Effective Decentralized Energy Restoration by a Mobile Robot

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***Abstract*—As most existing sensors are powered by batteries, the coverage provided by a sensor network degrades over time and eventually disappears if energy is not restored. A popular approach to energy restoration is to use a *robot* acting as a mobile battery charger/changer. The robot decides where to move next according to a predefined on-line energy restoration strategy. Since the goal of an energy restoration strategy is to maintain as much as possible of the network operational at any time, its *effectiveness* depends on the number of nodes it is able to maintain operational at any given time, as well as on for how long a node battery remains depleted.**

The ideal optimal on-line strategy (called `OPTIMAL`) occurs when the robot knows at any time the current status of all sensors, and it computes the best request to satisfy next, based on this information. Although optimal in terms of effectiveness, this centralized strategy constantly requires up-to-date global information; hence its high computational and communication costs make it not feasible.

We consider a drastically different on-line strategy (called `LIC`), which is simple and fully decentralized, uses only local communication, requires no computations, and is highly scalable. In our strategy, the robot visits the sensors in a predefined circular order, moving in a "clockwise" direction and only when aware of a pending request. A sensor whose battery is about to become depleted originates a recharging request and waits for the robot; the request is forwarded in a "counter-clockwise" direction until it reaches either the robot or another sensor waiting for the robot.

We show the perhaps unexpected result that, once the system becomes stable, in most networks the effectiveness of `LIC` is equivalent to that of `OPTIMAL`. In other words, in most cases, in spite of its simplicity and its extremely small (communication and computation) costs, the proposed decentralized strategy is as effective as the optimal centralized one. We augment our theoretical results with experimental analysis, showing among other things that the system stabilizes very quickly.

I. Introduction

A. Framework, Problem and Strategies

Wireless sensor networks are widely employed in a large variety of contexts and applications, mainly to monitor the conditions of the area in which they are deployed. Most existing sensors are powered by batteries whose lifetime is limited; once the battery becomes depleted, the node is no longer sensing and, in absence of redundant coverage, this *sensing hole* creates a *coverage hole* in the monitored area. Indeed, the coverage provided by the network degrades over time and eventually disappears if no action is taken. Extensive research has being carried out on how to address this problem, mainly concentrating on *energy management* strategies, whose goal is to prolong the lifetime of the network and delay the progressive coverage decay by balancing the energy levels among the sensors (e.g., see [1], [13]).

A very different line of research has been on *energy restoration*, with the ambitious goal to maintain the network operating perpetually. In this line are proposals to enhance the sensors with (radically different) additional capabilities. For instance, the sensors could be provided with the means to harvest energy from the environment and to convert it to electrical energy, enabling them to recharge their batteries (e.g., see [21], [25]). A different direction is to add mobility to the sensors, enabling them to move to recharge facilities deployed throughout the sensing area (e.g., [16], [22]). The drawback of these types of approaches is the increased complexity, and thus cost, of the sensor nodes; this at a time when technology trends are scaling sensors to be smaller and cheaper.

An alternative to adding more complexity to the nodes has been the proposal of using one or more external mobile devices, typically called *robots*, which would go around to restore energy to nodes with (near) depleted energy. The restoration can take place by either *recharging* the depleted batteries or by *replacing* them entirely with fully charged one. Fueled by the recent evolution in wireless power transfer techniques [8], the research on sensor recharging by mobile robots has been quite intensive (e.g., see [2], [3], [10], [15], [20], [27], [28]). The alternative of replacement has been considered in the literature (e.g., see [17], [26]), albeit with less intensity. For a comparison between these two alternatives see [17]. Notice that the idea of using a mobile robot in sensor networks is not new, as it has been proposed for data gathering and aggregation, for network repairs, as well as for other network maintenance tasks (e.g., see [9], [11], [18]).

In this paper we are interested in energy restoration by a single mobile robot. Regardless of whether restoration is by recharging or replacement, after servicing a node, the robot must choose where to move next. The algorithm followed by the robot to make this decision, here called *energy restoration strategy*, may prescribe the acquisition of information from the sensors (e.g., energy level, location, etc) and require possibly complex computations by the robot. Since the decision can be made based solely on current and older information, at an abstract level these strategies can be viewed as on-demand: a node whose battery is (about to become) depleted issues a request for the robot; the robot moves to service the requests so to optimize some cost parameters, based on the information currently available. Almost all the existing on-demand strategies are *centralized* (e.g., see [5], [6], [7], [12], [20], [26], [30]): the information about all the requests is communicated to the robot that then computes where to go next; alternatively, the information is communicated to the base station, which takes the decisions and provides the mobile robot with instructions. In addition to the high communication costs required, the optimization requirements to be met by the decision are typically accompanied by high computational complexity, which grows non-linearly with the number of sensor nodes; these factors imply a difficulty to scale for these strategies. The existing *decentralized* strategies are [2], [17], [19]; in [17] the concern is to maximize the time until the first interruption of the sensing activity of a single sensor; in [2], which is not on-demand (i.e., the order in which the nodes are recharged is fixed), the total amount of energy that can be put in the system is bounded (i.e. the energy restoration process is limited); in [19] the described technique applies only to linear sensor networks.

