s,d)}\sum_{0<n\le N_{sd}}\Delta_{w_{jic}}^{sdn}-T_i, \qquad (9)$$

$$g_{sdn}(\kappa)=\sum_{(p,q)}\sum_{0<z\le Z_{sd}}\upsilon_{sdn}^{pqz}G_{pqz}-C\alpha_{sdn}. \qquad (10)$$

The step size is given by

$$\alpha^{(h)}=\mu\times\frac{q^U-q^{(h)}}{g^T\left(z^{(h)}\right)g\left(z^{(h)}\right)} \qquad (11)$$

where q^U is an estimate of the optimal solution, and $q^{(h)}$ is the value of q at the hth iteration. Generally, the best value of the objective function J of the feasible routings obtained is used to be q^U. The parameters μ and q^U are changed adaptively as the algorithm converges. The details of the methods to speed up the convergence can be found in [55,56].

5.5 Constructing a Feasible RWA and Routing Scheme

Because some of the constraints are relaxed by the Lagrange multipliers, the solution to the dual problem is usually associated with an infeasible routing scheme, i.e., a scheme where the WC capacity constraint (1c), the converter capacity constraint (1d), the transmitter capacity constraint (1e) and the lightpath capacity constraint (1h) might be violated. Note that other constraints will be respected because of the way sub-problems of (5) are solved. To construct a feasible RWA and a routing scheme from the solution to the dual problem, a heuristic algorithm has to be employed to decide how to re-route the collided lightpaths and corresponding traffic demands. The main idea of the heuristic algorithm is to select the less profitable lightpaths and traffics to detour through other paths, or to reject them, when there are conflicts.

The whole heuristic algorithm can be described in the following steps:

1. Construct the wavelength graph (WG). The details of this procedure can be found in [54].
2. (*RWA Step*): Search for a feasible RWA solution for the lightpaths, based on the solution from the dual problem, using Modified Esau–Williams algorithm (MEWA) [56] to every s_{sdn}.
3. Construct the virtual topology: For all s_{sdn}, if $\alpha_{sdn}=1$, add a directed arch from node s to node d with capacity of C.
4. (*Routing Step*): The traffic demands are routed through the virtual topology.

   4.1. (*Rough search stage*) For every traffic demand x_{pqz}, apply the *Rough Search Algorithm*. The details of this algorithm can be found in the Appendix A.

   4.2. (*Check solvency stage*) For every s_{sdn}, if $\alpha_{sdn}=1$, calculate the total revenue from traffic $t_{sdn}=\sum_{(p,q)}\sum_{0<z\le Z_{sd}}\upsilon_{sdn}^{pqz}P_{pqz}G_{pqz}/h_{pqz}$, where $h_{pqz}=\sum_{(s,d)}\sum_{0<n\le N_{sd}}\upsilon_{sdn}^{pqz}$ is the hop number of x_{pqz}.
   If $t_{sdn}<D_{sdn}$, set $\alpha_{sdn}=0$.
   For every x_{pqz} with $\upsilon_{sdn}^{pqz}=1$, set $\gamma_{pqz}=0$ and apply *Rough Search Algorithm* to x_{pqz}.

   4.3. If the result obtained from step 4.1 $(J)>(1+l)\times J^*$, terminate the heuristic algorithm.
   Otherwise go to the next step;

   4.4. (*Extensive search stage*) Use another LR based algorithm to search for a better routing scheme, which consumes much more time than the *Rough Search Algorithm*. The details of the algorithm can be found in the Appendix B.

In the *RWA Step*, the routing and wavelength assignment of the lightpaths is decided (i.e., variables in A, Δ, Φ are determined). Based on the result from this step, the virtual topology is formed by the lightpaths connecting the nodes. In the *Routing Step*, the routing of the traffic demands over the virtual topology (i.e., variables in Γ, Y are determined). To minimize the computation time, we separate the heuristic algorithm of the *Routing Step* into two stages. In the first stage (*rough search stage*), a rough search of the feasible routing scheme is launched. Only if the result obtained in the first stage is within a certain range ($l > 0$) from the best result obtained (J^*), that the second stage, i.e., the extensive search (*extensive search stage*), is launched to search for a good routing scheme, based on the result from the dual solution and the virtual topology formed in *RWA Step*. In our current implementation, we chose $l = 0.05$.

To decide which lightpath should be deployed first, we define the rank $r_{sdn} = L_{sd} - Q_{sdn}$ for all the lightpaths, where Q_{sdn} represents the dual cost of the sub-problem corresponding to s_{sdn} (see Section 5.3), and $L_{sd} = \sum_{0<n\leq N_{sd}} t_{sdn}$ B.1) represent the “revenue” from the left traffic load between (s, d). At the beginning of the heuristic algorithm, L_{sd} is set to be the total traffic going from nodes s to d in the dual problem solution. If the lightpath s_{sdn} is deployed, L_{sd} is decreased by $m \times C$. If L_{sd} is less than 0, then L_{sd} will be set to 0. In our implementation, $m = 0.8$. The lightpath with the highest rank r_{sdn} is deployed first, and the tie is broken arbitrarily.

