Election in Arbitrary Networks

Mega-Merger

Yo-Yo

Some Considerations

Paola Flocchini

Election in Arbitrary Networks

(Gallager, Humblet, Spira ‘84)

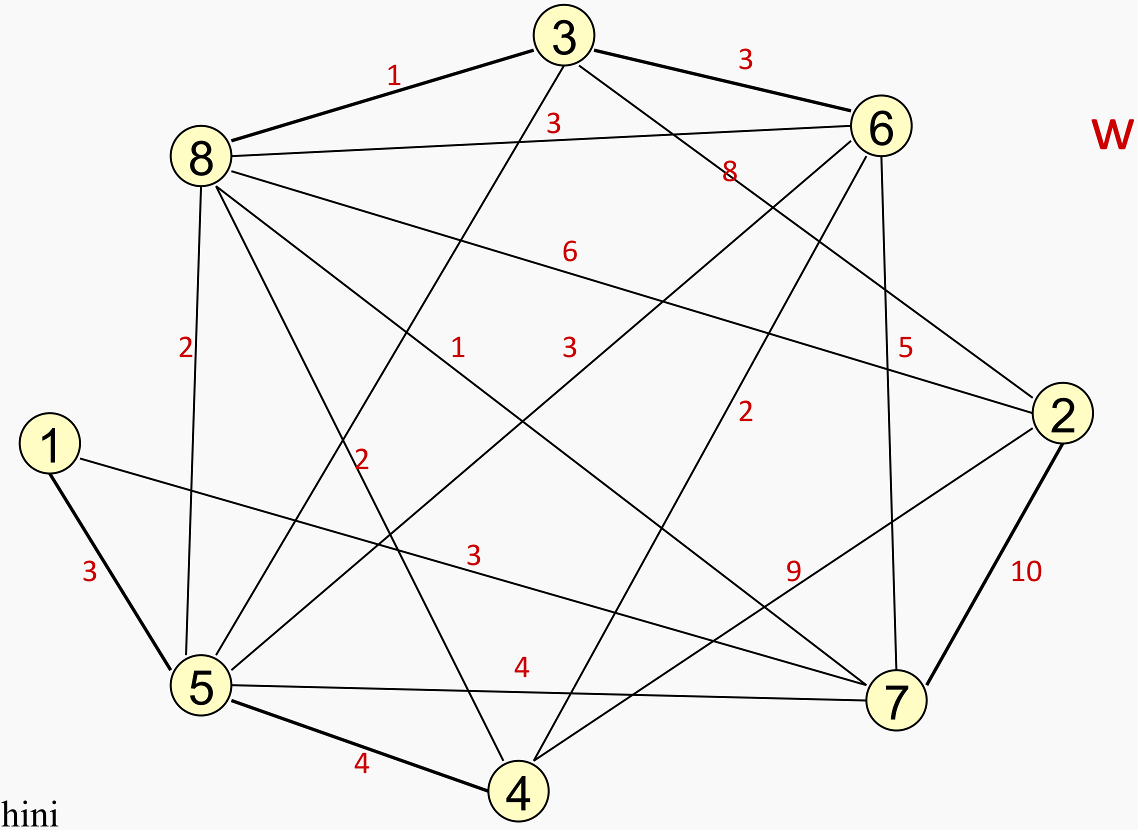
The Mega-Merger

In general networks, the election problem and the spanning tree construction problem are equivalent.

The Mega-Merger

Minimum spanning tree construction algorithm.

The root of the spanning tree is the leader

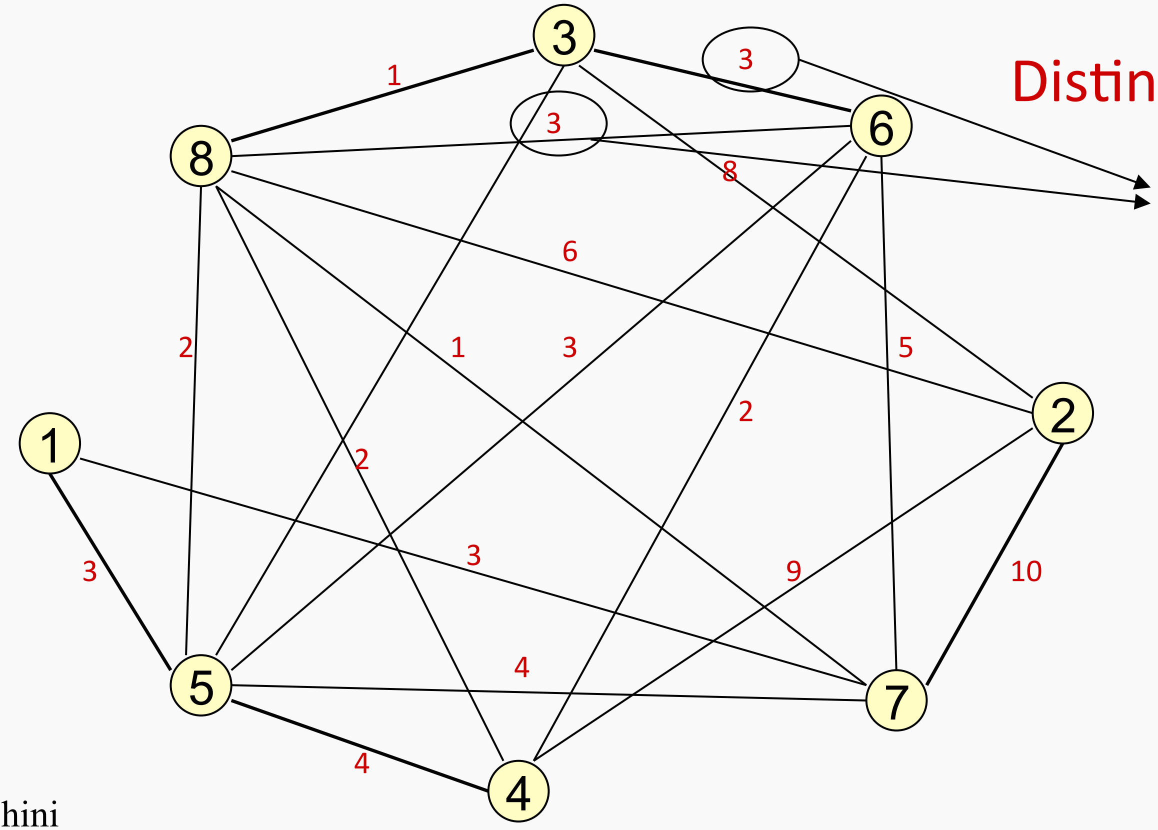


weights

The Mega-Merger

Minimum spanning tree construction algorithm.

The root of the spanning tree is the leader

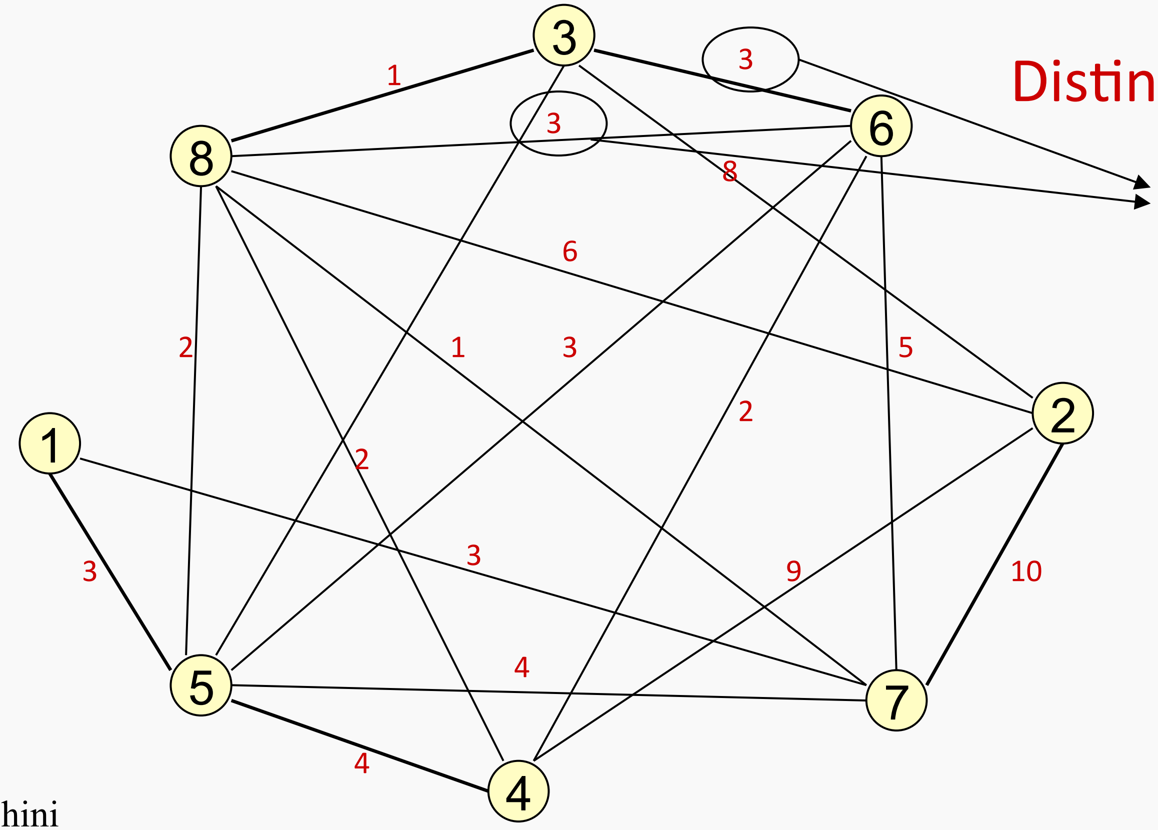


Distinct weights

The Mega-Merger

Minimum spanning tree construction algorithm.

The root of the spanning tree is the leader



Distinct weights

(3,6)

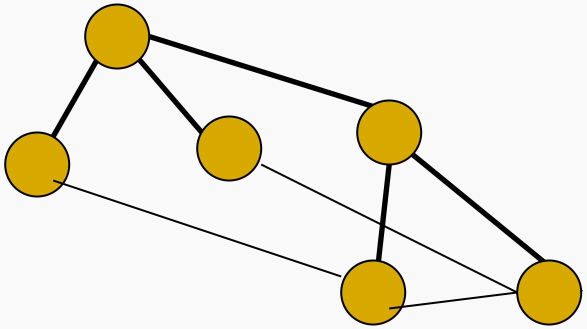
(8,6)

Equal weights can be made distinct

So, from now on we assume that

edges have distinct weights

and this is not a restriction since even if they are not distinct to start with, they can be made different.



node = village with a name
edge = road with a distance

names and distances are different

The goal

to merge all the villages into one mega-city

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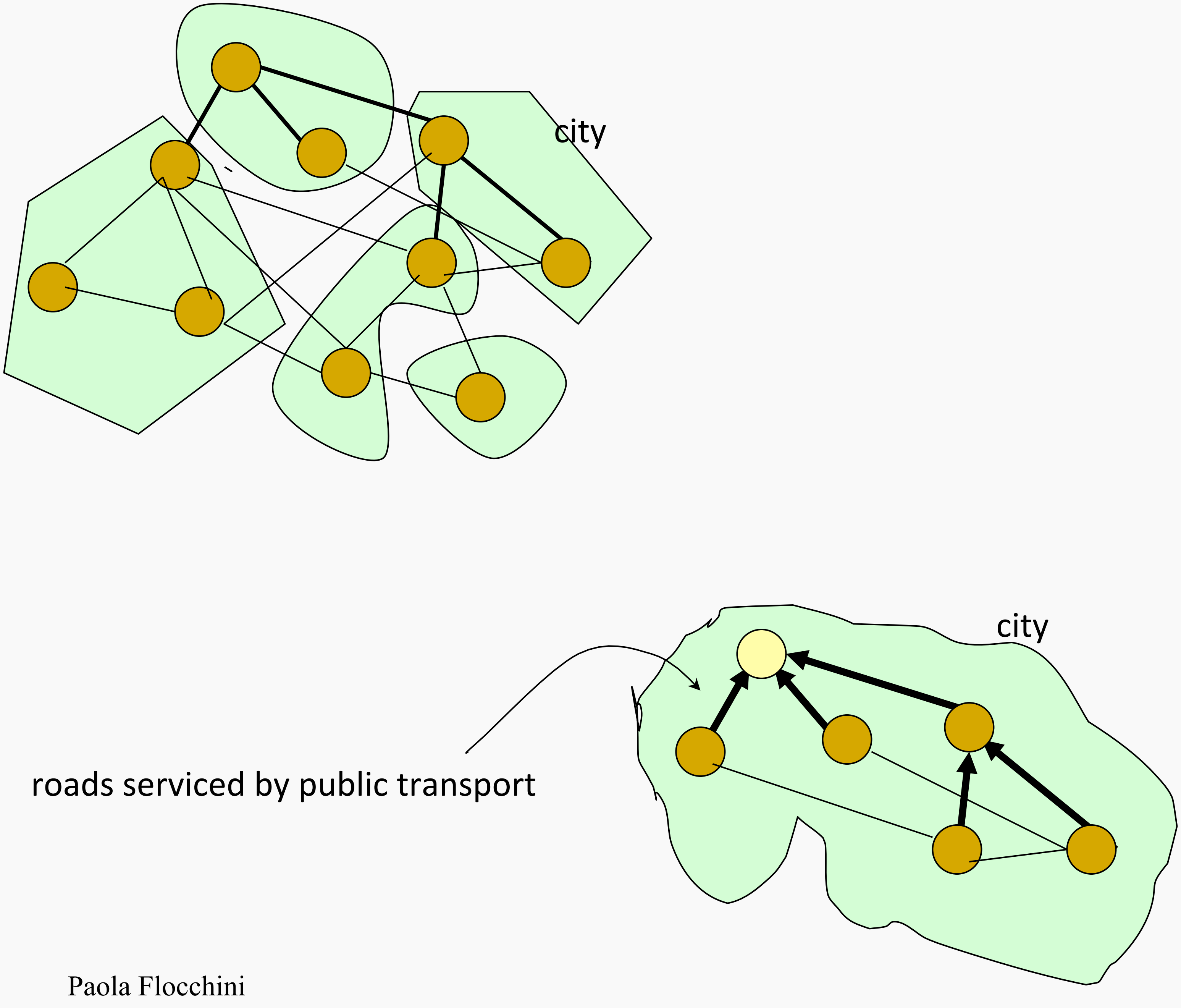
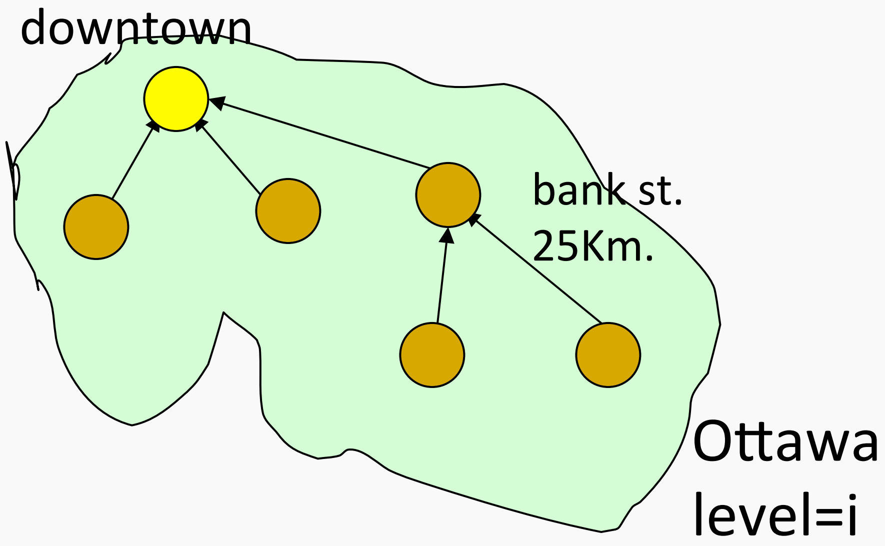


city

city

roads serviced by public transport

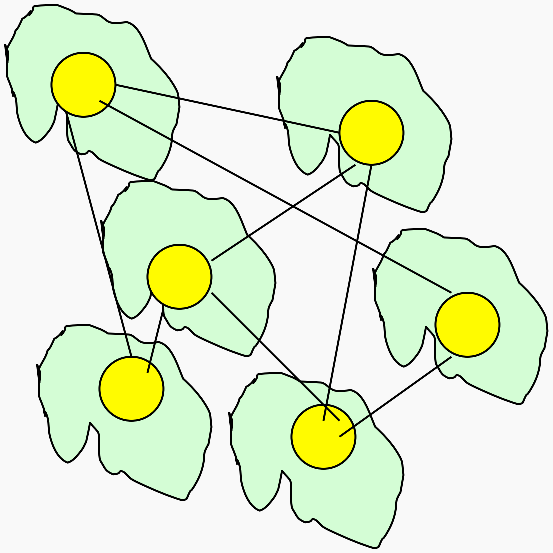
1) City is a subgraph,
its spanning tree
has public transportation,
root is downtown