B. Effectiveness, Costs and Contribution

The fact that some sensors might be inactive at some times is not a problem if an energy restoration strategy is in place. Indeed, the goal of an energy restoration strategy should be to maintain as much as possible of the network operational at any time. Its *effectiveness* towards this goal finally rests with the number of sensors it is able to maintain operational at any given time, in spite of battery depletions. We shall call this measure *operational size* or, with an abuse of notation, *coverage*. The other effectiveness measure of interest is the time from the moment a sensor becomes no longer operational to the time when the robot arrives to serve it; i.e., for how long a sensing hole lasts. We shall call this measure *disconnection time*.

Associated with each energy restoration strategy are also the computation and communication *costs* encurred when the robot operates in the network according to that strategy.

The effectiveness and the costs of the strategy employed by the robot to service the sensors depends on many factors, a crucial one being the amount of information about the network status available at any given time to the robot.

From the effectiveness point of view, the "ideal" optimal on-line strategy, which we shall call simply `OPTIMAL`, is clearly when the robot knows at any time the current status of all sensors, and it computes on-line which request it must satisfy next so to minimize the number of sensing holes and/or their duration. Even more that any other centralized on-line strategy, in addition to a high computational complexity, `OPTIMAL` requires constantly up-to-date global information; hence the communication costs required to implement it severely limit its feasibility.

In this paper, we propose a drastically different on-line strategy, which we shall call *Local Information and Communication* (`LIC`). In this strategy, the robot visits the sensors in a predefined circular order, moving in a "clockwise" direction when aware of a pending request. A node whose battery is about to become depleted originates a recharging request and waits for the robot; the request is forwarded in a "counter-clockwise" direction until it reaches either the robot or another node waiting for the robot. In other words, each node communicates only locally: with the neighbouring sensors in the circular order (to send or receive a request), or with the robot if currently there (to communicate the presence of a pending request); the robot moves only from one node to the next in the cyclic order, is aware only of whether or not there is a pending request, and has no need of memory or calculation.

In contrast to the existing centralized strategies, this simple on-line strategy is fully distributed and *decentralized*, uses only local communication, requires no computations, and it is highly scalable. However, it provides the robot only with the awareness of the existence of at least one current request.

Surprisingly, our results show that, in most networks, in spite of its simplicity and small costs, this decentralized strategy can be as effective as the ideal optimal centralized one.

C. Main Results

We study the effectiveness and costs of `LIC`, both analytically and experimentally in an abstract setting.

Like in every energy restoration strategy, effectiveness depends on two crucial system parameters: the battery life, i.e., the amount of time Δ a fully charged battery lasts under normal operations; and the recharging time, i.e., the amount of time ρ required for the charging/replacement once the robot is at the sensor's site. Let $\mathcal{N}(\Delta, \rho)$ denote the (infinite) class of sensor networks with those specific component characteristics.

We establish several results related to the *stability* of the system under `LIC`, whereas the network is deemed to be in a stable state if the order in which the nodes are charged in a round is the same in every round. In particular, we prove analytically that, for almost all networks in $\mathcal{N}(\Delta, \rho)$, once the system becomes stable, although the set of operational nodes changes in time, its size remains unchanged; furthermore this value is the same regardless of the initial network size. We also determine the disconnection time for such stable networks, providing a precise characterization of the performance and effectiveness of `LIC`. Due to space limitations, some of the proofs are omitted.

We then compare the effectiveness of `LIC` with that of the optimal (but expensive to implement) strategy `OPTIMAL`. We show the perhaps unexpected result that, in most networks, the effectiveness of `LIC` is equivalent to that of `OPTIMAL`. In other words, in most cases, in spite of its simplicity and its extremely small (communication and computation) costs, the decentralized strategy is as effective as the ideal optimal one.

We support our theoretical results with experimental analysis, showing that the system stabilizes very quickly and confirming all the theoretical bounds established for coverage size and disconnection time.

II. Model

Let $\mathcal{X} = \{x_0 \ldots, x_{n-1}\}$ be the set of *sensor nodes*, or simply *nodes*, forming the network. Each node has sensory equipment that allows it to monitor its surroundings; it also has provision for wireless communication. Let π be a cyclic order of the nodes; successive nodes in the order (e.g., $x_{\pi(i)}$ and $x_{\pi(i+1)}$, where all operations on the indices are modulo n) are called neighbours, and can communicate (possibly by multiple hops).

Normal operations require sensing and occasional communication; both operations consume energy, which is provided by an on-board battery of limited capacity. When the battery is nearly depleted, the node becomes non-operational, thus creating a sensing hole in the network, and the remaining energy is used for a small amount of emergency communication (e.g., forward a request). In the following, with an abuse of notation, we will say that the battery is depleted when the node becomes non-operational.

Let Δ denote the amount of time it takes for a fully charged battery to become depleted under normal operations. Each node monitors the energy level of its battery and determines whether it is below a fixed threshold. We denote by τ the amount of time, under normal operations, elapsed from the moment the battery falls below the threshold to the time it becomes depleted. In the following, for ease of discussion, we will sometimes refer to Δ as the battery life or capacity, and to τ as the threshold.