The heuristic algorithm to search for a feasible RWA for a lightpath is the same as *MEWA* in [56], and is omitted here for simplicity.

In the *Check solvency stage*, the lightpaths that have a higher cost (D_{sdn}) than the revenue (Pt_{sdn}) are deleted (set $\alpha_{sdn} = 0$) to minimize the overall objective function value. The traffic demands that are routed through those lightpaths are set to the rejected status ($\nu_{sdn}^{pqz} = 0$), and re-routed again using *Rough Search Algorithm* to see if there might be some other viable route.

After the lightpaths are deployed, the traffic demand should be routed through the virtual topology formed by the lightpaths. In the *rough search stage*, we simply route the traffic demands with a lower hop numbers in the dual solution. In the *extensive search stage*, a search algorithm employing the Lagrangian relaxation and the subgradient method is launched. Again, the detailed algorithms for the *rough search stage* and the *extensive search stage* can be found in the appendices.

5.6 Overall Structure of the Algorithm

Fig. 2 depicts the overall structure of the algorithm to solve the problem proposed in Section 4. We first solve the sub-problems independently using the RMCSLP and RSPA algorithms (Section 5.3). Note that solving all the sub-problems is to solve the dual problem (Section 5.2). After the solution for the dual problem is obtained, a heuristic algorithm (Section 5.5) is used to generate a feasible solution based on the solution for the dual problem. If the stopping criterion is not reached, the Lagrange multipliers are updated (Section 5.4), and another iteration of computation is launched. There are various criteria that can be used to terminate the algorithm, such as the duality gap [56], the number of iterations, the time of computation and the objective function value, etc. In the current implementation, we stop the algorithm when the duality gap does not decrease for 1000 iterations. The value of the objective function J of any feasible routing scheme obtained is an upper bound on the optimal objective J^*. The value of the dual function q^*, on the other hand, is a lower bound on J^* [50].

6 Numerical Results

To test the performance of our algorithm, we use the 14-node NSFNET topology as shown in Fig. 3. The randomly generated traffic demands are shown in Table 1 where the horizontal and vertical indexes are the source and destination nodes, respectively. Specifically, the number on the ith row and the jth column represents the number of traffic demands of specific types from node i to j. The total traffic is equivalent 1880 times of an OC-1 SONET traffic stream (i.e., 1880 * 51.84 Mbps = 97.46 Gbps). Since OC-48

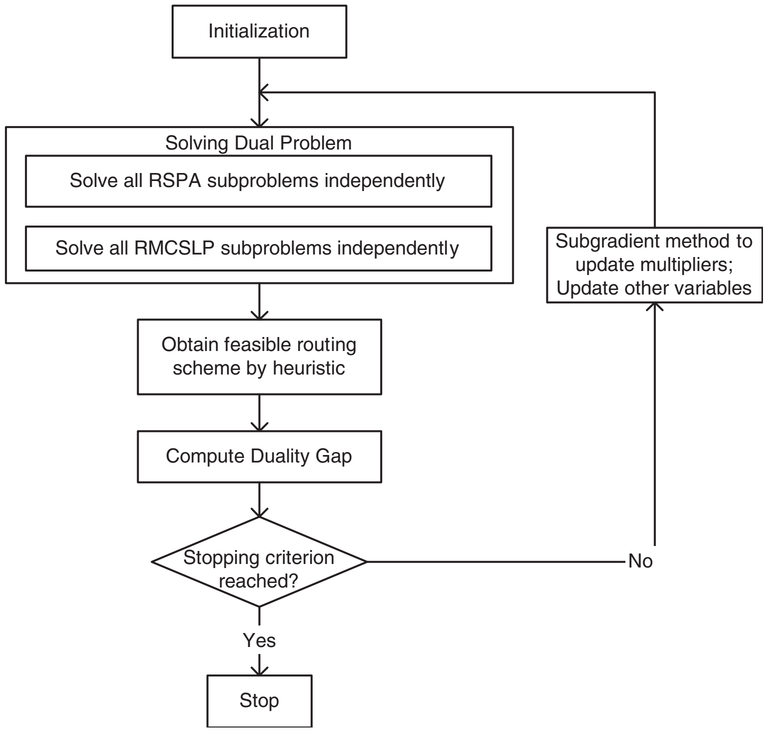

Fig. 2. Schematic depiction of the overall algorithm.

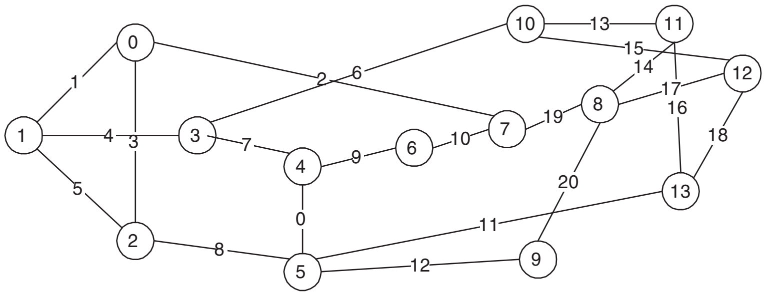

Fig. 3. 14-node NSFNET topology.

traffic is the well-studied RWA problem [48], and we can reuse the results, we need only to use the lower bandwidth traffic to study the traffic grooming behaviour of the algorithm.