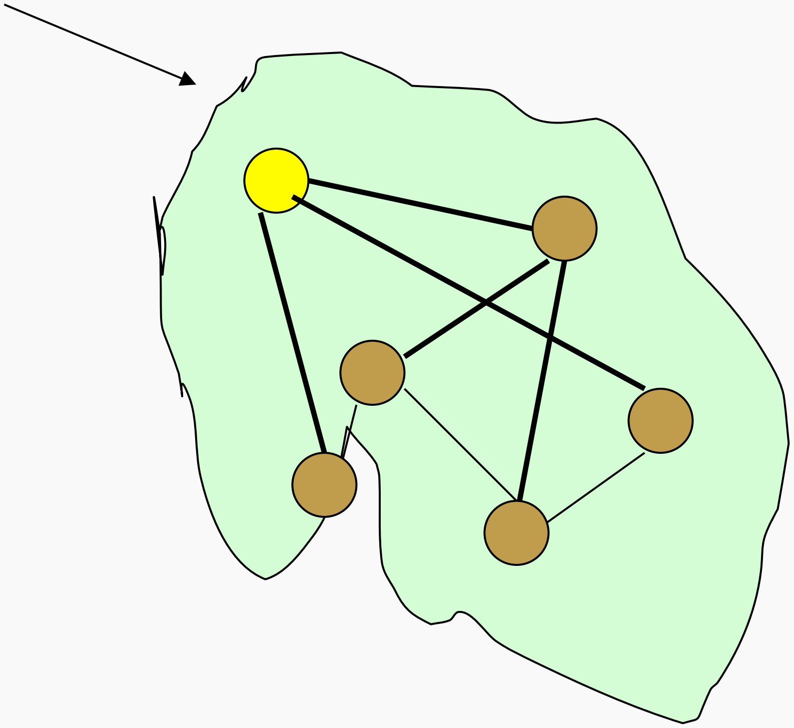
2) a city has a unique **name** and has a **level**
all districts eventually know the name of the city

3) Edges are roads with a distinct **name** and **distance**

4) **Initially**: each node is a city with just one district and no roads. All cities are at the same level (level 1).



Cities are merged into bigger cities until only ONE city is left.



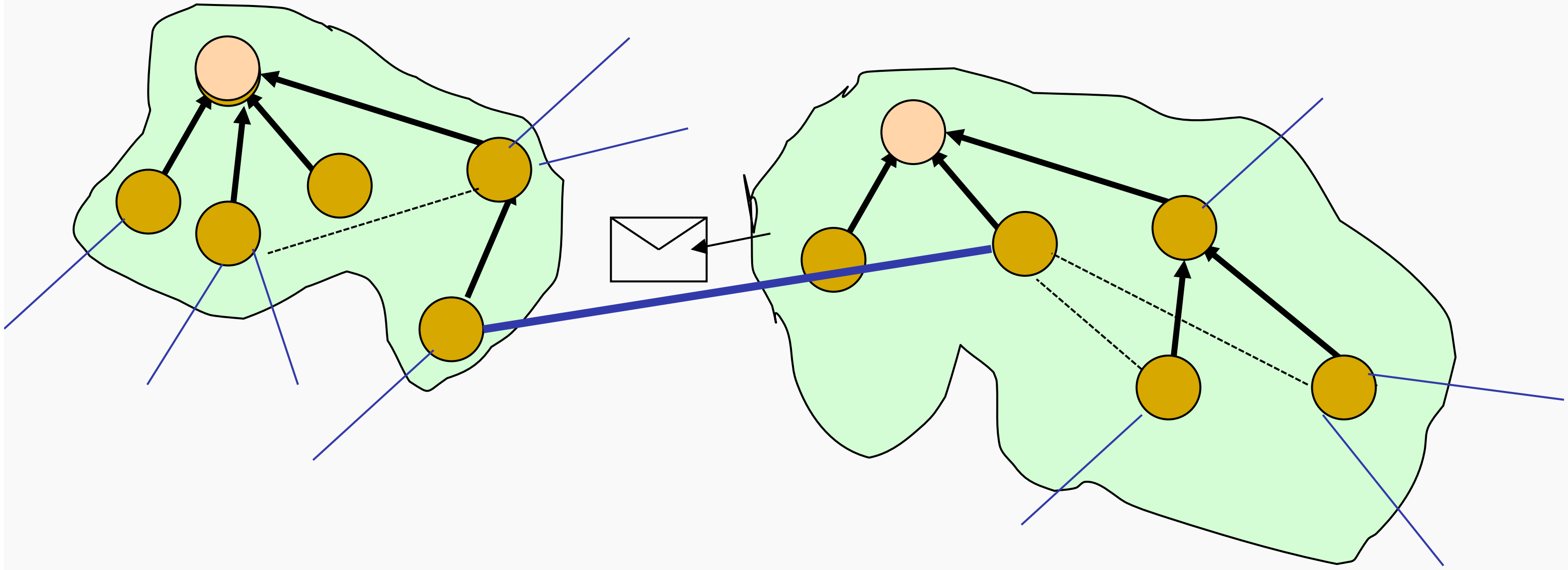
Issues to consider when merging two cities:

How to name the new city

will depend on several factors

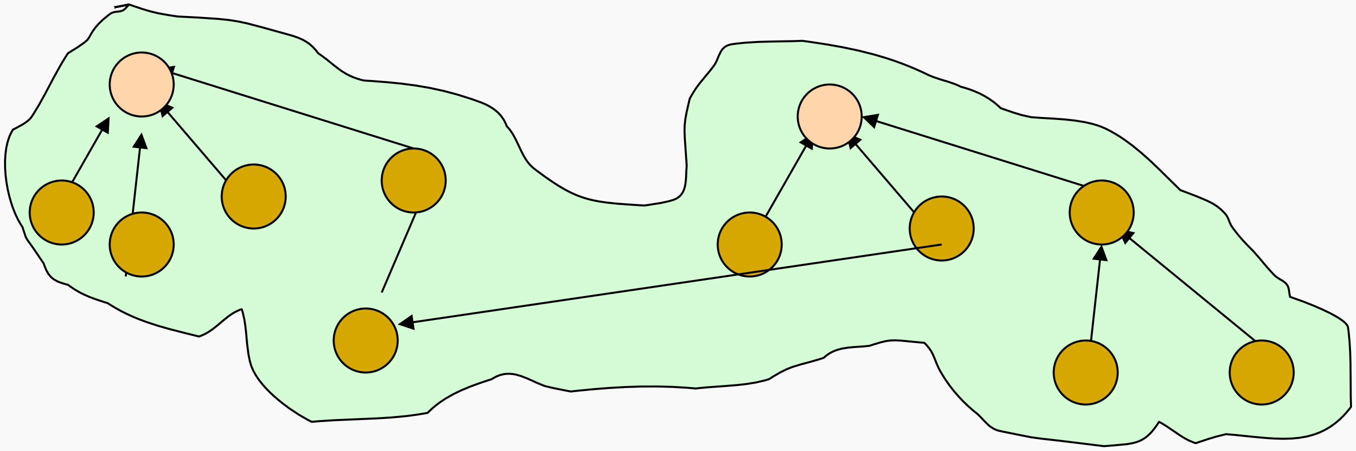
which roads of a city will be serviced by public transportation

[the roads serviced in the two cities plus a connecting road]



5) A city must merge with the closest neighboring city. To request a merge, it sends a *let-us-merge* message on the shortest road connecting the cities

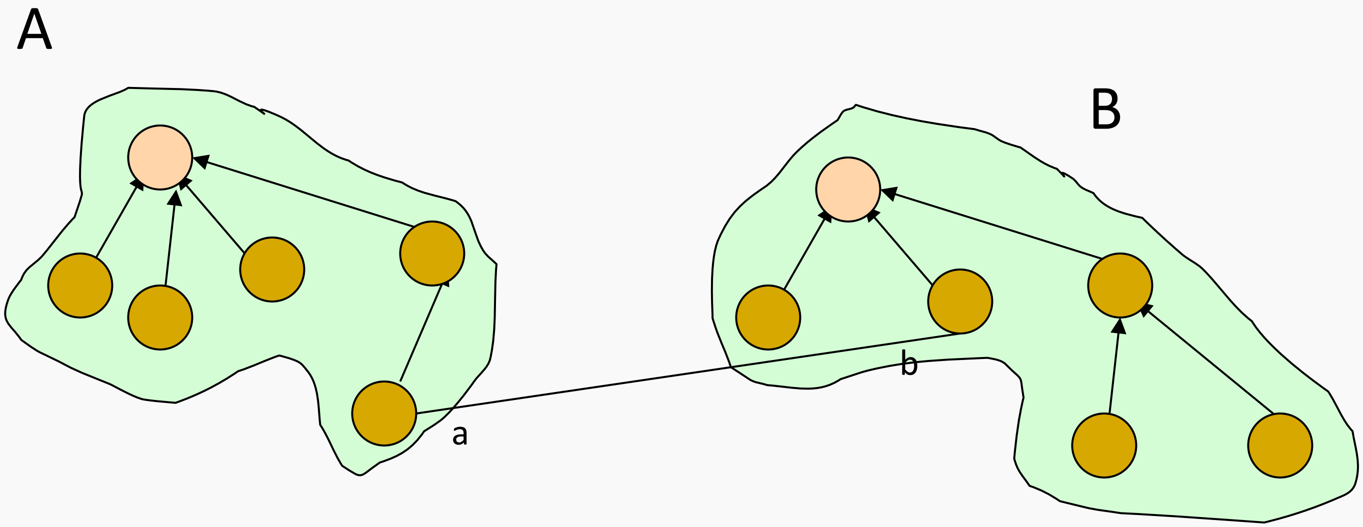
6)The decision to request a merge must come from downtown. There cannot be more than one request at a time



7) When the merge occurs, the roads of the new city serviced by buses will be the road of the two cities + the connecting road.
The new downtown will depend on several factors.

Note1: the level of a city is not equal to the numbers of nodes.
We will see later how the level changes.

A: city
D(A): downtown
level(A): level of city A
e(A) = (a,b): merge link with closest city (let it be B)



Note2: when the level of a city changes, it is communicated to all the nodes in the city

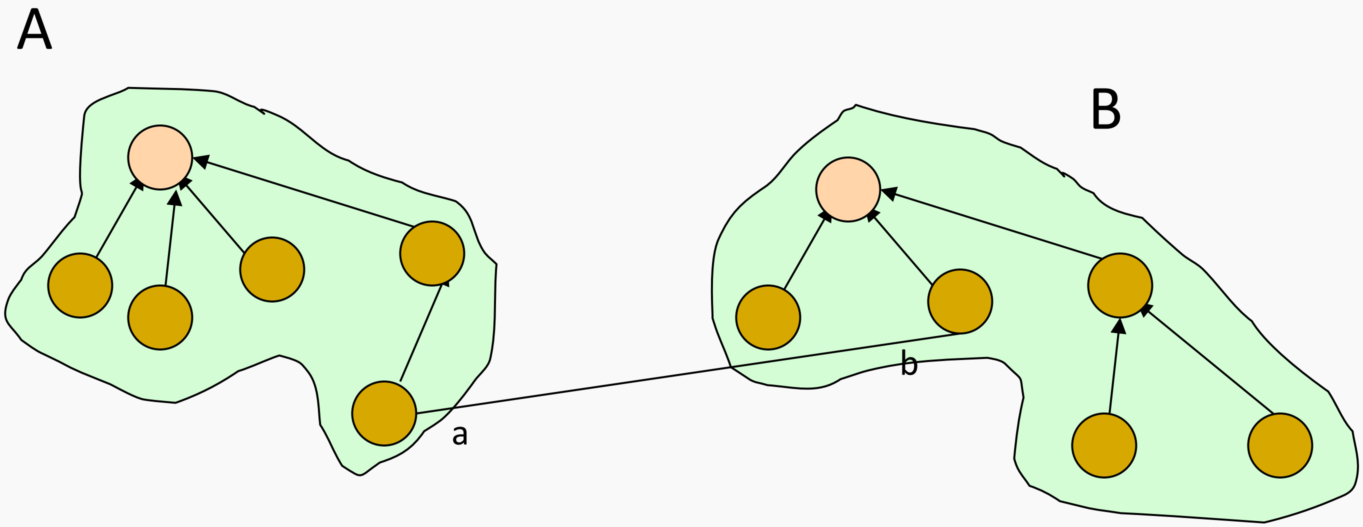
Important
Differences with respect to Algorithm Complete

A: city

D(A): downtown

level(A): level of city A

e(A) = (a,b): merge link with closest city (let it be B)



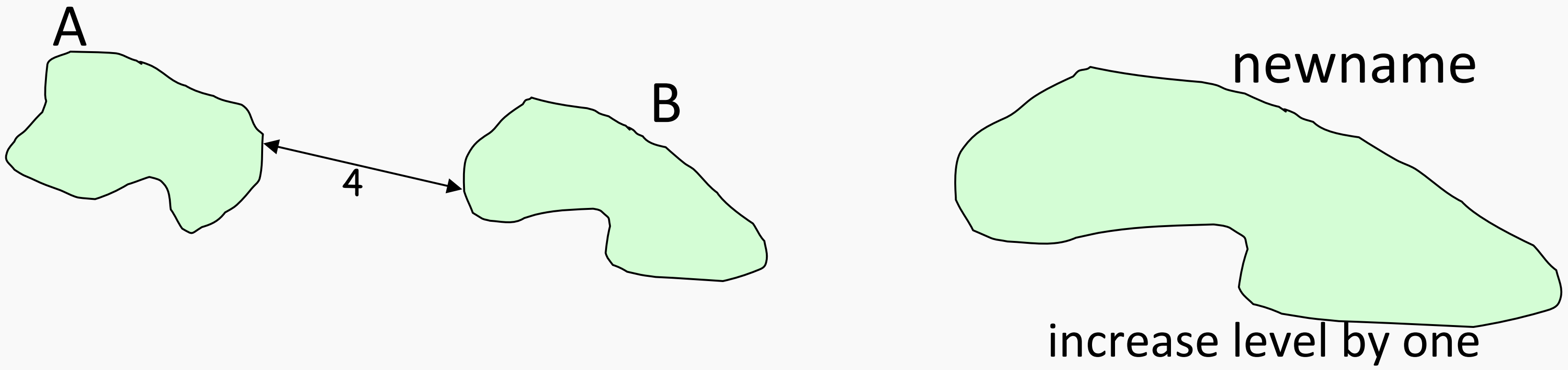
When the request arrives:

- the two cities have the same level
- the two cities have different levels

Let A send the *let-us-merge* message to B

8) If level(A) = level(B) AND the link chosen by A is the same as the one chosen by B (e(A)=e(B)), then:

friendly merger



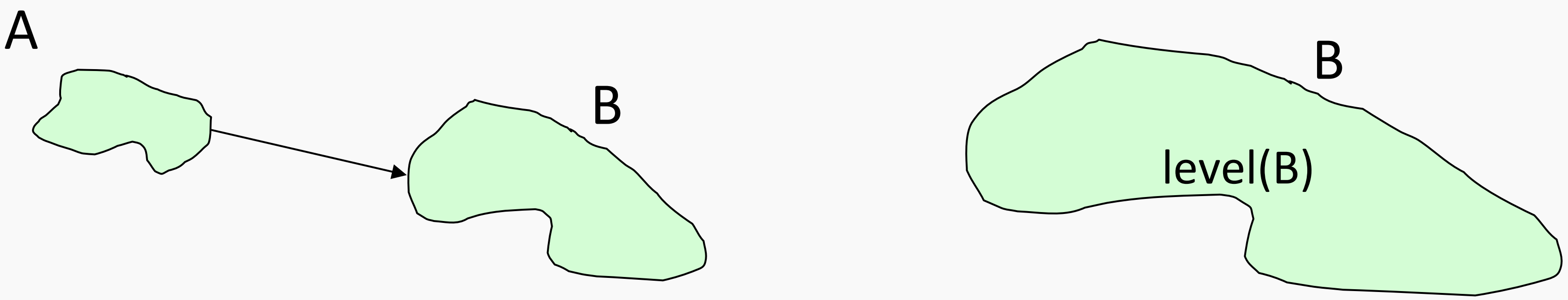
newname

increase level by one

Ex:
4 is the smallest out-going edge of A
and
the smallest out-going edge of B

Let A send the *let-us-merge* message to B

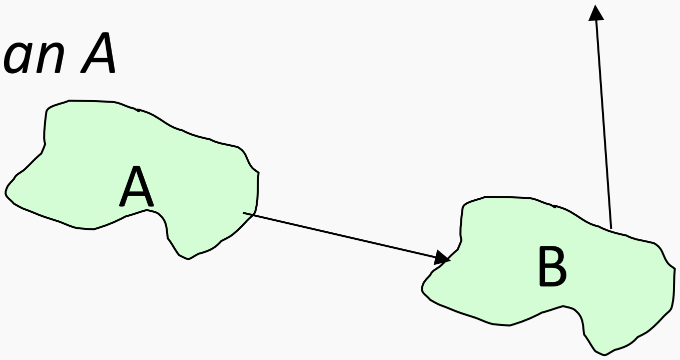
9) If level(A) < level(B)   A is absorbed in B



In the other cases the decision is postponed

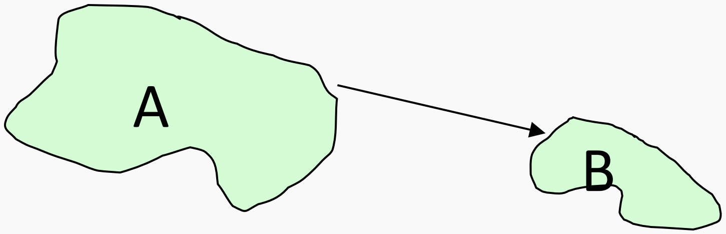
10) If level(A) = level(B) BUT e(A) ≠ e(B), then:

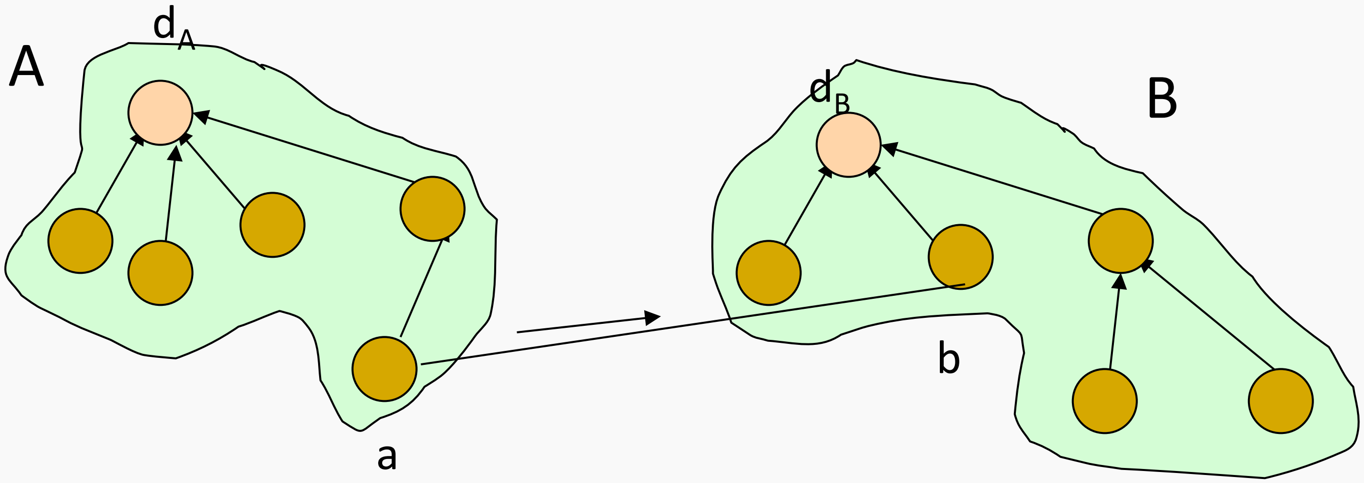
the merge is suspended until B
arrives at a level GREATER than A



11) If level(A) > level(B)   then:

the merge is suspended until B
arrives at the same level as A





Absorption (rule 9)

level(A) < level(B)

A will be absorbed by B

b notifies *a* about the absorption (putting B's name in the message)

a broadcast the info in A

flip all logical link direction to point to the new downtown

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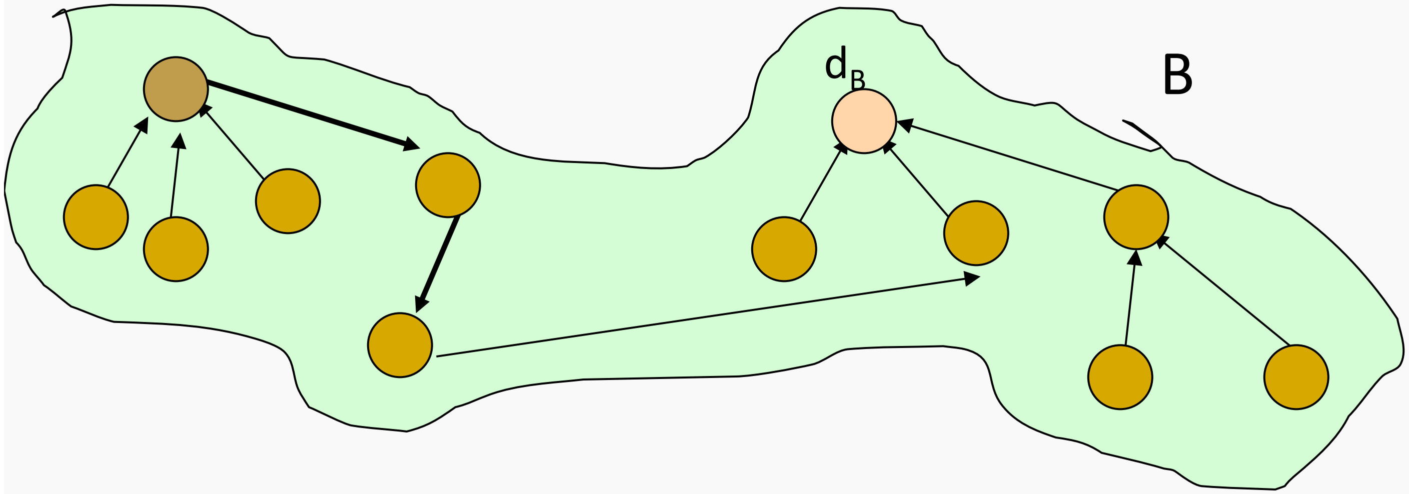

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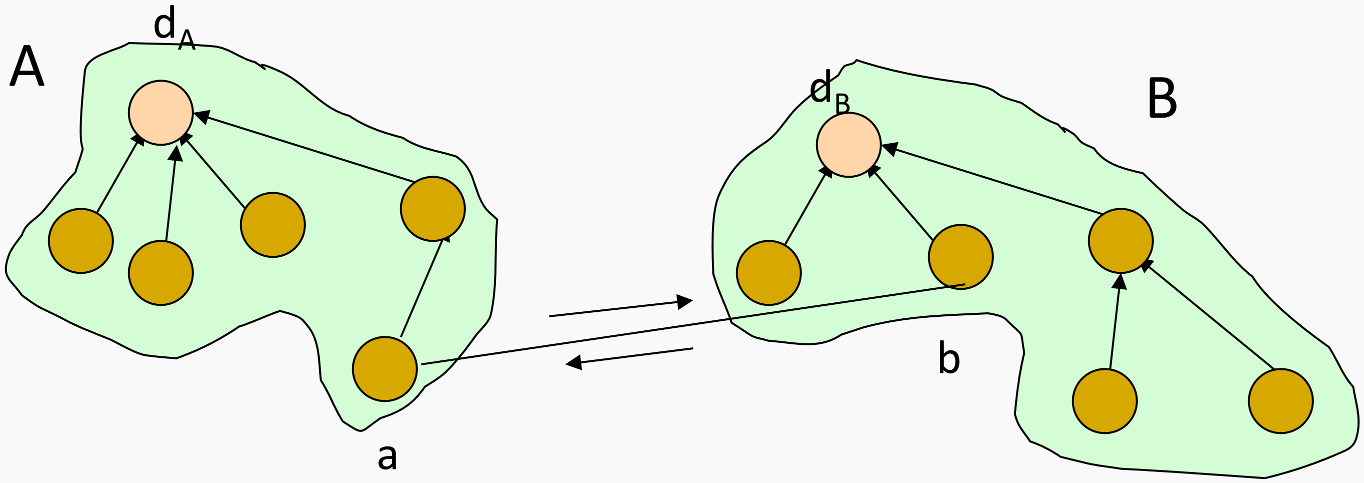
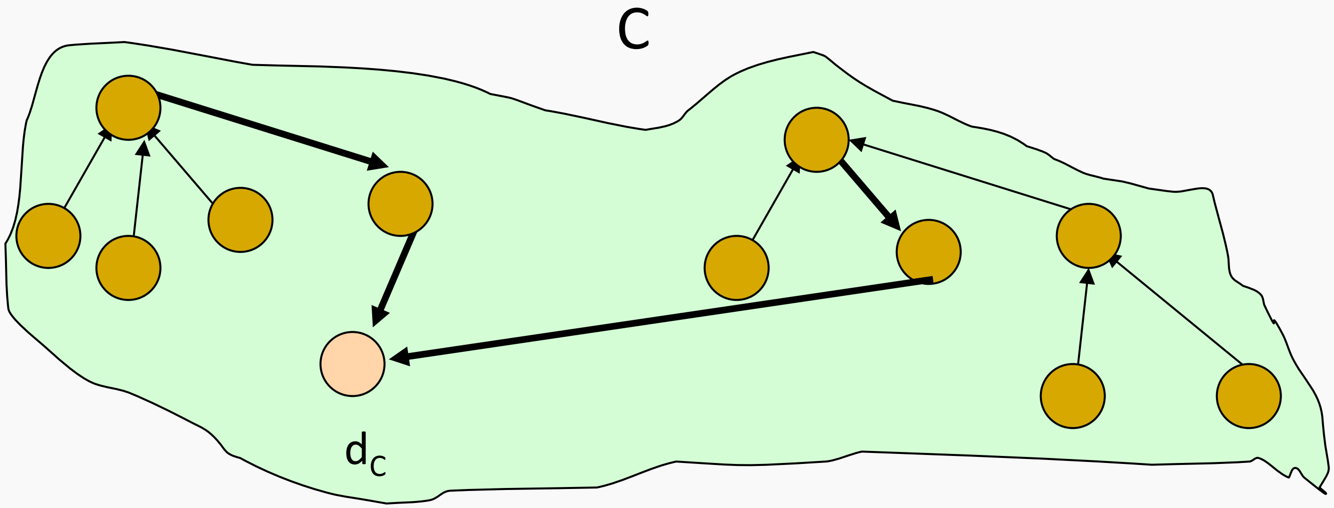


Friendly Merger (rule 8)

level(A) = level(B)
e(a)= e(B)

new downtown, new name, new level
downtown = min{*a,b*}
newlevel = oldlevel + 1
new name = name of the road connecting *a* and *b*



let a < b

Friendly Merger (rule 8)

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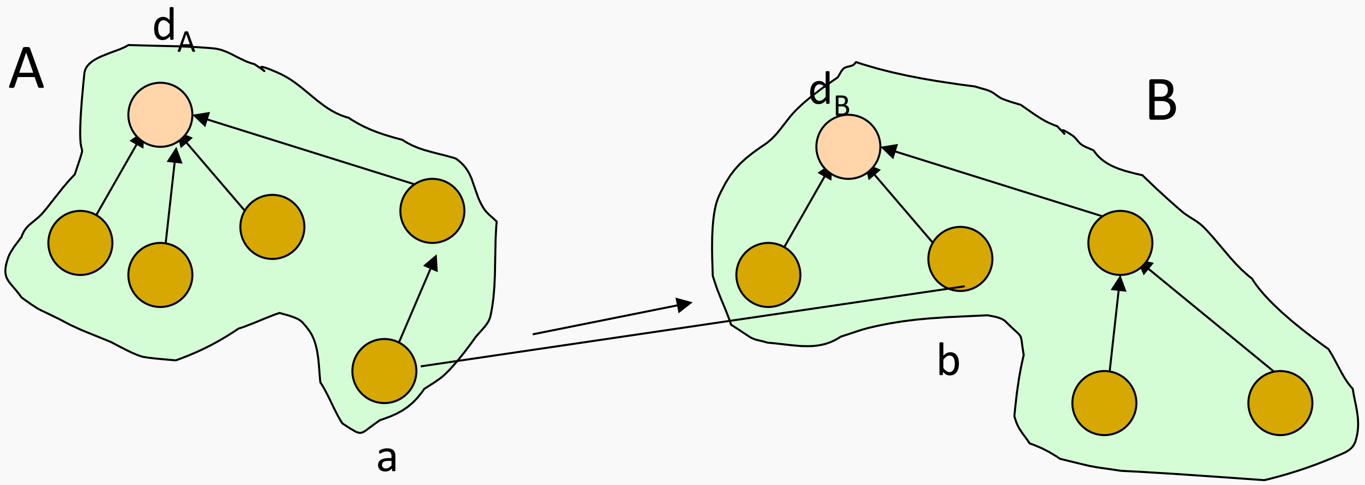
e(a)= e(B)

new downtown, new name, new level
downtown = min{*a,b*}
newlevel = oldlevel + 1
new name = name of the road connecting *a* and *b*

a and *b* compute the new info independently and broadcast
the appropriate links are flipped