A mobile *robot R* is available in the system to recharge/replace the sensors's batteries; once *R* reaches a node, if the energy level of the sensor is below the threshold, the robot will restore the energy. We denote by ρ the amount of time it takes for a battery to become replaced/recharged; i.e., if the robot reaches a node x whose battery is below the threshold at time t, x's battery will be fully charged at time $t + \rho$. The robot can move from node to node; we denote by $d_{i,j}$ the time it takes the robot to travel from node x_i to x_j. We assume uniform distances among neighbours, that is $d_{\pi(i),\pi(i+1)} = d \geq 1$ for $0 \leq i \leq n-1$.

We now introduce the measures we use to study the effectiveness of energy restoration strategies. Let $\mathtt{S}$ be an energy restoration strategy. The *operational size* or *coverage* at time t under $\mathtt{S}$ (denoted by $Coverage(\mathtt{S}, t)$) is the number of operational nodes (i.e., nodes with a non empty battery) at that time. Note that the operational size implicitly measures the number $n - Coverage(\mathtt{S}, t)$ of the sensing holes in the network at time t.

The *disconnection time* for a node x is the amount of time from the moment x becomes inactive to the time when the robot arrives to serve x. Disconnection time is, of course, zero if the node is charged before its battery is depleted (i.e., before it becomes inactive). More precisely, the disconnection time for node x at time t under $\mathtt{S}$ (denoted by $Disconnect(\mathtt{S}, t, x)$) is the amount of time x had been inactive when last serviced by the robot before or at time t. This measures indicates for *how long a sensing hole lasts*.

Since the focus of this study is on the effectiveness of the recharging strategies and on their computing and communication costs, we will not address how the robot acquires the means to service the sensor nodes (e.g., by stopping at a recharging station, by extracting it from the environment, etc.) and we assume (as in [24], [28]) that the robot is always capable of doing so.

III. Decentralized Strategy LIC

A. Description

We now describe a simple decentralized on-demand strategy, which uses only local information and requires only local communication, hence the name *Local Information and Communication* (LIC).

Starting from its initial position at an arbitrary node, the robot visits the sensors according to π (see Figure 1), moving in a "clockwise" direction (i.e., from $x_{\pi(i)}$ to $x_{\pi(i+1)}$) when aware of a pending request. A sensor whose battery is about to become depleted originates a recharging request and waits for the robot; the request is forwarded in a "counter-clockwise" direction (i.e., from $x_{\pi(i)}$ to $x_{\pi(i-1)}$) until it reaches either the robot or another sensor waiting for the robot, creating in this way a trail to be followed by the robot when it becomes available. Note that a request contains no specific information (e.g., id, location, etc) about the node issuing or forwarding it. All nodes have the ability to receive and forward a single request even if their sensor is no longer operational.

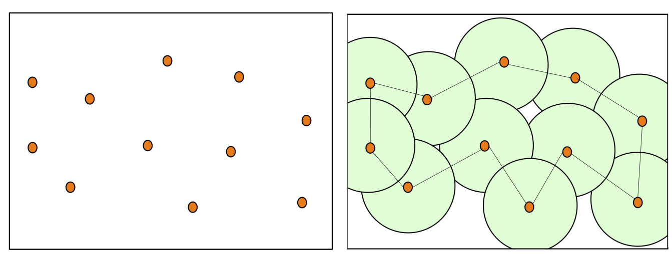

Fig. 1: *a*) A deployed sensor network. *b*) A cyclic order π of the sensors.

Summarizing, (1) each sensor communicates only locally: with the neighbouring sensors in the cyclic order (to send or receive a request), or with the robot if currently there (to communicate the presence of a pending request); (2) the robot moves only from one node to the next in the cyclic order, is aware only of whether or not there is a pending request, and has no need of additional memory or calculation.

The protocol LIC prescribing the behaviour of the sensor nodes and the robot is shown in Figure 2. With respect to the protocol, a sensor node x can be in one of two protocol states, Regular or Waiting, and keeps track of whether or not it has received a pending request from its predecessor in the order (Boolean variable $Q(x)$). Initially all nodes are in

REGULAR
when battery level reaches threshold
  if robot is not here **then**
    send request **to** counter-clockwise neighbour;
  become WAITING;
receiving request
  $Q(x) := 1$;
  send request **to** counter-clockwise neighbour;
  become WAITING

WAITING
receiving request:
  $Q(x) := 1$;
receiving Robot: (robot is here)
  if battery level at or below threshold
    `Be Charged`;
  if $Q(x) = 1$ **then**
    $Q(x) := 0$;
    send Robot **to** clockwise neighbour;
  become REGULAR;

Fig. 2: Protocol `LIC` executed by node x.

state REGULAR, $Q(x) = 0$ for all $x \in \mathcal{X}$, and the robot is at an arbitrary node.

B. Properties: Tours and Weakness

A *tour* from node x is defined as the visit of all the nodes by the robot starting from x (and possibly charging it) and ending when arriving again at x. Let $\hat{\Delta} = \Delta - \tau$ denote the amount of time before a fully charged battery falls below the threshold.