We set all penalty coefficients P_{pqz}'s to 1. This means the potential revenue generated by accepting a traffic demand is proportional to its bandwidth requirement. We will demonstrate the influence of different P_{pqz} values reflecting different pricing policy at the end of this section. We also set the transmitter cost t_i and the receiver cost r_i to the same value and refer to it as the transceiver cost. To simplify the simulation, we assume all fibres have the same number of wavelength (W) and all nodes have the same number of converters (F_i). Various computation results for different parameters ($W, V_{pqz}, r_i, t_i, d_{ijc}, T_i, R_i, F_i$ and c_i) are obtained. To study the influence on the network behaviour from different parameters in the optical domain, we first study the cases with $V_{pqz} = 0$, i.e., ignoring the electrical routing cost. Then we study the cases with various V_{pqz} values with the electrical routing cost.

Fig. 4 reflects the changes of primal value J and the dual value (i.e., bound) q with respect to the number of wavelength channels on a fibre

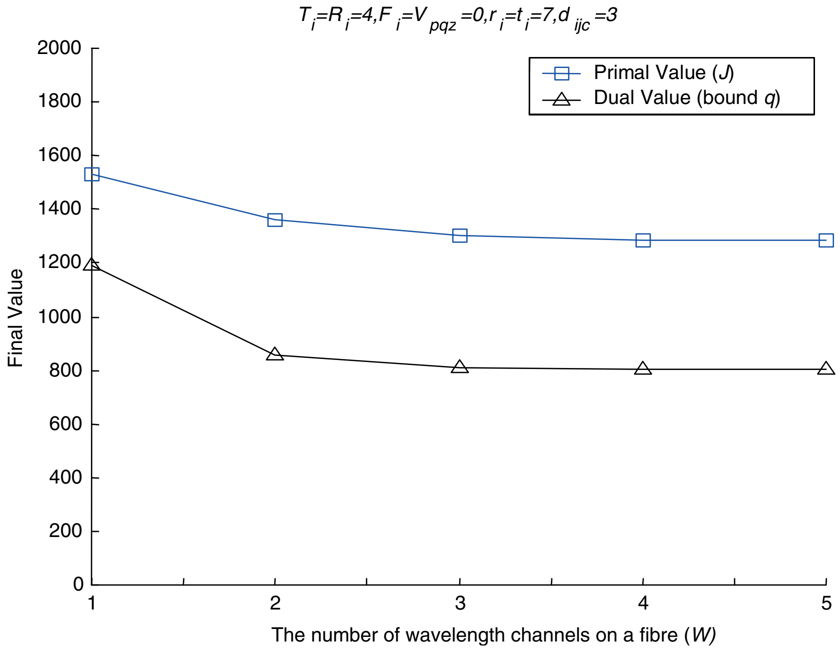

Fig. 4. Objective function J and the dual function q versus the number of wavelength channels on a fibre W.

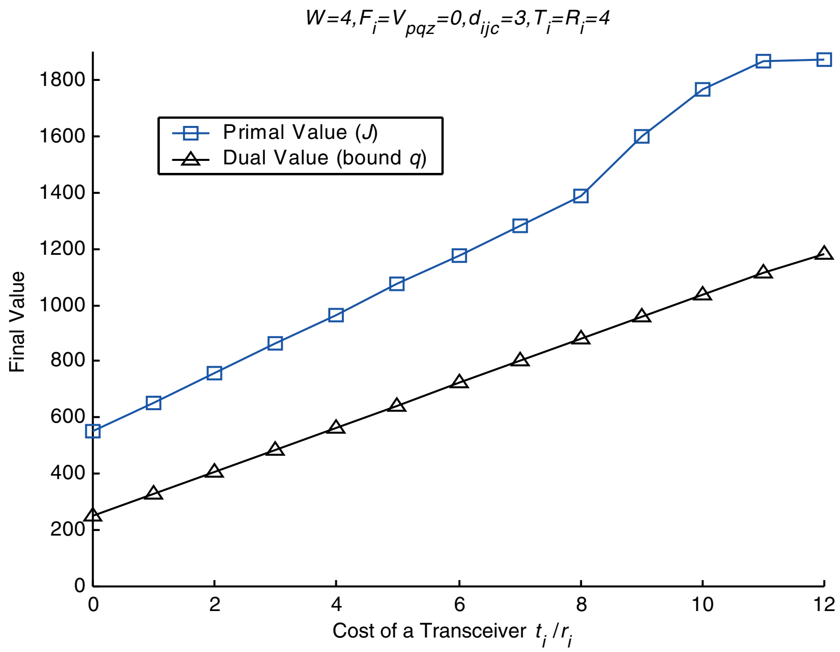

Fig. 5. Objective function J and the dual function q versus the cost of transceivers t_i (or r_i).