Suspension (rules 10,11)

level(A) = level(B) BUT e(A) ≠ e(B)
or
level(A) > level(B)

b locally keeps the necessary info for later

NOTICE: nobody in A knows about the suspension
no other request can be launched from A

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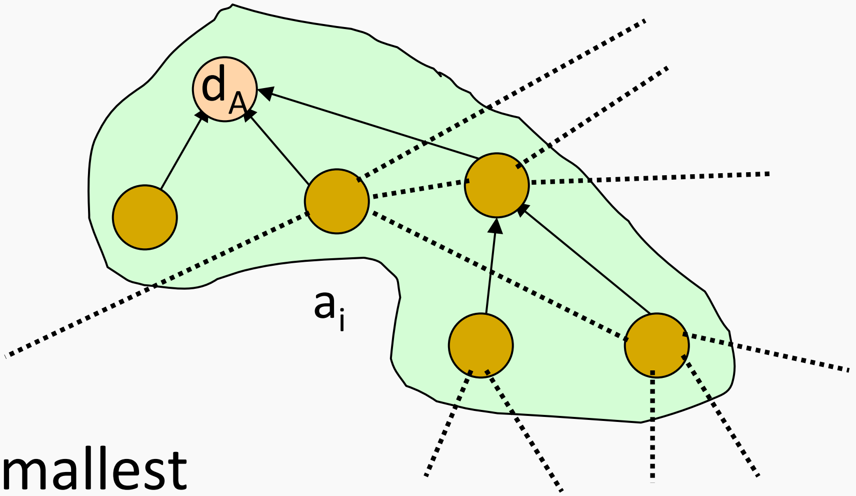
Choosing the merging link

d_A needs to find the min length among all edges exiting the city

5.1) each district a_i of A determines d_i of the shortest road going to another city (if none, $d_i = \infty$)



5.2) d_A finds the smallest (min in a rooted tree)

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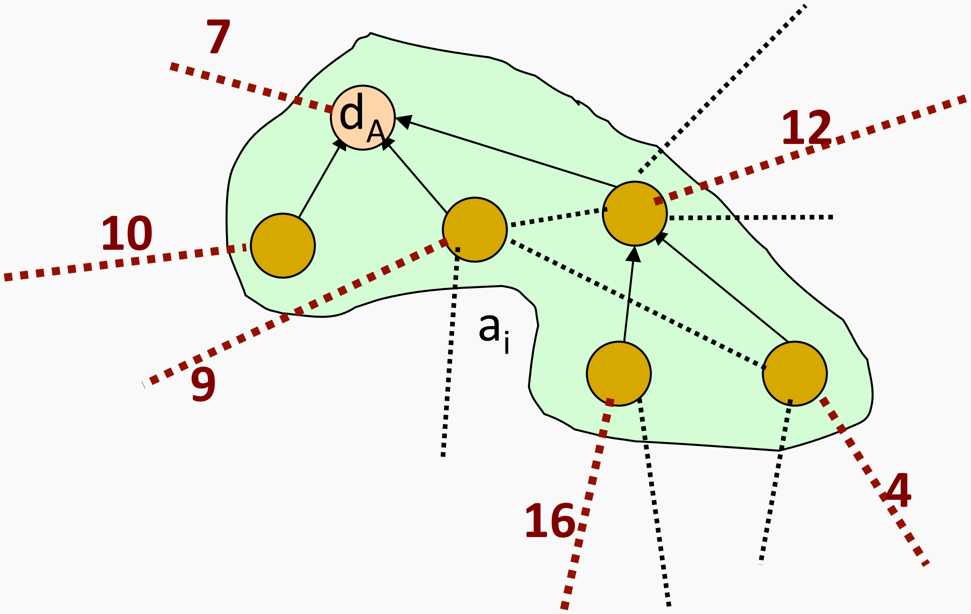
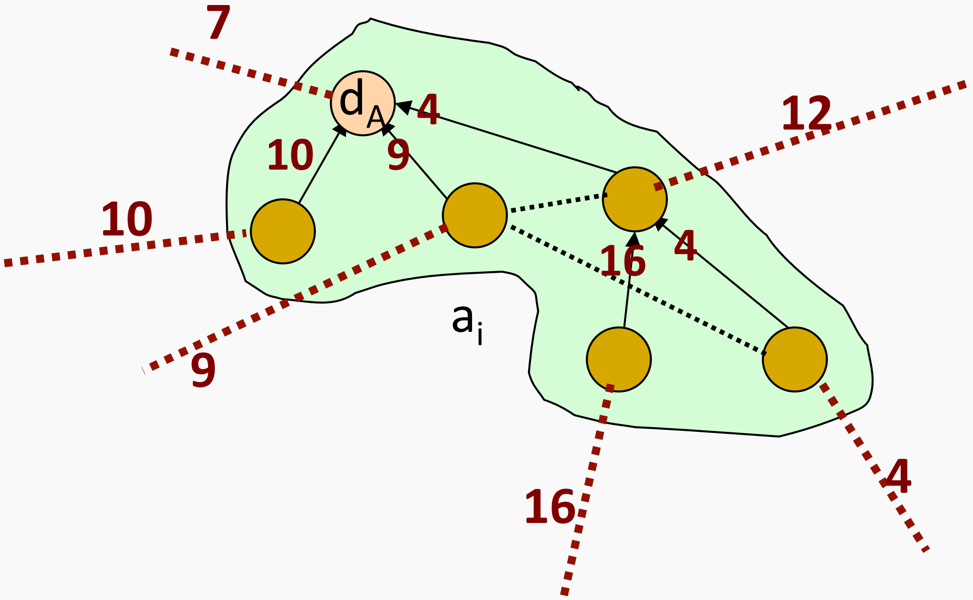


Choosing the merging link

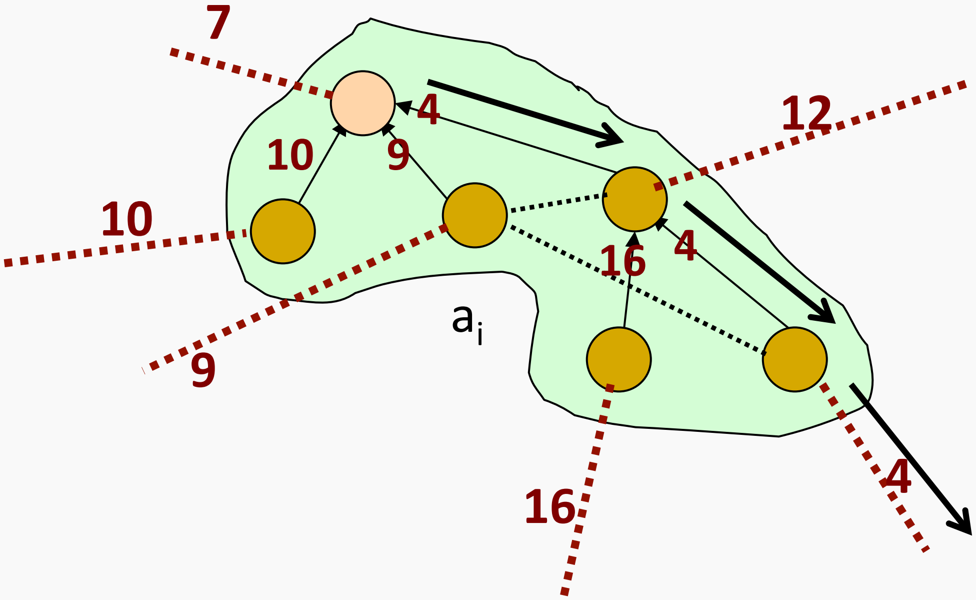
5.2) d_A finds the smallest (min in a rooted tree)

“4” is the minimum



Choosing the merging link

The request will exit the city from this link

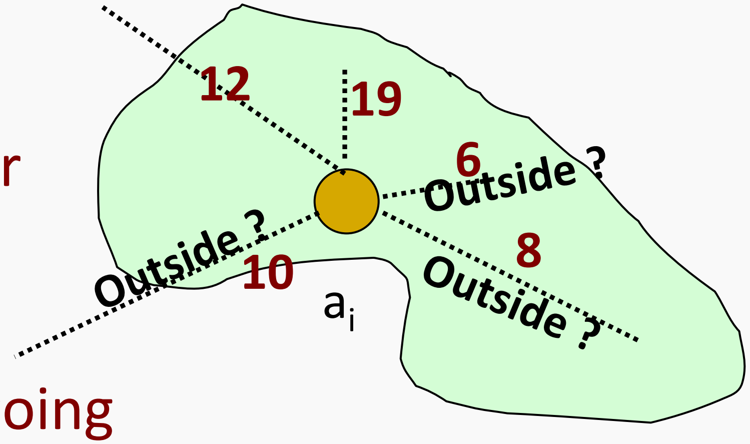


How to locally compute the shortest road going from yourself to another city ...

Note that a node cannot locally distinguish between internal and external incident links (except for tree links)

Ask one link at a time, from smallest to biggest

I stop asking as soon as
I receive a positive answer



This is my minimum outgoing link at this moment

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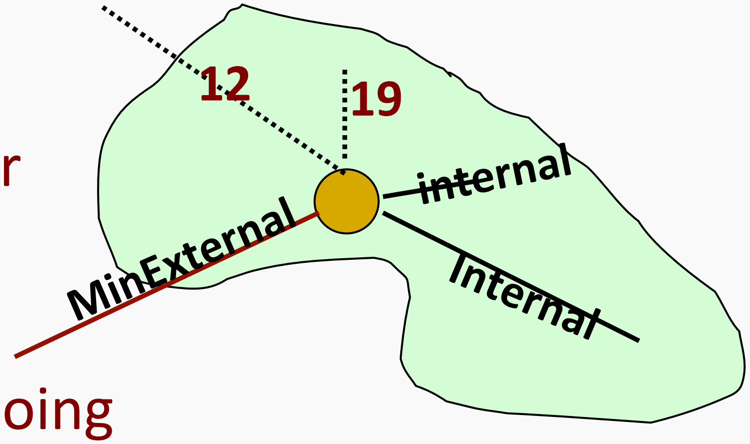
NOTE that I will never re-ask those links in the future.
The internal will always be internal.

I stop asking as soon as
I receive a positive answer



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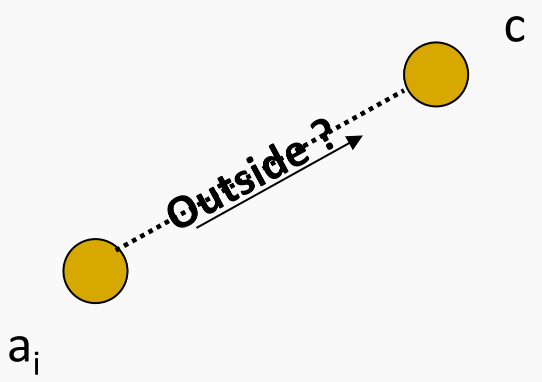
How to reply when receiving the request ...



if name(A) = name(C)

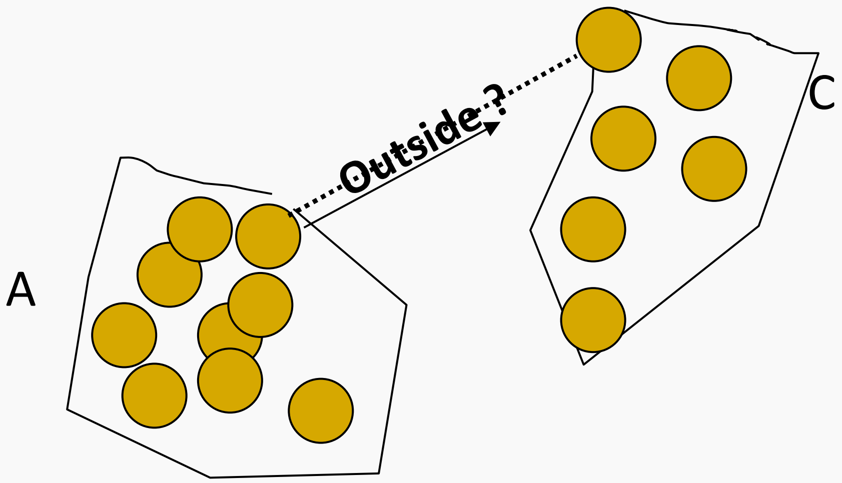
reply (internal)

if name(A) ≠ name(C)

the road is not necessarily external

maybe C has been absorbed by A and c does not know

BUT I DON'T KNOW YET



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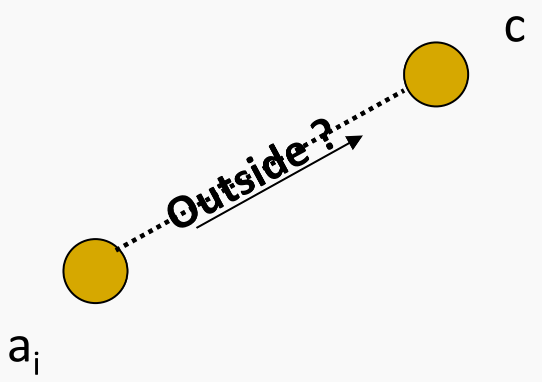

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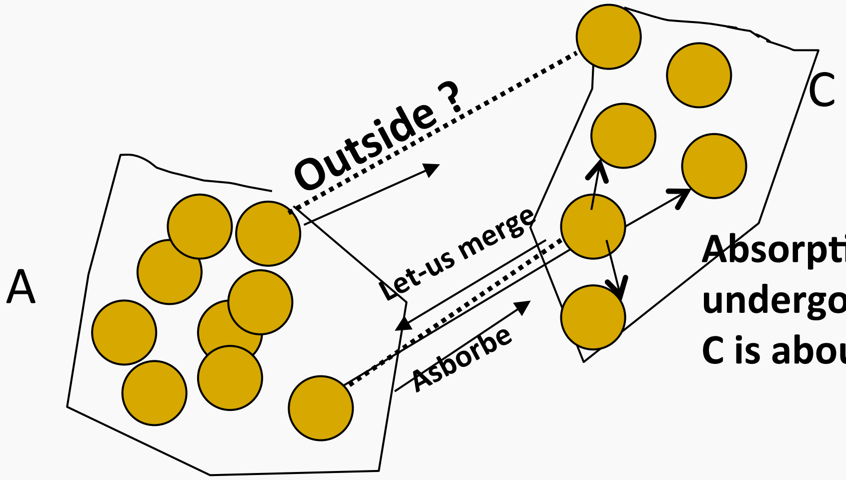
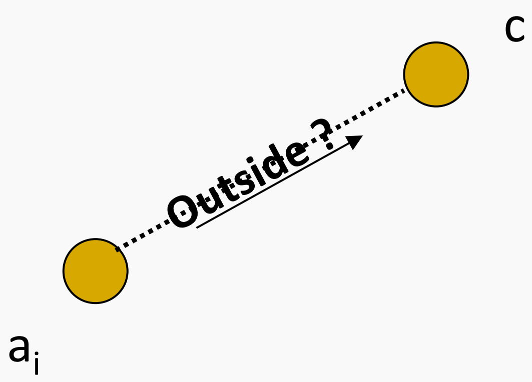


Absorption undergoing:
C is about to become part of A

How to reply when receiving the request ...



if name(A) = name(C)

reply (internal)

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the road is not necessarily external

maybe C has been absorbed by A and c does not know

but in such a case level(C) < level(A)

Absorption undergoing:
C is about to become part of A

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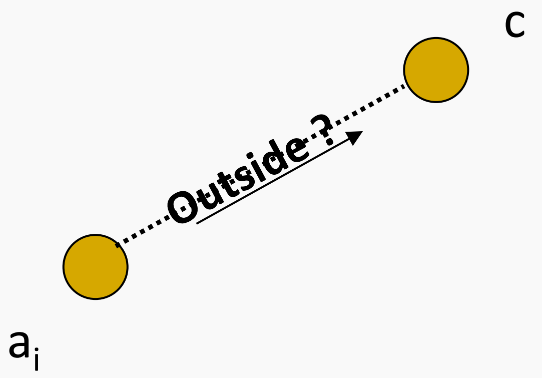
How to reply when receiving the request ...



if name(A) = name(C)

reply (internal)

if name(A) ≠ name(C)

the road is not necessarily external

maybe C has been absorbed by A and c does not know

but in such a case level(C) < level(A)

so:

If **name(A) ≠ name(C)**
and **level(C) ≥ level(A)** then

reply(external)

If **name(A) ≠ name(C)**
and **level(C) < level(A)** then

don't reply

More Details Discovering a friendly merger

level(A) = level(B) and e(A) = e(B)

To decide, b needs to know e(A) and e(B)

How does b know e(B) ?

e(B) is chosen by D(B), which will send the request through b

When receiving the request, b will know

So,

If e(A) =e(B), b will eventually know

If e(A) ≠ e(B), b is not the exit point, it will never know what e(B) is.