Lemma 1. *Let $x_{\pi(i)}$ require recharging both at the beginning and at the end of a tour from it; then also $x_{\pi(i-1)}$ requires recharging when reached by the robot in this tour.*

Proof. Let $x_{\pi(i)}$ be found to be needing recharging at time t_0, fully recharged at time $t_1 = t_0 + \rho$, and found empty again at the end of this tour, at time t_2.

By contradiction, let the previous node $x_{\pi(i-1)}$ be found not needing recharging when visited by the robot in this tour. Let $k \geq 0$ be the number of the nodes (other than $x_{\pi(i)}$) that have been recharged in this tour. In other words, the robot has spent in this tour $k\rho$ time units to charge them before reaching x_0 at time t_2; that is, $t_2 \geq t_1 + k\rho + nd$. Since $x_{\pi(i)}$ is found needing recharging at time t_2, we must have

$$k\rho + nd \geq \Delta - \tau = \hat{\Delta}.$$

On the other hand, the time elapsed between the time $t' = t_0 - d$ the robot left $x_{\pi(i-1)}$ and the time t'' it reached it again is at least $(k+1)\rho + nd$, since $x_{\pi(i)}$ was recharged in this interval. Since $x_{\pi(i-1)}$ is assumed to be found non needing recharging, we must have

$$(k+1)\rho + nd < \hat{\Delta}$$

a contradiction. □

Analogously,

Lemma 2. *Let $x_{\pi(i)}$ require recharging both at the beginning and at the end of a tour from it; then also $x_{\pi(i+1)}$ will need to be recharged when when reached by the robot in the next tour from $x_{\pi(i)}$.*

We say that x is *weak* if there exists a tour from x where x needs recharging both at the beginning and at the end of the tour. Let t_{weak} be the first time when this happens. The two Lemmas, 1 and 2, together prove the following important property:

Theorem 1. *If there is a weak node then from time t_{weak} every node visited by the robot is found to need recharging. This holds regardless of the threshold τ.*

This, in turns, has important consequences on the size of the coverage of the network:

Theorem 2. *Let $n > m = \lceil \frac{\Delta}{(\rho+d)} \rceil$. If there is a weak node, then there exists a time t, such that, $\forall t' > t$:*

$$m \leq Coverage(\texttt{LIC}, t') \leq m + 1$$

and all sensors have the same disconnection time:

$$\forall x \in \mathcal{X}, Disconnect(\texttt{LIC}, t', x) = (n-1)(\rho + d) + d - \Delta.$$

Proof. Let $x_{\pi(i)}$ be weak; thus, there is a tour where $x_{\pi(i)}$ will be recharged both at the start and the end of that tour; let t be the time when the recharging at the end of that tour will be completed. By Theorem 1, from this time on the robot finds only nodes needing recharging; since it takes ρ time units for the robot to recharge a sensor and d time units to move to the next sensor, by time $t' = t + \Delta$ the robot has recharged the consecutive nodes $x_{\pi(i+1)}, x_{\pi(i+2)}, ..., x_{\pi(i+m-1)}$ where $m = \lceil \frac{\Delta}{(\rho+d)} \rceil$ and all operations on the indices are modulo n; if Δ is not a multiple of $\rho + d$, then it is currently recharging $x_{\pi(i+m)}$, otherwise also $x_{\pi(i+m)}$ has been fully recharged. But, at this time $t' = t + \Delta$, $x_{\pi(i)}$'s battery is totally depleted, and so obvioully is the battery of all the sensors after $x_{\pi(i+m)}$ up to and including $x_{\pi(i)}$. This means that, at time $t_0 = t + m(\rho+d)$, exactly $n - m$ nodes have their battery completely empty; hence $Coverage(\texttt{LIC}, t_0) = m$.

Observe that, when $x_{\pi(i+m+1)}$ is reached and fully recharged, at time $t_1 = t_0 + (\rho + d)$, $x_{\pi(i+1)}$'s battery is depleted; that is $Coverage(\texttt{LIC}, t_1) = m$. More generally, when $x_{\pi(i+m+j)}$ is reached and fully recharged, at time $t_j = t_0 + j(\rho + d)$, $x_{\pi(i+j)}$'s battery is depleted; that is, $Coverage(\texttt{LIC}, t_j) = m$. On the other hand, at any time $t_j < t' < t_{j+1}$ $x_{\pi(i+j)}$'s battery might not yet be depleted; that is, $Coverage(\texttt{LIC}, t') \leq m + 1$.

Since after time t the robot keeps charging every node it encounters, it will spend $n(\rho+d)+d$ time units to complete a tour. During that time, each node is not disconnected for $\rho+\Delta$ time units. Therefore, every node will have disconnection time $(n-1)(\rho + d) + d - \Delta$. □

C. Properties: Rounds and Stability

Let us call *round* from node x any sequence of consecutive tours from x where at the beginning of the first tour and at the end of the last x's battery needs to be recharged, and in all others it does not. Clearly a round from x might include several tours from x.

We will denote by $r(x,j)$ the j-th round from x, $j \geq 1$; by $t_s(x,j)$ and $t_e(x,j)$ the starting and ending time of $r(x,j)$, respectively. When j is clear from the context, we will indicate a round from x simply by $r(x)$ and the corresponding starting and ending time by $t_s(x)$ and $t_e(x)$ respectively.