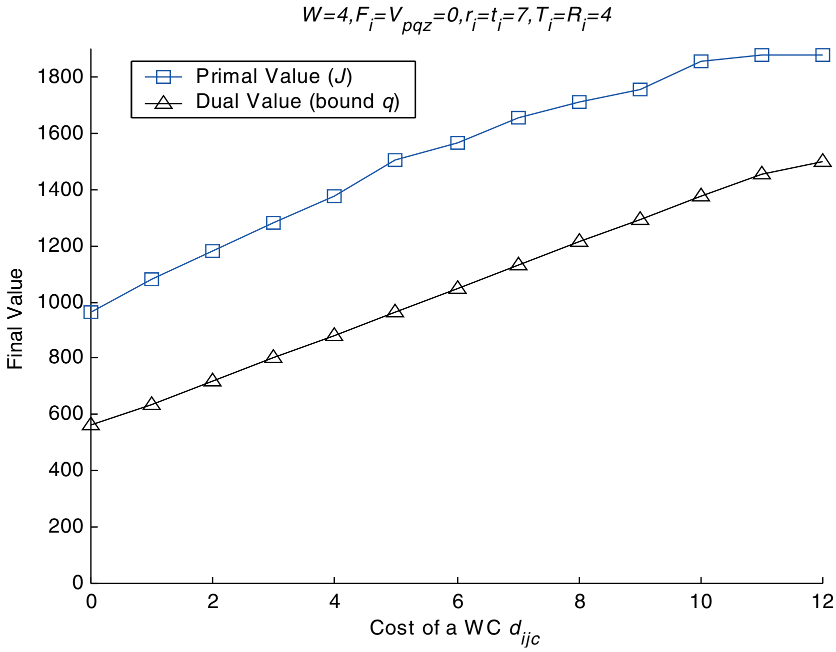

Fig. 6. Objective function J and the dual function q versus the cost of a wavelength channel d_{ijc}.

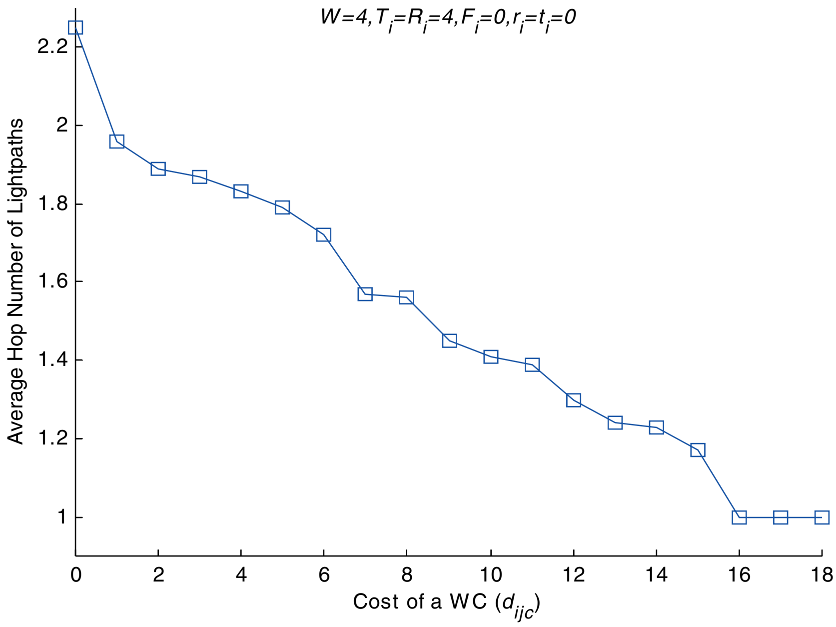

Fig. 7. Average hop number of lightpaths versus the cost of a wavelength channel d_{ijc}.

(W). The other parameters values are $T_i = R_i = 4$, $F_i = V_{pqz} = 0$, $r_i = t_i = 7$, and $d_{ijc} = 3$. In the following figures, all the parameters values will be listed on the top of the graphs as in Fig. 4. We can see from Fig. 4 that as W increases, J and q decrease. However, when W reaches a certain value (4 in this case), J cannot be further improved by increasing W. It indicates that when a certain number of resources are installed, no further profit can be achieved by installing more resources. This is because when the resources (fibres, transceivers and wavelength converters) are abundant, J is limited by the resource cost, instead of the amount of resources. Most of the results can be obtained in two hours on a personal computer configured with 1.4 GHz Intel® CPU, 512MB RAM and Windows XP® operating system.