More Details Discovering a friendly merger

level(A) = level(B) and e(A) = e(B)

Receiving a let-us-merge:

If b has already received a let-us-merge from D(B) to be sent to a

both b and a will know that this is a friendly merger

Otherwise

b waits

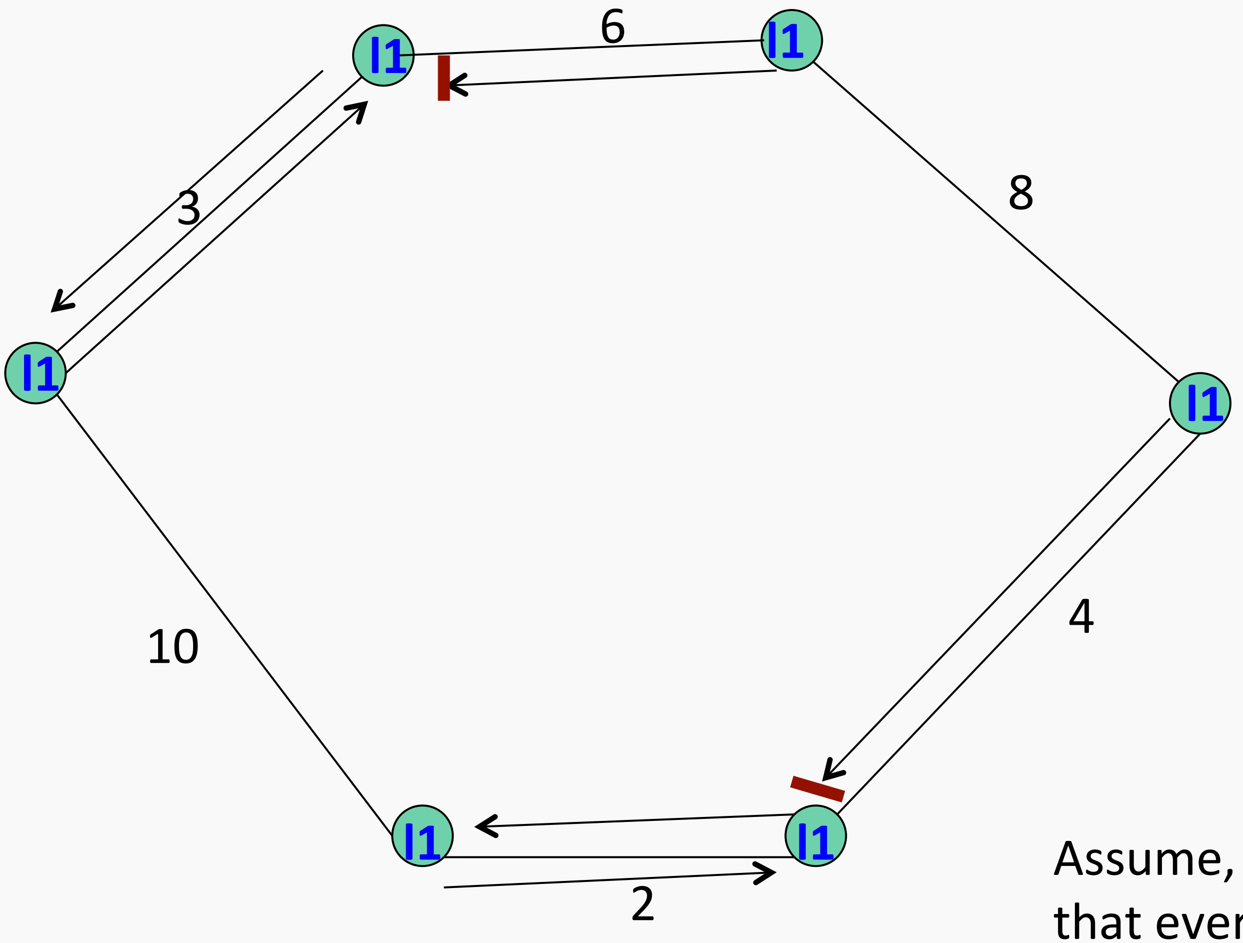
eventually, either it will know that it is a friendly merger
or its level will be increased (because of
requests from B to other cities)
and level(B) will become greater than level(A).

[absorption]

[Note: A is waiting,its level cannot increase]

Examples of merges

In the example, we are not worrying about the minimum outgoing link finding part.



Assume, for this example, that everybody is initially awake at level 1

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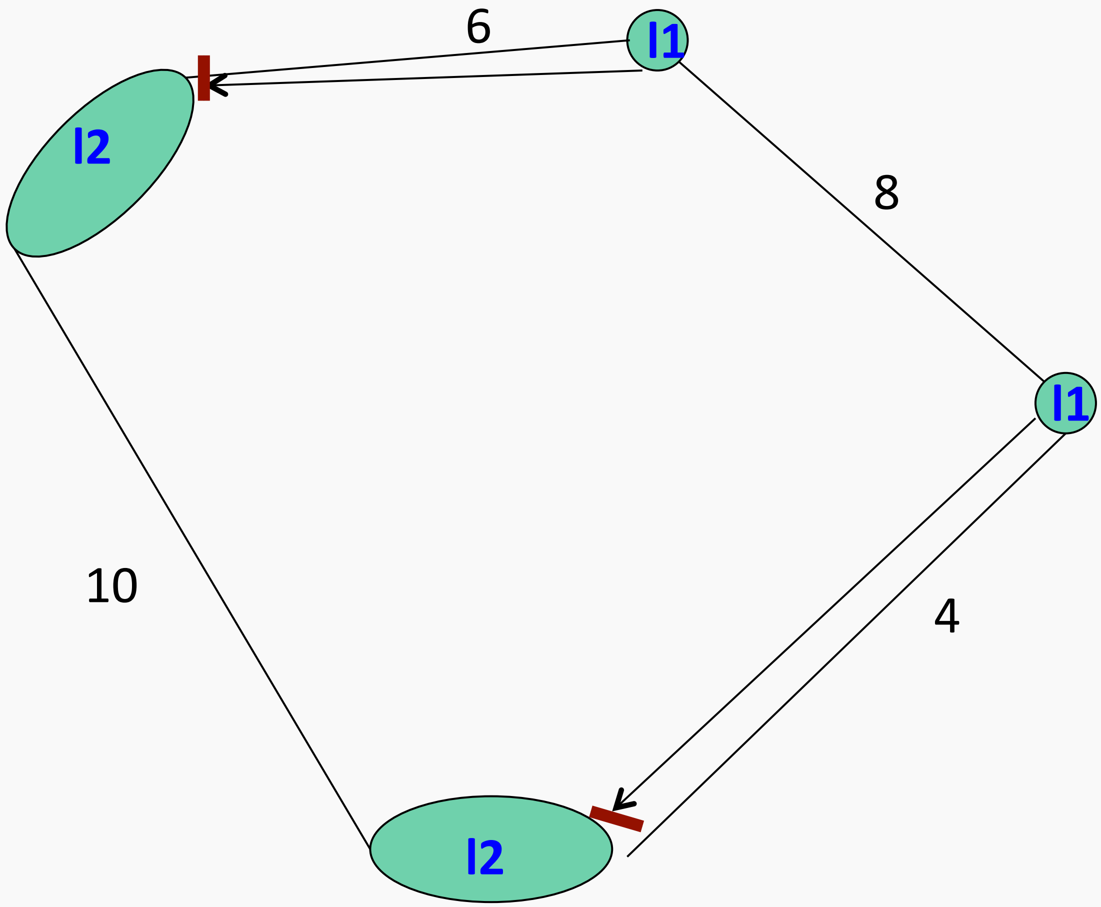

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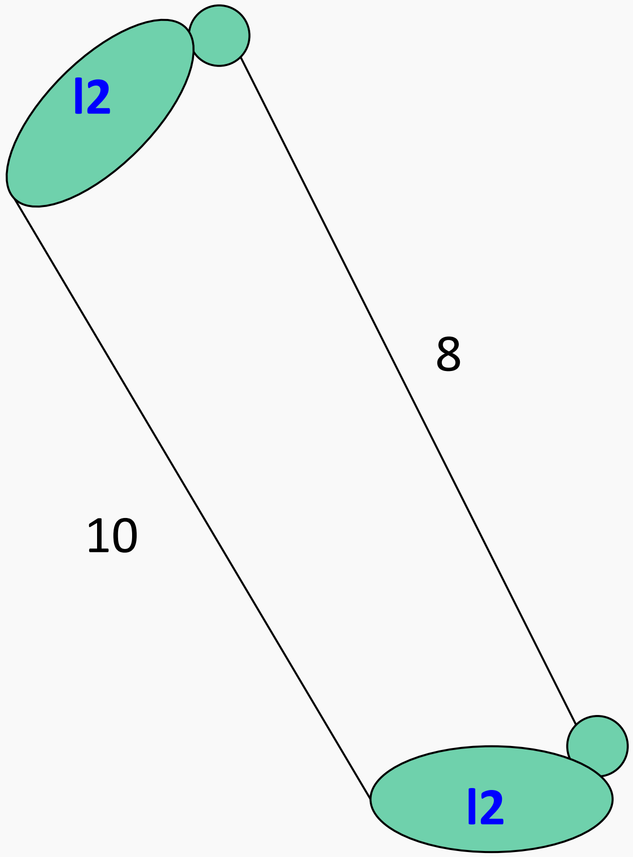

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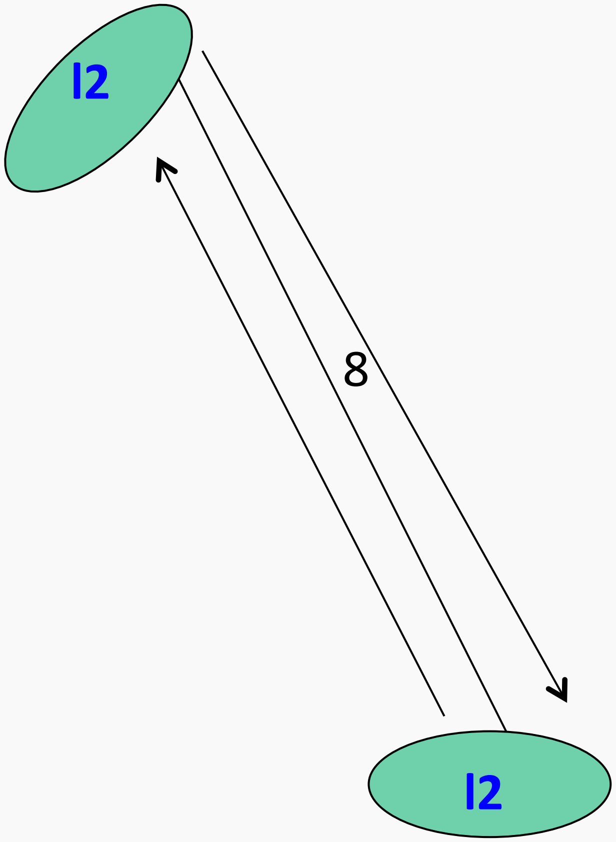

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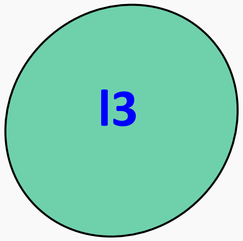
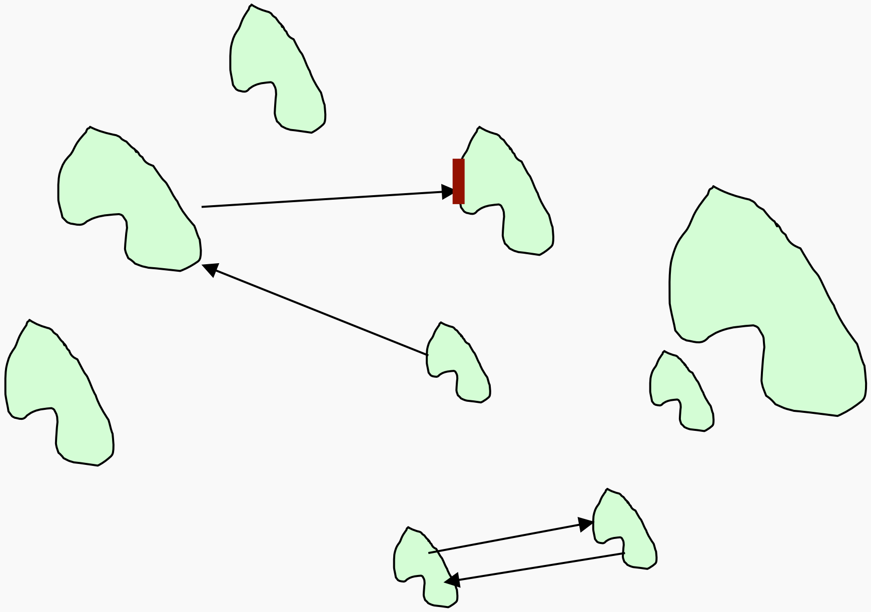


Examples of merges

Note: here the level is indicated by the size of the blob just for simplicity.