Let $\sigma(x,j)$ denote the ordered sequence of the nodes charged during $r(x,j)$; notice that $\sigma(x,j)$ starts with the node charged after x and ends with x; further notice that a node may appear more than once in $\sigma(x,j)$ while some may be absent.

We say that the system is *stable* if $\exists j \geq 1$ such that $\forall x \in \mathcal{X}, \forall j' > j$, $\sigma(x,j) = \sigma(x,j')$. That is, in a stable system, the order in which the nodes are charged is the same in every round; hence, in a stable system, we can omit the indication of the round and denote $\sigma(x,j)$ simply as $\sigma(x)$.

When a system is (or has become) stable, it enjoyes particular properties. Among them:

Lemma 3. *Let the system be stable. Then*
(i) $\sigma(x)$ is a permutation of the elements of $\mathcal{X}$.
(ii) $\forall x,y \in \mathcal{X}$, $\sigma(x)$ is a cyclic shift of $\sigma(y)$
(iii) Every round from any node $x \in \mathcal{X}$, is composed of the same number s of tours.

Theorem 3. *If the system is stable and $n > 2\frac{\Delta}{\rho}+1$, each round is composed by a single tour.*

Proof. Let the system be stable; then, by Lemma 3 (iii), every round from any node x is composed of the same number s of tours. We want to show that $s=1$ if $n > 2\frac{\Delta}{\rho}+1$.

By contradiction, let $s>1$. Consider a round from node x and let $f(x,s)$ be the number of nodes charged in the last tour of this round. We will consider three cases depending on the value of $f(x,s)$.

Case 1 : $f(x,s) < \frac{n}{2}$. In this case, the number of nodes charged in the first $s-1$ tours is $k = n - f(x,s) \geq \lfloor\frac{n}{2}\rfloor + 1$.

Consider the amount of time T elapsed from the moment x has been charged at the beginning of the round, to the beginning of the last tour of this round.

By definition, and since $k \geq \lfloor\frac{n}{2}\rfloor + 1$ and $s>1$, we have

$$T = (k-1)\rho + (s-1)d\,n \geq \lfloor\frac{n}{2}\rfloor\rho + (s-1)d\,n \geq \lfloor\frac{n}{2}\rfloor\rho + d\,n.$$

Since, by definition of round, x is found by the robot not to need recharging at this time, we have $\Delta - \tau = \hat{\Delta} \geq T$, that is:

$$\Delta \geq \hat{\Delta} \geq T \geq \lfloor\frac{n}{2}\rfloor\ \rho + d\,n > \lfloor\frac{n}{2}\rfloor\ (\rho+1)\ .$$

But $n > 2\frac{\Delta}{\rho}+1$ by hypothesis; that is, $\frac{(n-1)}{2}\rho > \Delta$; a contradiction.

Case 2 : $f(x,s) > \frac{n}{2}$. In this case, there must exist a node y such that $f(y,s) < \frac{n}{2}$. By considering the round from y (instead than from x), by Case 1 the contradiction occurs.

Case 3 : $f(x,s) = \frac{n}{2}$. If $s>2$ then there must exist a node y such that $f(y,s) < \frac{n}{2}$; by considering the round from y (instead than from x), by Case 1 the contradiction occurs.

Finally, let $s=2$. In this case, n is even, the round $r(x)$ is composed of two tours, and $f(x,1) = f(x,2) = \frac{n}{2}$. Note that the nodes charged in the second tour of $r(x)$ are the complement of the ones charged in the first tour.

Consider now the node y, next in the cycle, visited by the charger right after x. We have two cases (see Figure 3) depending on whether or not y needs to be recharged at this time. Case $(3a)$: y needs to be charged. In this case, consider the round $r(y)$ starting after charging y (see Figure 3 (3a), bottom). Round $r(y)$ must also be composed of two tours each containing exactly $\frac{n}{2}$ nodes in need of charge (i.e., we must have $f(y,1) = f(y,2) = \frac{n}{2}$), otherwise a contradiction would arise because of the previous reasoning applied to y. However, the nodes in need of charge in the first tour of $r(y)$ are the same as the ones in the first tour of $r(x)$ except for y itself; thus $f(y,1) = \frac{n}{2}-1$, a contradiction. Case $(3b)$: y does not need to be charged. In this case, consider the round $r(y)$ starting from y the last time it was charged before time $t_s(x)$. By definition, the nodes in need of charge in the first tour of $r(y)$ are the complement of the ones charged in the first tour of $r(x,1)$ excluding y, thus $f(y,1) = \frac{n}{2}-1$, a contradiction. $\square$

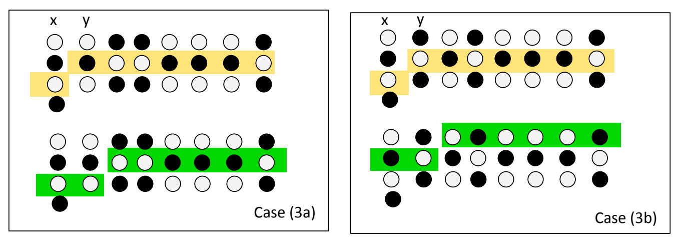

Fig. 3: Cases (3a) and (3b) of Theorem 3 in a stable system with $n=8$. Each row corresponds to a tour starting from x; black (white) circles represent sensors charged (not charged) in that tour. A round $r(x)$ consists of two consecutive tours starting after a black x. Highlighted in the top (resp. bottom) is the first tour of $r(x)$ (resp. $r(y)$) in the two cases.