In Fig. 5 and 6, we can see that as the cost of transceivers (t_i or r_i) and the cost of a WC (d_{ijc}) increase, the objective value J increases almost linearly. In Fig. 5, we can see that as t_i and r_i reach a certain value (11 in this case), because the cost of setting up the lightpaths is so high that most traffic demands are rejected, J stays close to 1880, which is the penalty value for rejecting all the traffic demands (the total bandwidth of the traffic is OC-1880 and $P_{pqz} = 1$). However, J cannot become worse when t_i and r_i increase more. The abnormal increase of J when t_i (or r_i) goes from 8 to 10 maybe is because of the imperfection of the heuristic algorithm in our framework. The similar phenomenon is observed in Fig. 6.

Fig. 7 shows the influence of d_{ijc} on the average hop number of the lightpaths. When the cost of a WC increases, the hop number of a lightpath tends to be lower. We can see from Fig. 7 that as d_{ijc} increases, the average hop number of the lightpaths decreases, until it reaches the minimum hop number (i.e., 1). Due to the randomness of the heuristic algorithm in our framework, the curve is not very smooth.

Fig. 8 shows the influence of the OC-3 traffic's penalty coefficient (P_{pqz}) on the percentage of the rejected OC-3 traffic, while the P_{pqz} value for the OC-1 and OC-12 traffic remains 1. We show in Fig. 8 that as P_{pqz} increases, the percentage of OC-3 traffic demands rejected decreases. This is because as the revenue of OC-3 traffic increases (i.e., P_{pqz} goes higher), it is more likely for the

Table 1. Traffic demand matrices.

OC-1 TRAFFIC DEMANDS

0	1	3	1	5	1	3	0	2	0	1	2	0	3
0	0	2	2	2	11	1	1	1	2	1	0	1	3
3	2	0	3	0	1	2	3	1	3	1	2	2	0
3	1	4	0	1	1	2	3	2	2	1	2	1	3
1	3	0	2	0	1	0	2	0	3	0	1	1	3
1	2	1	3	2	0	1	3	3	11	0	6	10	9
2	2	3	1	10	3	0	0	3	1	2	0	3	7
3	10	2	3	1	4	1	0	0	3	2	0	3	0
3	0	12	3	3	3	1	0	0	2	1	1	1	0
0	0	0	1	2	0	2	0	1	0	1	0	0	3
1	0	0	10	0	3	0	1	0	3	0	3	1	3
2	3	1	1	3	2	3	2	10	2	2	0	1	3
13	0	1	2	0	1	2	0	9	0	2	1	0	3
10	14	0	15	9	3	1	3	0	12	2	1	30	0

OC-3 TRAFFIC DEMANDS

0	1	0	1	1	1	0	2	2	0	1	2	0	1
0	0	0	2	2	2	1	1	1	2	1	0	1	1
1	2	0	1	0	1	2	1	1	1	1	0	2	0
1	1	0	0	1	1	2	1	2	2	1	2	1	1
1	1	0	2	0	1	0	2	0	1	0	1	1	1
1	2	1	1	2	0	1	1	1	1	0	1	1	2
2	2	1	1	1	1	0	0	1	1	2	0	0	1
2	1	2	1	1	0	1	0	0	1	2	0	1	0
1	0	1	1	1	1	1	0	0	2	1	1	1	0
0	0	0	1	2	0	2		1	0	1	0	0	1
1	0	0	2	0	0	0	1	0	1	0	1	1	2
2	1	1	1	1	2	1	2	2	2	2	0	1	1
0	0	1	2	0	1	2	0	1	0	2	1	0	1
1	1	0	2	1	0	1	1	0	1	2	1	0	0

OC-12 TRAFFIC DEMANDS

0	1	0	1	0	1	0	0	1	0	1	1	0	0
0	0	0	1	0	0	1	0	0	1	1	0	1	0
0	1	0	0	0	1	0	0	1	0	1	1	0	0
0	1	0	0	1	1	1	0	1	1	0	1	1	0
1	0	0	1	0	1	0	1	0	0	0	0	0	0
1	1	0	0	1	0	1	0	0	1	0	1	0	1
1	1	0	1	0	0	0	0	0	1	1	0	0	0
0	1	1	0	1	0	1	0	0	0	1	0	0	0
0	0	1	0	0	0	1	0	0	1	1	0	1	0
0	0	0	1	1	0	0	0	1	0	1	0	0	0
1	0	0	0	0	0	0	1	0	0	0	0	1	0
1	0	0	1	0	1	0	1	0	1	1	0	1	0
0	0	1	0	0	1	1	0	0	0	1	1	0	0
1	1	0	1	1	0	1	0	0	1	0	1	0	0

Table 2. Influence of the number of wavelength converters on a node F_i on the objective function J.