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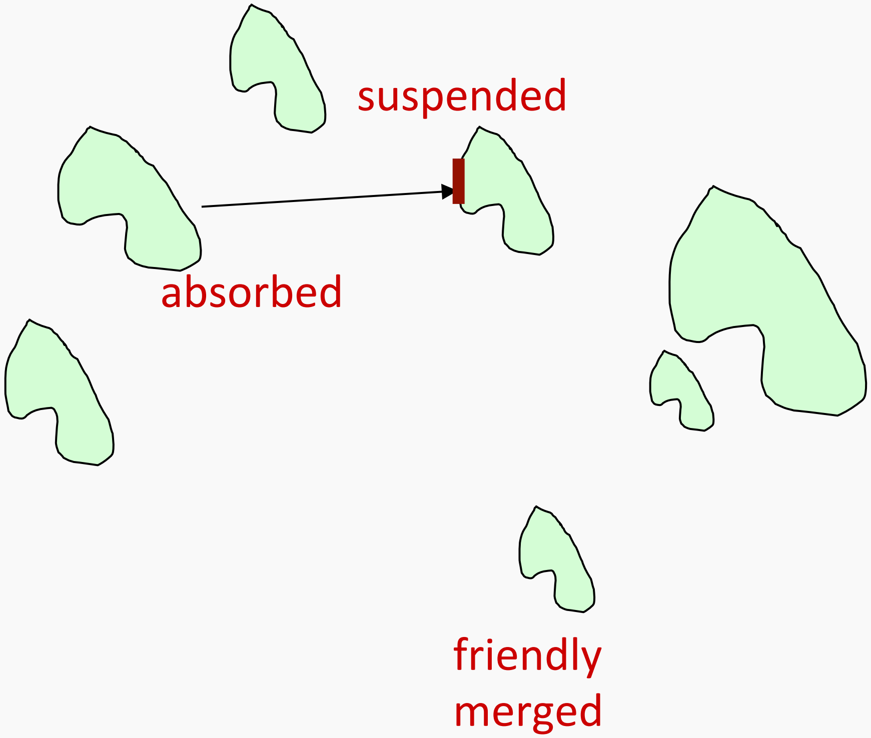

suspended

absorbed

friendly merged

Examples of merges

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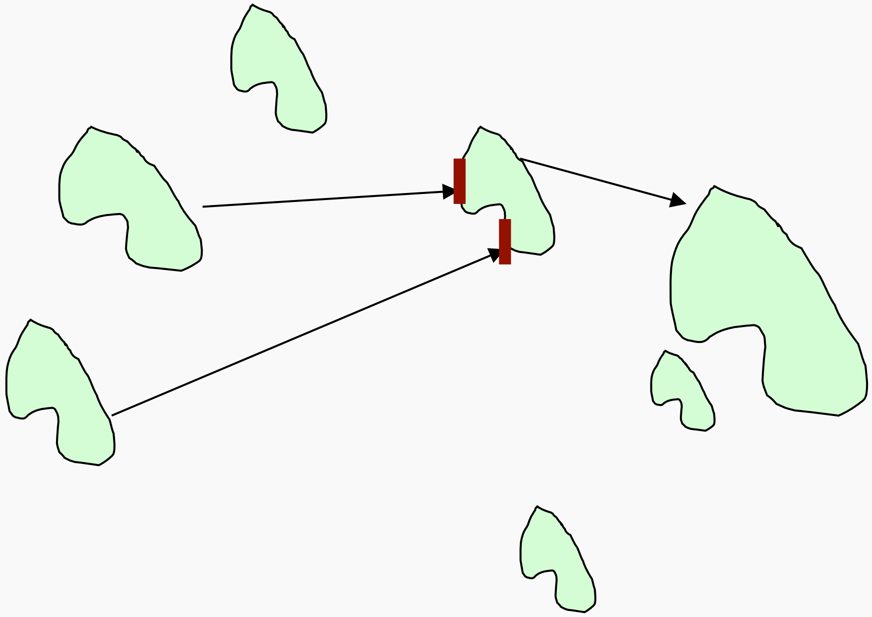

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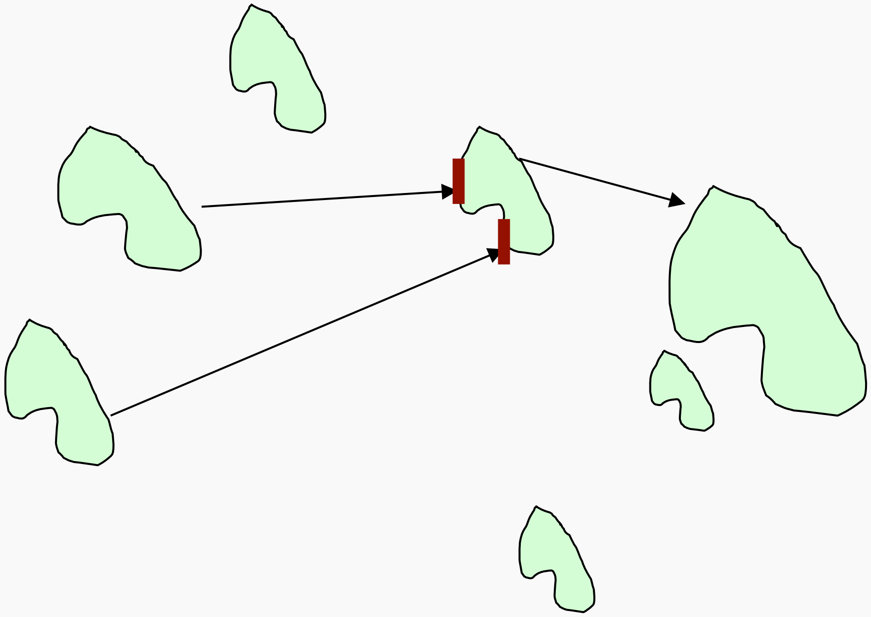

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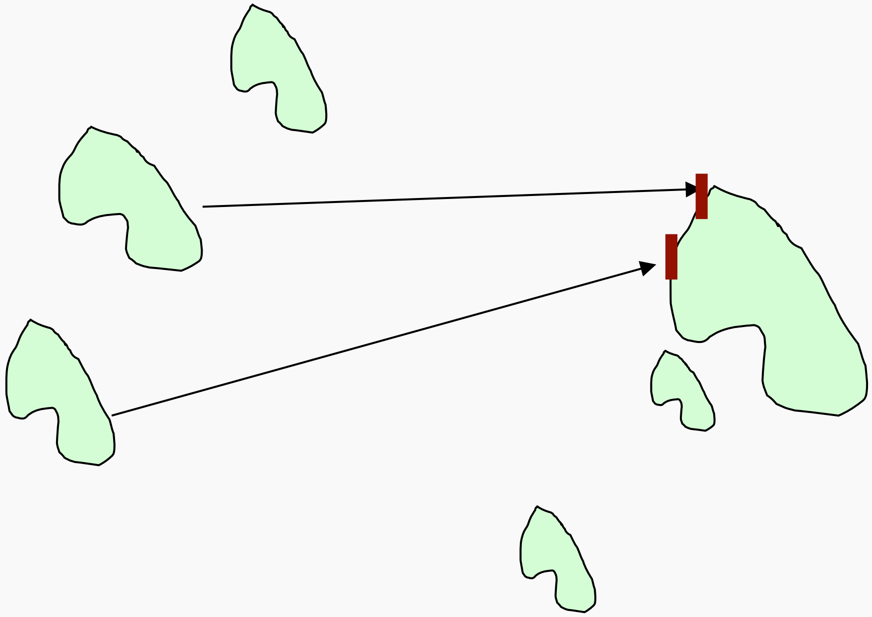

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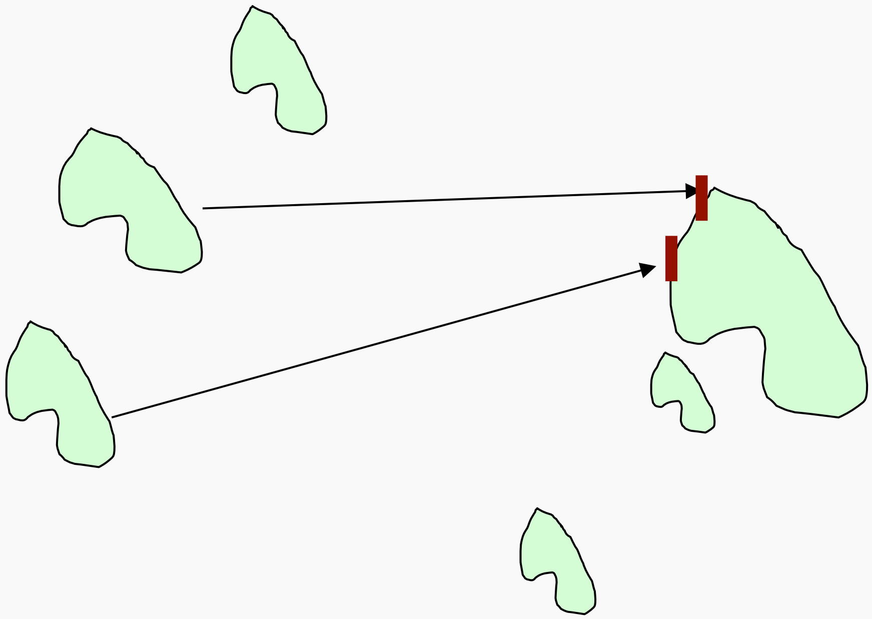

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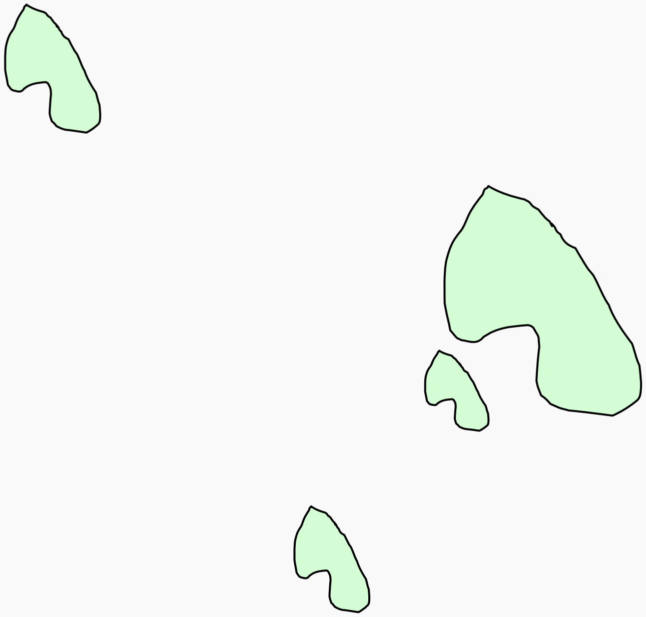

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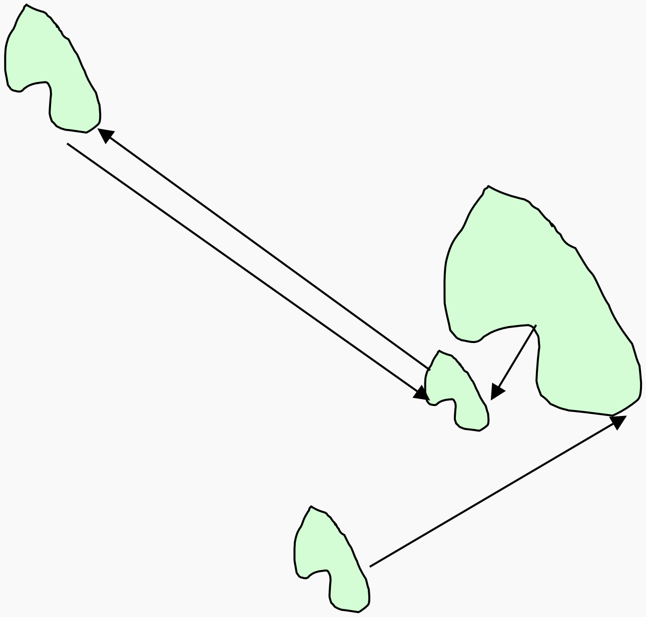

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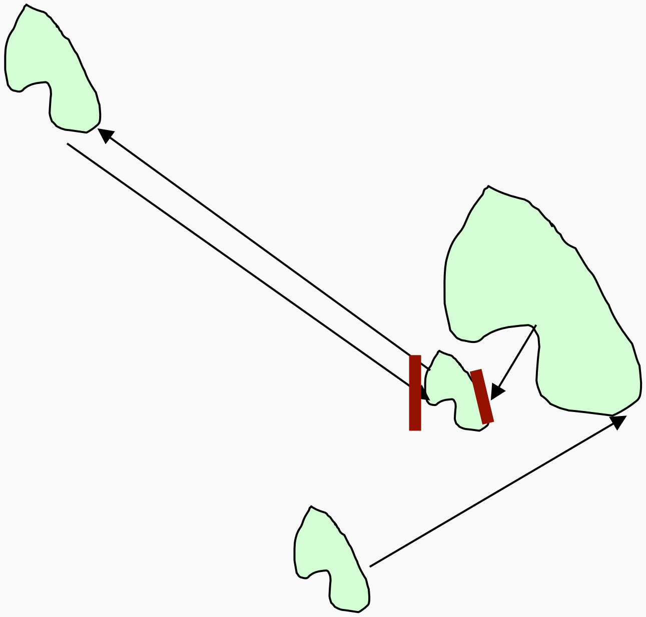

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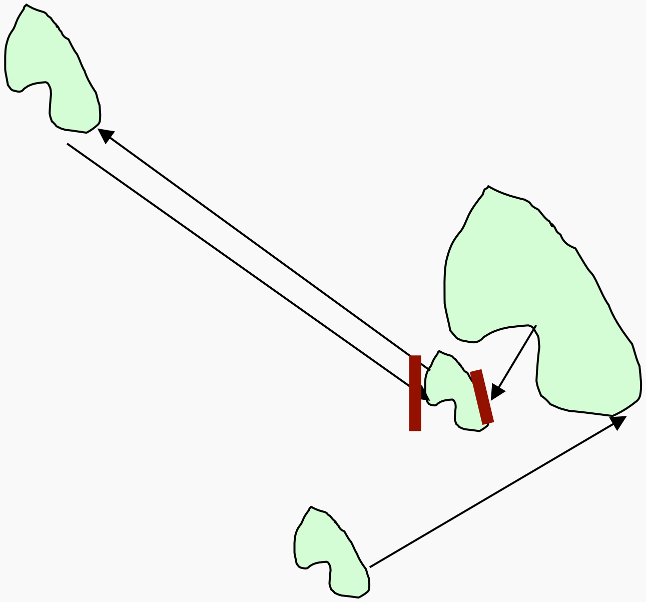

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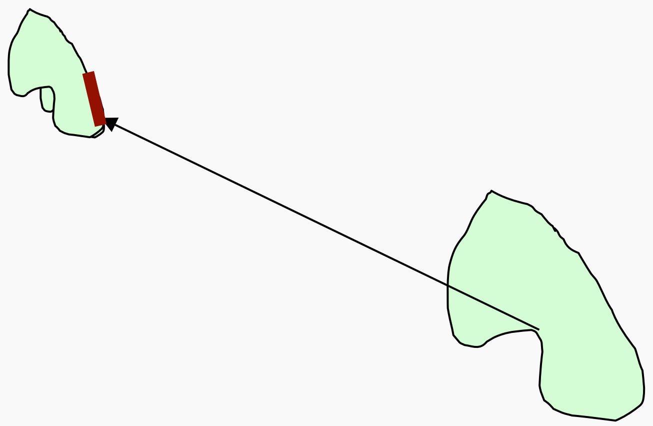

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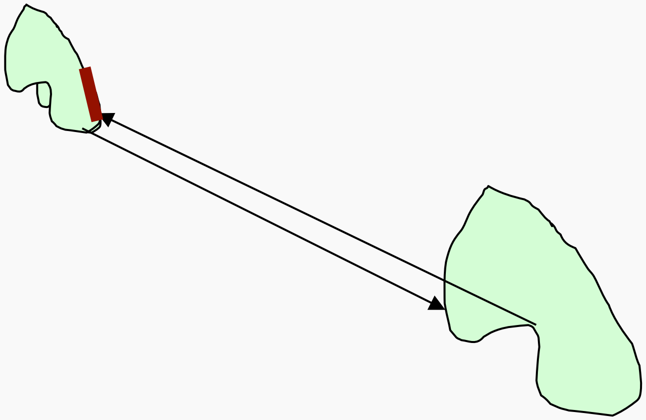
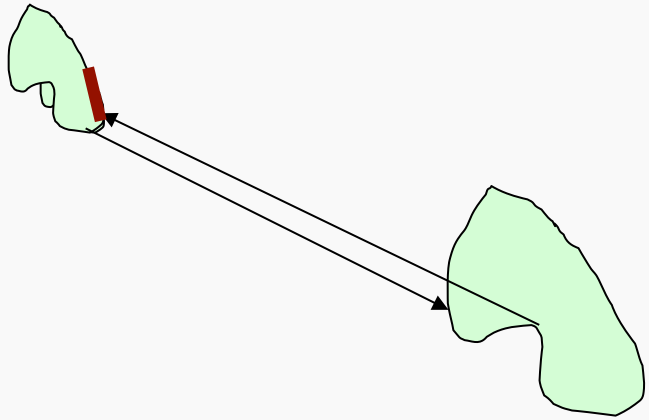


Examples of merges

Note: here the level is indicated by the size of the blob just for simplicity.