By bringing the observations on weakness and stability together, we have:

Theorem 4. *Let the system be stable and let $n > \frac{2\Delta}{\rho}+1$. Then there exists a time t, such that, for all $t' > t$ we have*

$$m \leq Coverage(\texttt{LIC}, t') \leq m+1,$$

where $m = \lceil\frac{\Delta}{(\rho+d)}\rceil$; moreover, for all $x \in \mathcal{X}$

$$Disconnect(\texttt{LIC}, t', x) = (n-1)(\rho+d) + d - \Delta.$$

IV. Stability and Optimality

A. On the Stability of LIC

All the analytical results we have established on the effectiveness of LIC hold once the network has become stable under LIC.

It is not difficult to prove that all networks, regardless of their size, become stable under LIC when starting from specific initial battery levels. This is for example the case when all nodes start with a fully charged battery, or when initially all batteries are empty:

Theorem 5. *Let $\delta(x)$ denote the amount of time elapsing before the battery of node x reaches the threshold for the first time. If $\delta(x) = \delta(y)$ for all $x, y \in \mathcal{X}$, then the network become stable under* `LIC` *after one round.*

This is also true if the initial battery levels are different but increasing with respect to the cyclic order starting from the initial position of the robot:

Theorem 6. *Let initially the robot be at node $x_{\pi(i)}$. If $\delta(x_{\pi(j)}) \leq \delta(x_{\pi(j+1)})$ for all $i \leq j \leq i+n-1$, then the network becomes stable under* `LIC` *after one round.*

We conjecture that, in all networks with $n > \frac{2\Delta}{\rho} + 1$, stability under `LIC` is inevitably achieved; furthermore this occurs with a small number of rounds.

Conjecture. Let $n > \frac{2\Delta}{\rho} + 1$; then the system becomes stable within a constant number of rounds.

As we will see, this conjecture is supported by the strong experimental evidence presented in Section V.

B. `LIC` *versus* `OPTIMAL`

We are going to compare the effectiveness of `LIC` with that of the optimal on-line strategy `OPTIMAL`.

In `OPTIMAL`, each request message is sent by the sensor to the charger; the robot processes all the current request messages, and it computes which request to satisfy next so to minimize the number of sensing holes and their duration. We are actually going to consider the ideal cost settings for `OPTIMAL`: every request from every node reaches the robot directly, regardless of its current location; the robot can reach any node from any node in the same amount d of time, regardless of its distance; and the robot's processing time is negligible regardess of the complexity of the computation.

Notice that the behaviour of the robot under `OPTIMAL` in this setting is easy to describe: the robot just processes and services the request messages in the order they arrive; if two or more requests arrive at the same time, ties are arbitrarily broken (e.g., by Ids).

The effectiveness of `OPTIMAL` is also simple to derive for most networks:

Theorem 7. *If $n > (\Delta + \rho)/(\rho + d)$ then, under the* `OPTIMAL` *strategy, there exists a time t such that, for all $t' > t$ and all $x \in \mathcal{X}$ we have:*

$$\lceil \frac{\Delta}{(\rho + d)} \rceil \leq Coverage(\texttt{OPTIMAL}, t') \leq \lceil \frac{\Delta}{(\rho + d)} \rceil + 1;$$

$$Disconnect(\texttt{OPTIMAL}, t', x) = (n-1)(\rho + d) + d - \Delta.$$

This theorem has an immediate very strong consequence for the effectiveness of `LIC`:

Theorem 8. *Let the system become stable under* `LIC`*. If $n > \frac{2\Delta}{\rho} + 1$ then there exists a time t such that for all $t' > t$ and all $x \in \mathcal{X}$*

$$Coverage(\texttt{LIC}, t') = Coverage(\texttt{OPTIMAL}, t'),$$

$$Disconnect(\texttt{LIC}, t', x) = Disconnect(\texttt{OPTIMAL}, t', x)$$

Proof. Since $\frac{2\Delta}{\rho} + 1 > \frac{\Delta + \rho}{\rho + d}$, the claim follows directly from Theorems 4 and 7. □

In other words, for all networks with $n > \frac{2\Delta}{\rho} + 1$, the recharging strategy `LIC`, with its low communication and computations costs, performs as well as the optimal stategy.

V. Experimental Analysis

The results of the previous Section describe the behaviour of the system once it stabilizes in time. We run extensive simulation to determine the stability of the system under `LIS` and to observe its behaviour in terms of Coverage Size and Disconnection Time (already studied theoretically).

A. Simulation Environment

The experiments were implemented in the simulator discrete-event MAS toolkit MASON [14].