W	T_i, R_i	d_{ijc}	r_i, t_i	V_{pqz}			F_i	c_i	J
				OC-1	OC-3	OC-12			
4	4	0	0	0	0	0	0	0	227
4	4	0	0	0	0	0	1	0	216
4	4	0.2	0	0	0	0	0	0.01	262
4	4	0.2	0	0	0	0	1	0.01	235
4	4	3	7	0	0	0	0	0	1284
4	4	3	7	0	0	0	1	0	1284
4	4	1	0	0.1	0.3	1.2	0	0.01	580
4	4	1	0	0.1	0.3	1.2	1	0.01	580

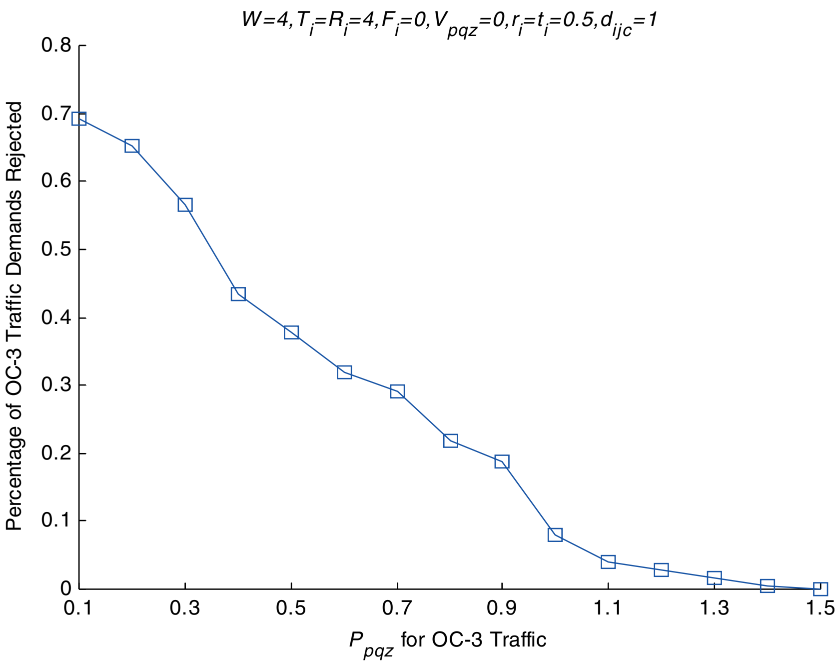

Fig. 8. Percentage of OC-3 traffic rejected versus the OC-3 traffic's penalty coefficient P_{pqz}.

network to gain profit to route OC-3 traffic rather than other traffic (e.g., OC-1 and OC-12). Also, as the revenue goes higher, it is more likely to set up profitable lightpaths. After P_{pqz} reaches a certain value, the majority of the OC-3 traffic demands are accepted, and there are still some residual OC-3 traffic demands that need some expensive lightpaths to be set up, which might not be profitable, if the P_{pqz} is not high enough.

We have also extensively studied the benefit of using wavelength converters. Some of the results under different parameters are listed in Table 2.

Table 2 attempts to demonstrate how F_i influences J. We can see that the benefit on J of having wavelength converters is marginal, even if we assume that the wavelength converter is free, by setting the cost of wavelength converters (c_i) to 0. The cases with wavelength conversion capacity

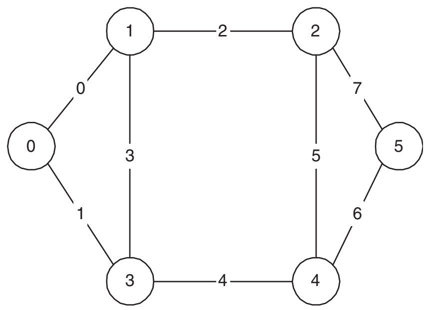

Fig. 9. A six-node network.

(F_i) larger than 1 are not listed in Table 2, because we do not observe any improvement by setting F_i to values larger than 1. This observation is in accordance with the results obtained in [48], which concludes that F_i does not have significant influence on the total RWA cost in the optical domain.

6.1 Special Case

When we set all the costs for the lightpath routing, i.e., r_i, t_i, d_{ijc}, and c_i, to 0, and set the F_i to 0. The problem we solve is exactly the same as the multi-hop case studied in [34].

To have a fair comparison with the existing algorithm, we used the same six-node network and same traffic matrix used in [34]. The topology and the traffic matrices are shown in Fig. 9, and Table 3, respectively.

The comparison between the results provided in [34] and our algorithm is shown in Table 4. Results are numbers of equivalent OC-1 bandwidth routed by different algorithms. We can see that the amount of traffic that can be routed by our algorithm is much more than either of the heuristic algorithms provided in [34], and in most cases, is even better than the result generated from the CPLEX software. This proves that the results from CPLEX in [34] are not optimal, because after a period of time, the CPLEX computation was terminated and the optimal value was not yet found.

Table 3. Traffic matrices for the 6-node network (Equivalent to 988 OC-1 bandwidth in total).