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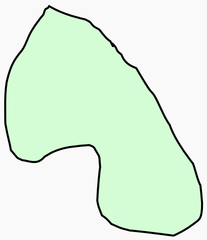
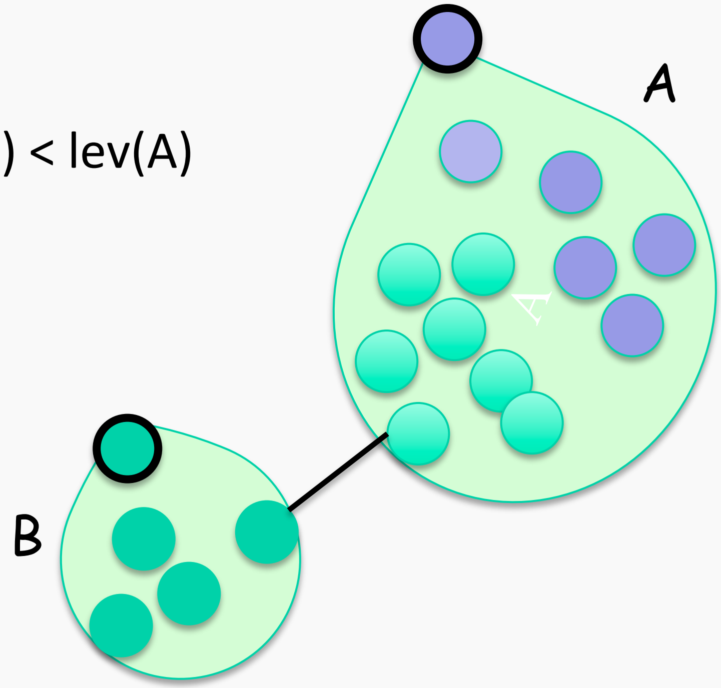
Examples of merges



Some Scenarios

Lev(B) < lev(A)

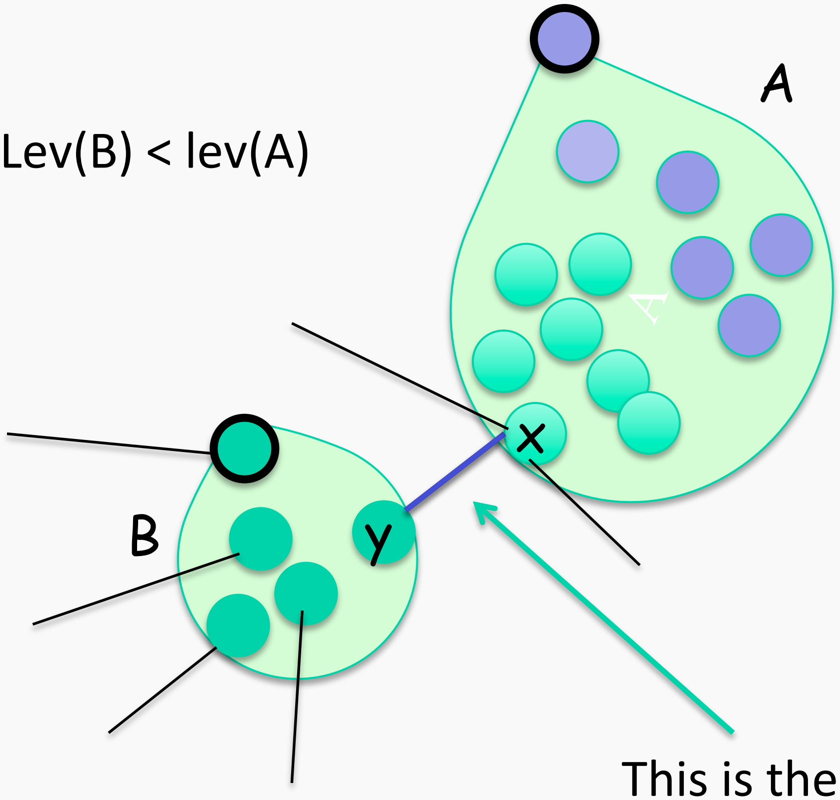


● Still computing min-outgoing

● Already computed min-outgoing.
Now ready to start a merge request

Is it ok for B to get absorbed into A?
If B gets absorbed, does the new city A lose
a better min-outgoing edge, possibly exiting from B ?

Lev(B) < lev(A)



● Still computing min-outgoing

● Already computed min-outgoing.
Now ready to start a merge reques

This is the smallest
out-going edge of city B

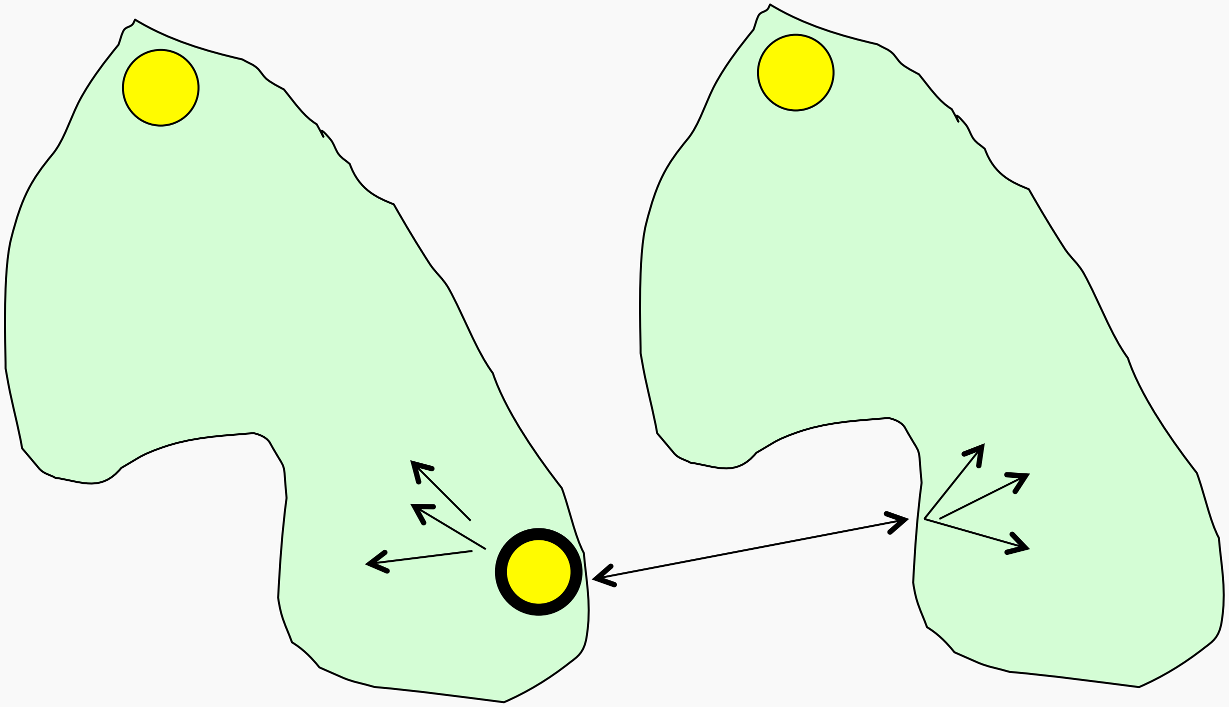
The min-outgoing for x is surely smaller than (x,y). **Why ?**

If x had asked y "*Outside?*", the request would have been suspended because lev(B)<lev(A)

Since x has terminated its selection of min, x has chosen something smaller than (x,y)

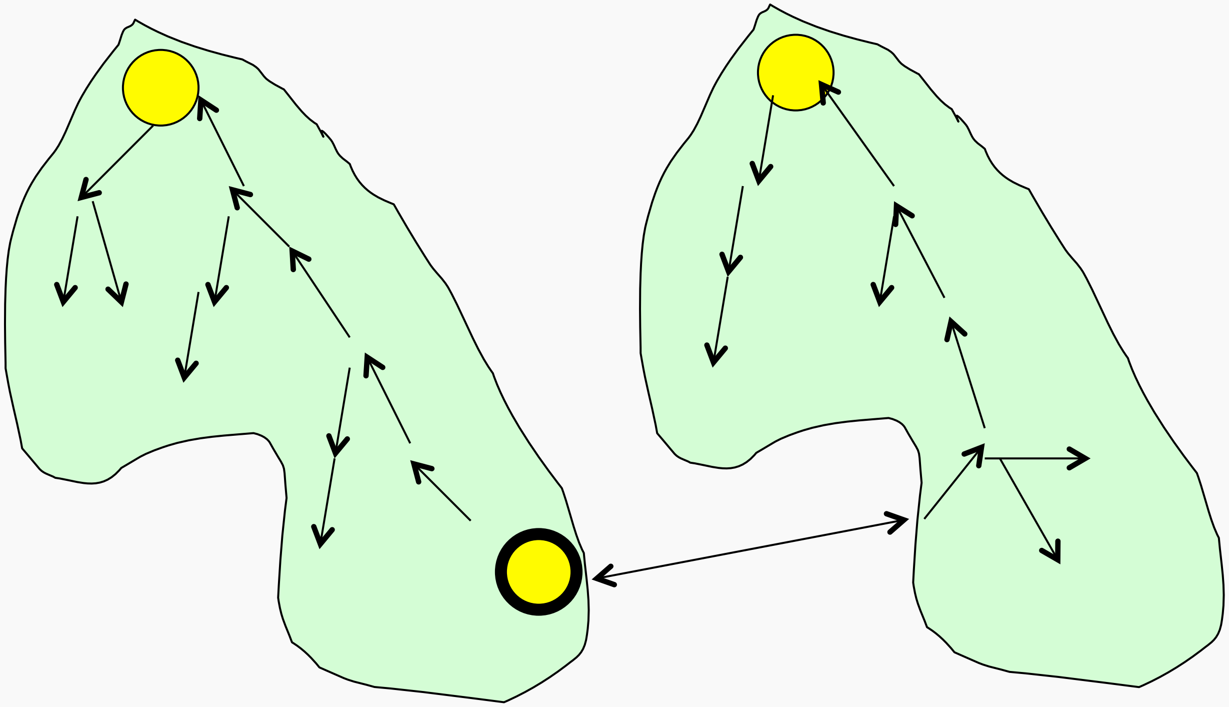
Some Scenarios

The two steps: choosing the link+ sending the merge request



When a merge occurs: during the broadcast /edge flipping
also the request-for-minimum-link is performed

The two steps: choosing the link+ sending the merge request



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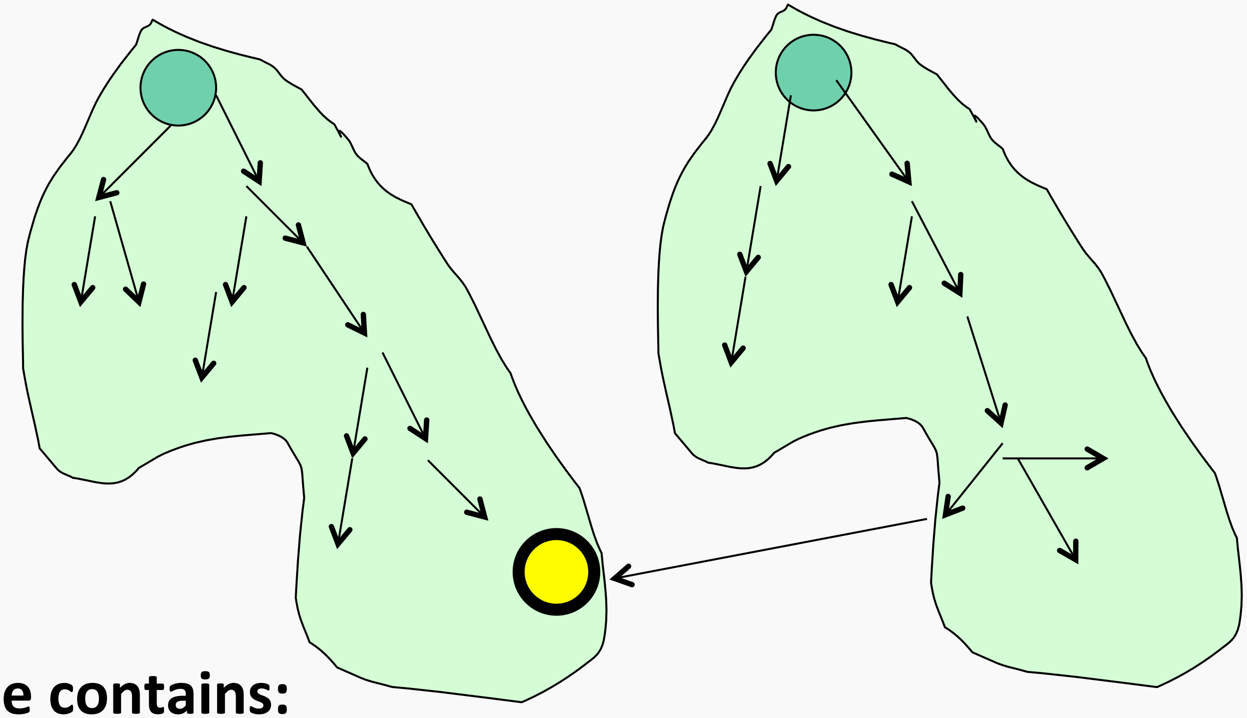


Broadcast message contains:

NEW INFO about new city

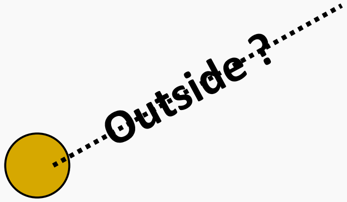
EDGE-FLIPPING and

give-me-new-minimum

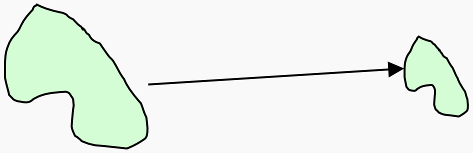
More Details Deadlocks

waiting cases:

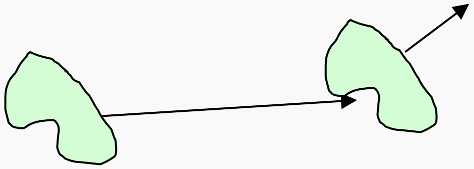
1) c send Outside? to d (level(D)< level(C)



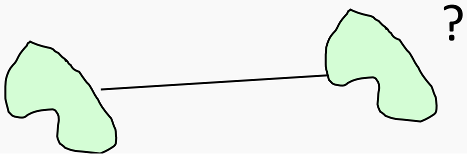
2) receiving let-us-merge on e(C)=(c,d), d knows that level(D) < level(C)



3) receiving let-us-merge on e(C)=(c,d), d knows that level(C)=level(D)
but it is not friendly



4) receiving let-us-merge on e(C)=(c,d), d knows that level(C)=level(D)
but does not know if it is friendly