The variable parameters involved in the experiments are: the number of nodes n, the battery life Δ, the charging time ρ, and the travel time d from a node to the next.

The following table shows the values considered for each parameter, where the temporal values are all in the same scale:

Parameters	*Values*
Number of nodes n	100, 200, 300, 400, 500
Battery Lifetime Δ	2000, 3000, 4000
Threshold τ	30% of Δ
Charging Time ρ	1, 10, 20, 30, 40, 50
Travel Time d	1, 5, 10, 20, 30

Each sensor x has initially an amount of energy level chosen uniformly at random in the range $[\tau, \Delta]$; the charger's initial placement is at a node chosen uniformly at random.

For each combination of the values of the parameters, we have executed 20 executions and computed the average coverage size and disconnection time. To detect stability we have also maintained the information about the charging order of the nodes under `LIC`.

B. Simulation Results

Starting with arbitrary initial charge levels, the experimental results show that, for all values of the parameters, the charging order becomes periodic and the system stable; moreover stabilization occurs within two rounds. In other words, *under* `LIC` *the networks become stable within two rounds*. See Figure 4 where stability is shown for some choices of the parameters; the results for all the other parameters' combinations are consistent with these. Note that, for smaller capacities, as well as for larger networks, stabilization occurs even sooner, within one round.

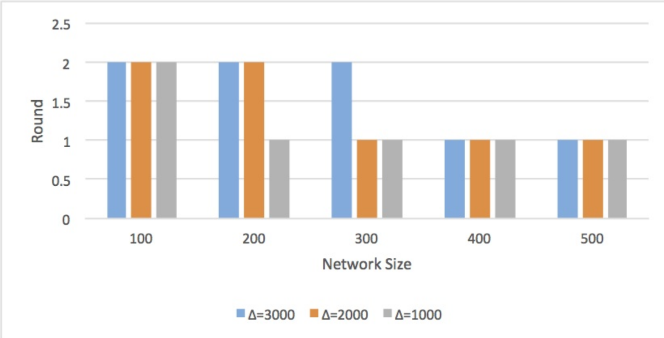

Fig. 4: Stability of the system varying n for $\Delta = 1000, 2000, 3000$, $\rho = 20$, $d = 1$.

Once the system stabilizes, the theoretical bounds on coverage size and disconnection time established analytically (Theorem 4) hold; indeed, the simulation results confirm all these bounds.

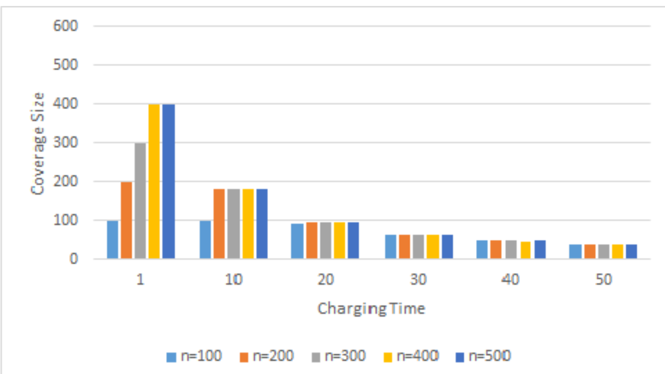

Fig. 5: Effect of Charging Time ρ on Coverage (Δ=2000, d=1)

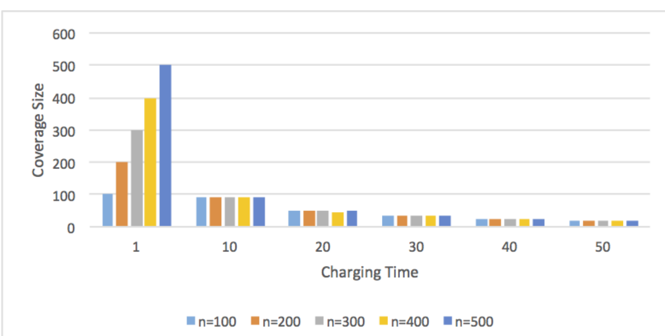

Fig. 6: Effect of Charging Time ρ on Coverage (Δ=1000, d=1)

For example, Figure 5 and Figure 6 show how the coverage size changes at the varying of the charging time ρ and the network size n, showing that the average coverage size coincides with the theoretical value derived in Theorem 4, even for n smaller that $\frac{2\Delta}{\rho}+1$. The figures correspond to two different choices of Δ for the same ρ; the same phenomenon is observed for the other values of the parameters.