OC-1 TRAFFIC DEMANDS					
0	5	4	11	12	9
0	0	8	5	16	6
14	12	0	9	6	16
4	11	15	0	1	5
10	2	3	3	0	9
2	1	8	15	13	0

OC-3 TRAFFIC DEMANDS					
0	6	2	1	5	4
8	0	8	6	7	8
1	3	0	0	2	7
5	7	3	0	2	6
6	4	5	0	0	2
5	4	4	2	0	0

OC-12 TRAFFIC DEMANDS					
0	1	1	1	0	0
1	0	1	1	0	2
0	1	0	2	1	0
2	0	2	0	2	0
1	2	0	2	0	1
1	1	2	2	2	0

7 Conclusions

Traffic grooming in SONET-over-WDM networks involves the routing of individual low-bandwidth traffic demands in the SONET layer and the routing of lightpaths in the WDM layer. Although the integrated design of both layers offers better results than the two-step design for the two layers, the optimization problem is very complex and thus solving the optimization problem is challenging. By relaxing some of the constraints, we are able to solve the problem by using Lagrangian-relaxation technique and subgradient methods. The overall dual function is decomposed into two sets of sub-problems that are solved independently. An effective heuristic algorithm is also proposed to generate the feasible result based on the solution to the dual problem. The decomposition of the dual function into sub-problems, simplifies the inner problem solution. The outer problem is solved by applying the subgradient method. As a special case of our formulation, the comparison between our results and some other research shows the high effectiveness of our algorithm.

Besides the contribution of solving the traffic-grooming problem by the Lagrangian-relaxation and subgradient methods, we propose a novel objective function taking into account the rejection penalty, the routing cost associated with the electrical and optical domains. By optimising this cost function, the potential profit of network operation is maximized. Compared to other

Table 4. Comparison between the results in [34] and our results.

	Result from CPLEX [34]	Result from Heuristic 1 [34]	Result from Heuristic 2 [34]	Result from our algorithm	Upper bound from our algorithm
T = 2, W = 3	–	–	–	516	576
T = 3, W = 3	738	701	666	751	851
T = 4, W = 4	927	883	925	930	976
T = 5, W = 3	967	933	933	969	987
T = 7, W = 3	967	933	933	969	987
T = 3, W = 4	738	701	666	751	851
T = 4, W = 4	933	920	925	930	976
T = 5, W = 4	988	988	988	988	988

formulation accepting traffic demands without considering the network cost, we have adjusted our objective to accept only the most profitable traffic demands for a given network capacity. As a result, the traffic demands are identified as profitable or not. By doing a series of computations for different parameters, the impact of network resource allocation on the network profit is obtained.

Appendix A: Rough Search Stage

The *Rough Search Algorithm* is a simple heuristic algorithm to obtain an estimate of the overall cost in the electrical domain (the switching cost and the rejection penalty) for the routing of the traffic demands, i.e., $\sum_{(p,q)}\sum_{0<z\leq Z_{pq}} P_{pqz}G_{pqz}(1-\gamma_{pqz}) + \sum_{(p,q)}\sum_{0<z\leq Z_{sd}} E_{pqz}$, subject to constraints (1 g) and (1 h).

u_i is the label associated with node i. To simplify the description, the source node is set to node 1, and u_1 is set to 0.

Step 1 (Initialization):

(1.1) $u_1 := 0$; n := false;

(1.2) If $i \neq 1$ and there exist an arch (an admitted lightpath) from node 1 to i, then $u_i := 1$, $(\forall i)$.

Step 2 (Search all vertices):

(2.1) Node m := next node (the node 1 for the first run; until all nodes have been searched).

Step 3 (Find the improvement):

(3.1) For all every node $q \neq 1$, if there exists an arch from node m to node q, $u_q > (u_m + V_{pqz})$ and the capacity of this arch is not all used, then $u_q := (u_m + V_{pqz})$, n := false; If node q is the destination node of the traffic demand considered, terminate the algorithm and assign the route to this traffic demand;

(3.2) If $!n$, n := true; Go to *Step 2*; else end the algorithm and set the traffic demand rejected ($\gamma_{pqz} = 0$). "n = true" means that there is no more possible label change.

Appendix B: Extensive Search Stage

Only if the estimate from *Rough Search Stage* is within a certain range from the best result obtained, the *Extensive Search Stage* is launched. *Extensive Search Stage* generally takes much more time than the *Rough Search Stage*. In *RWA Step*, the variables in A, Δ, Φ are decided. The *Extensive Search Stage* is to minimize $\sum_{(p,q)}\sum_{0<z\leq Z_{pq}} P_{pqz} G_{pqz}(1-\gamma_{pqz}) + \sum_{(p,q)}\sum_{0<z\leq Z_{sd}} E_{pqz}$, subject to constraints (1 g) and (1 h), with Γ, ν as variables.