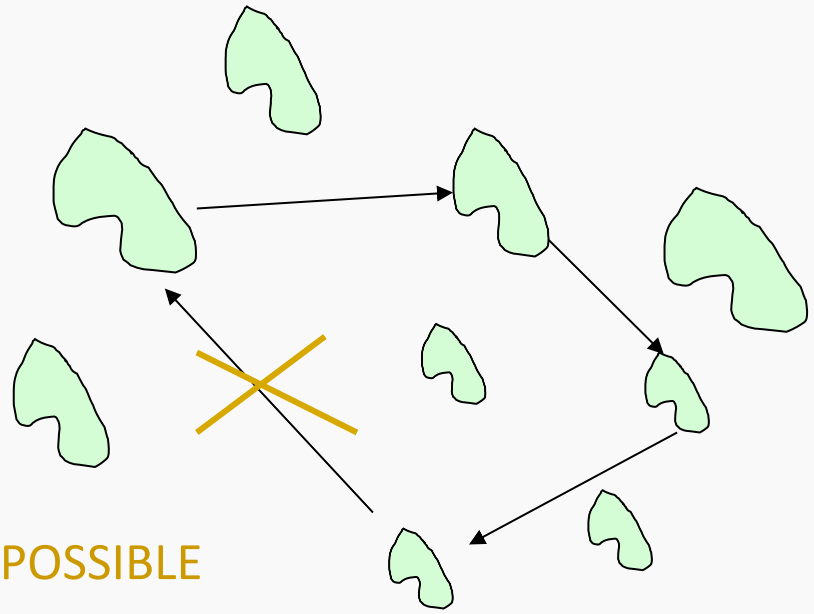
Correctness

If a city of level l will not be suspended, its level will increase
(unless it is the mega-city)

Let city C at level l be suspended by a district d in D.
If the level of D becomes greater than l, C will no longer be suspended

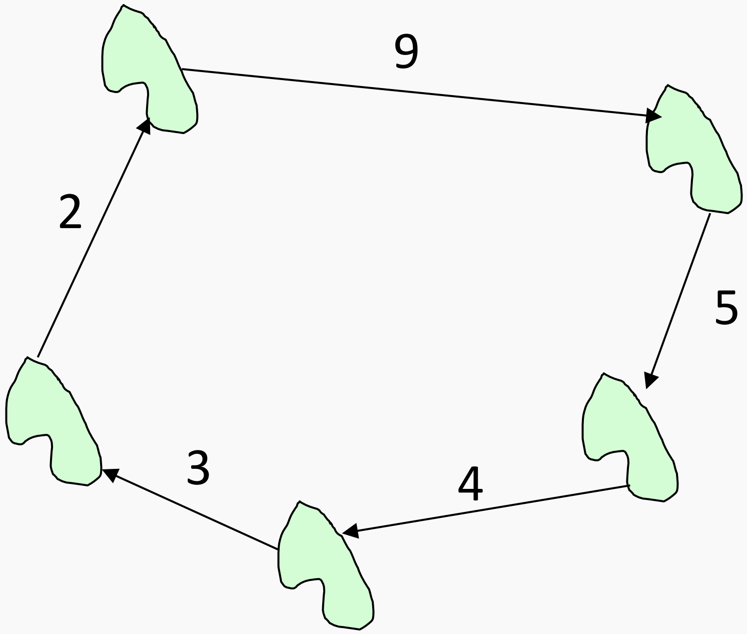
No city in C will be suspended by a city of higher level

Protocol Mega-merger is deadlock-free



IMPOSSIBLE

IMPOSSIBLE



Termination

If A is the mega-city, there are no other cities.
All the unused links are internal

The minimum finding will return a special value (∞)

D(A) understands and broadcasts termination

Complexity: for level i

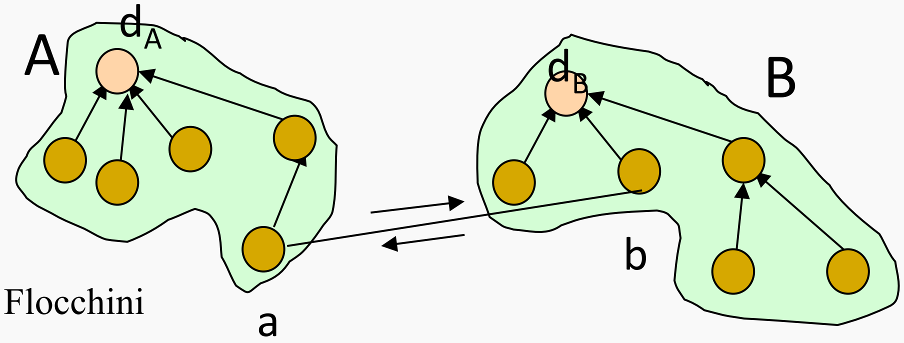
Number of messages per level: CITY A

For each friendly merger from level i-1 to level i

Computation of merge links: (n(A)-1)

Forwarding of let-us-merge from D(A) to e(A): n(A)

Broadcast info about new city: n(A)-1



TOT: 3 n(A) - 2

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Complexity: for level i

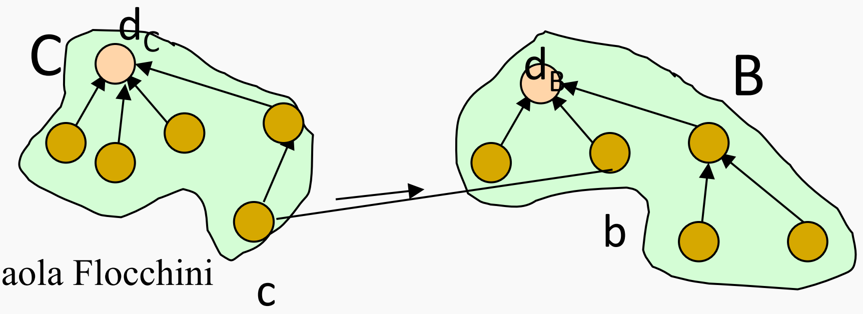
Number of messages per level : CITY C

C absorbed at level i

Computation of merge links: (n(C)-1)

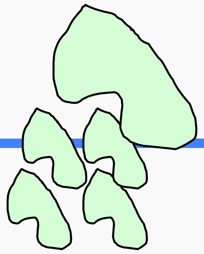
Forwarding of let-us-merge from D(C) to e(C): n(C)

Broadcast info about new city: n(C)



TOT: 3 n(C)-1

Complexity: for level i



As usual: *virtual level*.
It is not happening simultaneously!

disjoint cities, so: $\sum_{B \in City(i)} n(B) \leq n$

$City(i) = Merge(i) \cup Absorb(i)$

Number of messages per level

$$\sum_{A \in Merge(i)} (3n(A)-2) + \sum_{C \in Absorb(i)} 3(C)-1 \leq 3\sum_{B \in City(i)} n(B) -1$$

$$\leq 3n$$

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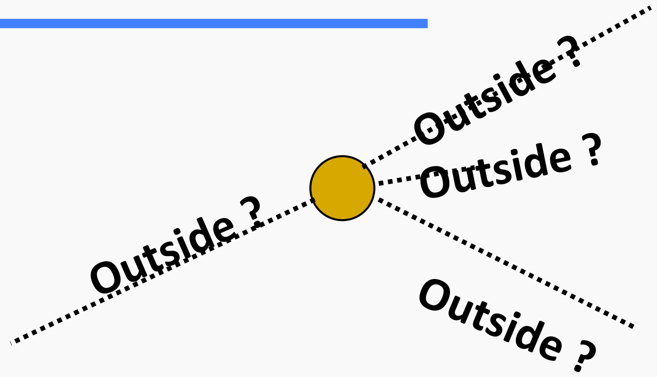
Complexity: for level i

Outside? with answer external



≤ 2n

I ask one at a time until
I find the smallest
External link.
While in level i, I am not
going to ask anymore.

→ One positive answer per node.

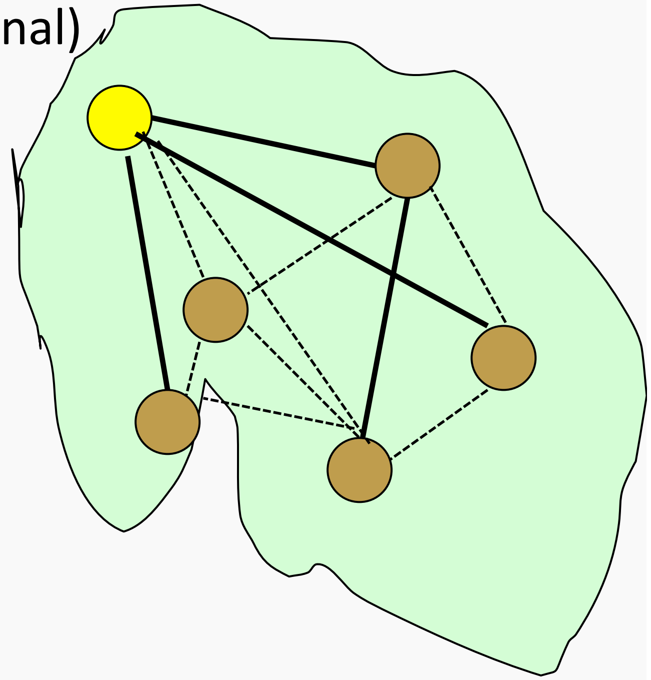
Total Cost(i) ≤ 3n+ 2n = 5n

Complexity – more

overall during the entire execution,
not just for level i

Useless messages **Outside?** with answer **internal**

- On a link, the answer "internal" can occur only once during the entire execution (because then it will stay internal)
- We have an **Outside?** with answer **internal** for each road that does not belong to the final city



2(m - (n-1))

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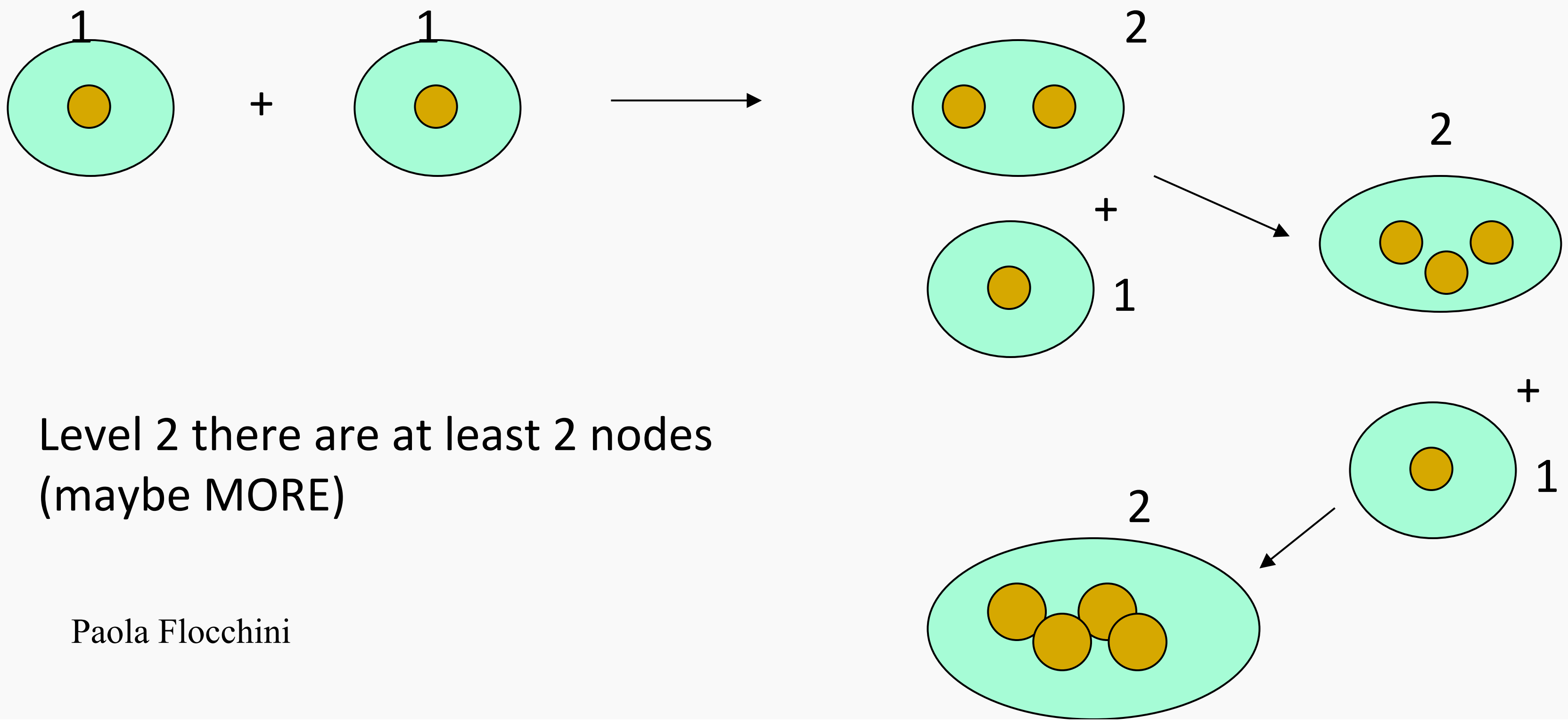
Complexity

Broadcasting Termination: n-1

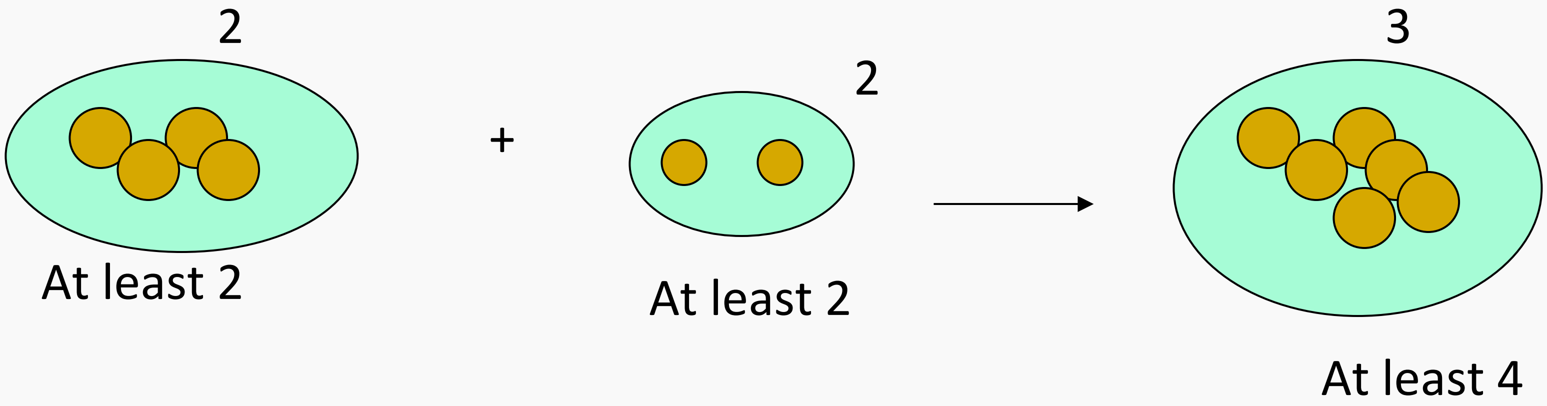
Complexity

How many levels ?

The level is incremented only if
the merger is between two cities with the same level



Level 2 there are at least 2 nodes
(maybe MORE)



2

At least 2

+

2

At least 2

3

At least 4

Complexity

In general, at Level i there are at least 2^i nodes (maybe MORE)

$$\text{Nodes at level } i \geq 2^i$$

$$n \geq 2^i$$

$$i \leq \log n$$

Total: $\leq \overbrace{2(m - (n-1))}^{\text{useless}} + \overbrace{n-1}^{\text{notification}} + \overbrace{5n \log n}^{\text{merges}}$

$$\leq 2m + 5\, n \log n - n+1$$