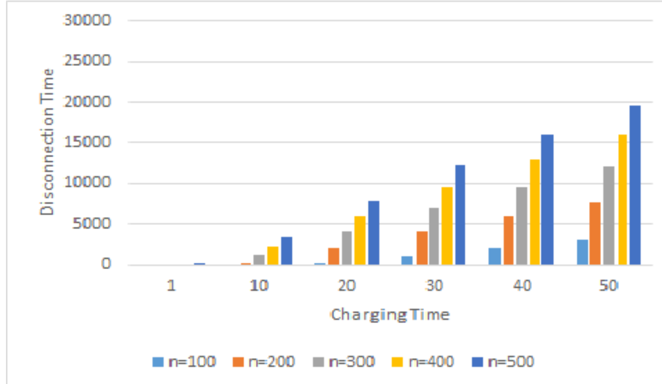

Fig. 7: Effect of Charging Time ρ on Disconnection time (Δ=2000, d=1)

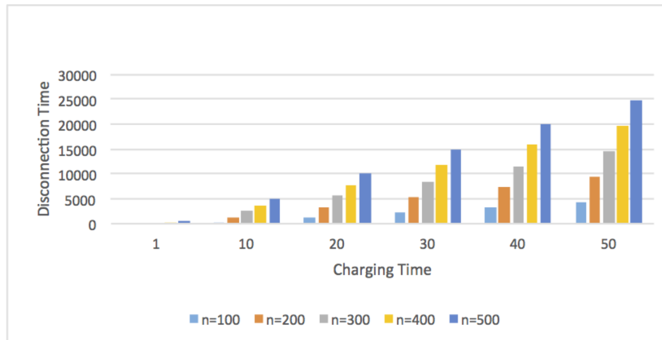

Fig. 8: Effect of Charging Time ρ on Disconnection time (Δ=1000, d=1)

Figure 7 and Figure 8 show how disconnection changes varying the charging time ρ and the network size n. Also these experiments confirm the theoretical bound established in Theorem 4 even for n smaller than $\frac{2\Delta}{\rho}+1$. As for the case of coverage size, the figures correspond to two different choices of Δ ($\Delta = 1000, 2000$) maintaining $\rho = 20$; the same phenomenon is observed for the other values of the parameters.

C. Comparison with `OPTIMAL`

We now turn to the comparison between `LIC` and `OPTIMAL`. We already established that, when $n > \frac{2\Delta}{\rho}+1$, the two strategies are equivalent both in terms of coverage size and disconnection time (Theorem 8). Extensive experimental results, varying Δ, ρ, d, and n, confirm the theoretical findings.

For example, Figures 9 and 10 show coverage and disconnection time of the two strategies for different network sizes when $\Delta = 2000$, $\rho = 20$ and $d = 1$. Notice that Theorem 8 does not hold when $n < \frac{2\Delta}{\rho}+1$; in fact, as shown in Figure 9, `OPTIMAL` has a much better coverage than `LIC` for the case $n = 100$.

VI. Concluding Remarks

In this paper, we introduced the notion of effectiveness of energy restoration strategies. We proposed a very simple decentralized battery recharging strategy, which, in spite of its simplicity and of the use of very limited resources, achieves optimal effectiveness in most cases. The technique is based

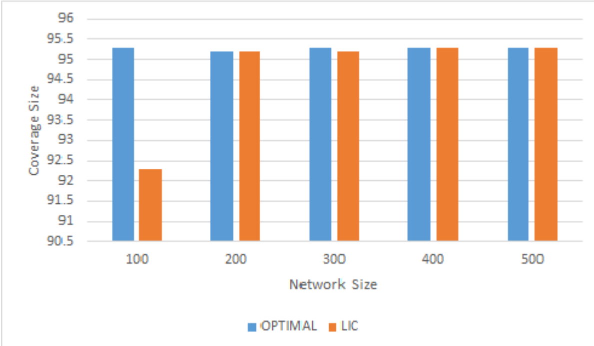

Fig. 9: LIC vs OPTIMAL: Coverage. ($\Delta = 2000$, $\rho = 20$, $d = 1$).

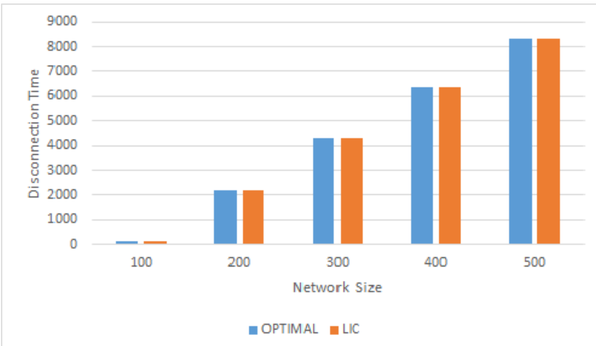

Fig. 10: LIC vs OPTIMAL: Disconnection time. ($\Delta = 2000$, $\rho = 20$, $d = 1$).

on the on-demand visit of the sensors by a mobile robot in a predefined circular order only when aware of a pending request. The optimality of the strategy is proven for sufficiently large networks ($n > \frac{2\Delta}{\rho} + 1$). It would be interesting to consider also the case of smaller n, where our strategy is not optimal; the detailed analysis of the charging dynamics for that case will be the object of future study.

Our studies, both analytical and experimental, have been carried out in an abstract setting, with several simplifying assumptions. Among them, we assumed that the time necessary for the robot to move from a node to its successor in the cyclic order is uniform. We did run experiments with variable distances between sensors; all these experiments do not show any significant difference with the results obtained in the paper with uniform distances; the theoretical validation is however left for future work.

Another assumption that would be interesting to lift is the one of constant charging rate ρ, the same for all sensors. The case of variable charging rates, possibly depending on the current battery level, as well as other physical factors (e.g., battery capacity decay) are important open research directions.

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