The Lagrangian relaxation and subgradient methods are again used to solve this problem. We can simply relax the lightpath capacity constraints (1 h) by using Lagrangian multiplier ω_{sdn}, and obtain the dual problem (DP1):

$$\max_{\omega\geq 0} p \equiv \min_{\Gamma,y} \left\{ \sum_{(p,q)} \sum_{0<z\leq Z_{pq}} [P_{pqz}G_{pqz}\left(1-\gamma_{pqz}\right) + E_{pqz}] + \sum_{(s,d)} \sum_{0<n\leq N_{sd}} \omega_{sdn} \left[\sum_{(p,q)} \sum_{0<z\leq Z_{sd}} \nu_{sdn}^{pqz} G_{pqz} - C\alpha_{sdn} \right] \right\}, \text{subject to}\,(1\,g).$$

By using the fact that $\nu_{sdn}^{pqz} = \nu_{sdn}^{pqz}\gamma_{pqz}$, DP1 can be rewritten as:

$$\max_{\omega\geq 0} p \equiv \min_{\Gamma,Y} \left\{ \sum_{(p,q)} \sum_{0<z\leq Z_{pq}} \left[\left(1-\gamma_{pqz}\right) P_{pqz}G_{pqz} + \gamma_{pqz} \sum_{(s,d)} \sum_{0<n\leq N_{sd}} \left(V_{pqz}+G_{pqz}\omega_{sdn}\right)\nu_{sdn}^{pqz} \right] \right\} - \sum_{(s,d)} \sum_{0<n\leq N_{sd}} C\omega_{sdn}\alpha_{sdn}$$

$$= \sum_{(p,q)} \sum_{0<z\leq Z_{pq}} \min_{\gamma_{pqz},Y_{pqz}} \left[\left(1-\gamma_{pqz}\right) P_{pqz}G_{pqz} + \gamma_{pqz} \sum_{(s,d)} \sum_{0<n\leq N_{sd}} \left(V_{pqz}+G_{pqz}\omega_{sdn}\right)\nu_{sdn}^{pqz} \right] - \sum_{(s,d)} \sum_{0<n\leq N_{sd}} C\omega_{sdn}\alpha_{sdn}$$

$$= \sum_{(p,q)} \sum_{0<z\leq Z_{pq}} \min_{\gamma_{pqz}} \left\{ \left(1-\gamma_{pqz}\right) P_{pqz}G_{pqz} + \gamma_{pqz} \min_{\nu_{pqz}} \left[\sum_{(s,d)} \sum_{0<n\leq N_{sd}} \left(V_{pqz}+G_{pqz}\omega_{sdn}\right)\nu_{sdn}^{pqz} \right] \right\} - \sum_{(s,d)} \sum_{0<n\leq N_{sd}} C\omega_{sdn}\alpha_{sdn}$$

subject to (1 g).

Note that α_{sdn}'s are already decided in *RWA Step*, so the *Routing Step* in the heuristic algorithm sees only the virtual topology formed by all the nodes and the lightpaths with $\alpha_{sdn} = 1$.

To obtain the minimum for every sub-problem $S_{pqz} = \min_{\gamma_{pqz}} \left\{ \left(1-\gamma_{pqz}\right) P_{pqz}G_{pqz} + \gamma_{pqz} \min_{\gamma_{pqz}} \left[\sum_{(s,d)} \sum_{0<n\leq N_{sd}} \left(V_{pqz}+G_{pqz}\omega_{sdn}\right)\nu_{sdn}^{pqz} \right] \right\}$, we can simply use the same Revised Shortest Path Algorithm (RSPA) provided in Section 5.4, assuming the cost of using lightpath s_{sdn} by traffic demand x_{pqz} is $V_{pqz} + \omega_{sdn}G_{pqz}$.

The dual solution is thus obtained by solving all the sub-problems. The lightpath constraint (1 h) are relaxed in the dual problem, so that the solution to the dual problem is generally not feasible. To obtain the feasible result, we can simply apply the same *Rough Search Algorithm*.

Subgradient method is used to maximize the dual solution.

The multiplier vector ω is updated using the following formula:

$$\omega^{(i+1)} = \omega^{(i)} + \beta^{(i)} g(\omega^{(i)}),$$

where $\omega^{(i)}$ denote the value of vectors ω obtained at the ith iteration, and $\beta^{(i)}$ denote the step size at the ith iteration. The notations $g(\omega)$ is the subgradients of p with respect to ω. The vector $g(\omega)$ is composed of $g_{sdn}(\omega)$, where

$$g_{sdn}(\omega) = \sum_{(p,q)} \sum_{0<z\leq Z_{sd}} \nu_{sdn}^{pqz} G_{pqz} - C\alpha_{sdn}. \qquad (10)$$

The step size is given by

$$\beta^{(i)} = \theta \times \frac{p^U - p^{(i)}}{g^T(\omega^{(i)})g(\omega^{(i)})},$$

where p^U is an estimate of the optimal solution, and $p^{(i)}$ is the value of p at the ith iteration. Generally, the best value of the objective function of the feasible routings obtained is used to be p^U. The adaptation of the parameters in the algorithm is the same as Section 5.2.

Here in current implementation, we terminate the *Extensive Search Stage* after 2000 iterations.

Acknowledgments

The authors want to thank the anonymous reviewers for their constructive comments to improve the quality of our paper. This research is supported in part by the National Science and Engineering Research Council (NSERC) and industrial partners, through the Agile All-Photonic Networks (AAPN) Research Program.

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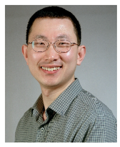

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