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The sampling of NTSC color signals is studied with the aid of the three-dimensional spectrum. The effect of various spatio-temporal sampling patterns on the spectrum is derived and some experimental measurements are given. A number of sampling structures which allow a $2f_{sc}$ horizontal sampling rate are identified, and possible reconstruction filters are described.

Introduction

The task of a television system is to communicate a three-dimensional function: the time-varying scene imaged by the camera. Although it is necessary to sample (scan) this function and map it into a one-dimensional signal for transmission and display purposes, the basic signal remains three-dimensional. Any processes acting on this signal, in particular those utilizing multiple lines or frames, are best understood in three dimensions. The one-dimensional spectrum description of the video signal commonly used is inadequate to describe any but the simplest three-dimensional processes. Thus, an accurate characterization of the three-dimensional spectrum of television signals is important in such applications as sampling, coding, component separation and enhancement.

For color signals, the function of interest is a three-dimensional vector valued function (the vector of appropriate tri-stimulus values). Although this vector is the one most natural for processing purposes, it is often considered desirable to process directly a composite color signal, such as the NTSC color signal. In such cases, it is necessary to characterize the spectrum of this composite color signal.

In this paper, the three-dimensional spectrum of the composite NTSC signal is investigated, with particular emphasis on the question of sampling. The NTSC signal in analog form is already sampled in vertical and temporal dimensions by the scanning process. The vertical and temporal sampling intervals are in general beyond the control of the digital system designer, being specified by the television standard in use in the particular country. This vertical-temporal sampling is usually effected in a hexagonal pattern known as 2:1 line interlace. Thus, in sampling the NTSC signal to obtain a digital video signal, the horizontal sampling frequency, as well as spatial (i.e. horizontal-vertical) and horizontal-temporal projections of the sampling pattern remain to be specified; these parameters determine the three-dimensional sampling grid.

This paper analyzes three-dimensional sampling grids which can be constructed as a superposition of rectangular sampling grids. Most sampling grids of interest are of this class. The three-dimensional spectrum of the NTSC signal is derived and the effect of various three-dimensional sampling schemes on this spectrum is shown. Considerations of reconstructing the analog signal from the samples are also discussed.

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Preliminary — Sampling on a Superposition of Rectangular Grids

Let $u(x_1,x_2,t)$ be a continuous three-dimensional function, with Fourier transform $U(f_1,f_2,f_3)$. Sampling u on a rectangular grid is represented by multiplying $u(x_1,x_2,t)$ by a three-dimensional array of delta functions

$$u_S(x_1,x_2,t) = u(x_1,x_2,t) \cdot d_R(x_1,x_2,t) \quad (1)$$

where

$$d_R(x_1,x_2,t) = \sum_{i_1=-\infty}^{\infty} \sum_{i_2=-\infty}^{\infty} \sum_{i_3=-\infty}^{\infty} \delta(x_1 - i_1T_1, x_2 - i_2T_2, t - i_3T_3) \quad (2)$$

The spectrum of the sampled signal is then given by the convolution of the Fourier transforms of u and d_R

$$U_S(f_1,f_2,f_3) = U(f_1,f_2,f_3) * D_R(f_1,f_2,f_3) \quad (3)$$

where

$$D_R(f_1,f_2,f_3) = \frac{1}{T_1T_2T_3} \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} \sum_{k_3=-\infty}^{\infty} \delta(f_1 - k_1/T_1, f_2 - k_2/T_2, f_3 - k_3/T_3) \quad (4)$$

The convolution of U with D_R has the effect of replicating U in horizontal, vertical and temporal dimensions at intervals $1/T_1$, $1/T_2$ and $1/T_3$ respectively.

Figure 1a shows the spatial projec-

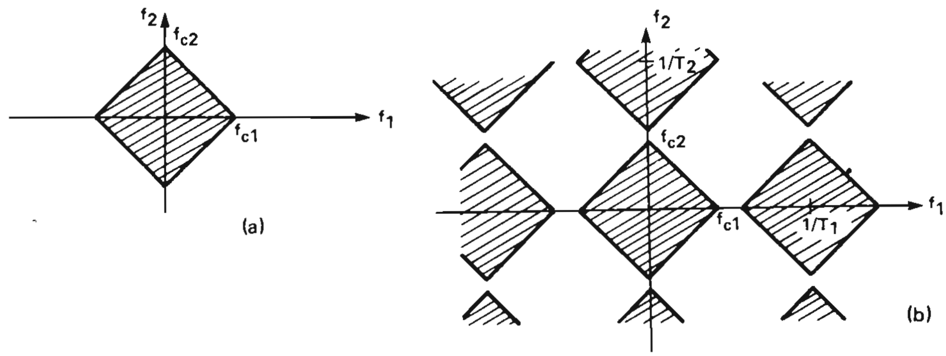


Figure 1. (a) Spatial projection of region of support for the spectrum of a three-dimensional function. (b) Spatial projection of spectrum of signal shown in (a), sampled with a rectangular sampling grid of period T_1, T_2, and T_3.

tion of the region of support for the spectrum of a three-dimensional signal. This hypothesized spectrum is non-zero in a diamond-shaped region. The spatial projection of the spectrum of the sampled signal is shown in Fig. 1b. It is evident that to avoid overlap of the spectra (or aliasing), it is necessary that $1/T_1 \geq 2f_{c1}$ and $1/T_2 \geq 2f_{c2}$. The basic principle of efficient multidimensional sampling is to choose the sampling grid such that the number of samples per unit volume is minimized, subject to the absence of aliasing.[1] It can be seen from Fig. 1 that for the spectrum shape shown, rectangular sampling does not make efficient use of spectral space; if T_1 and T_2 are chosen as large as possible while avoiding aliasing, i.e. $T_1 = \frac{1}{2}f_{c1}$, $T_2 = \frac{1}{2}f_{c2}$, large areas of the f_1, f_2 plane remain vacant.

An alternative to rectangular sampling, which can allow more efficient use of spectral space, is to sample along three arbitrary axes (non-coplanar).[1] For example, for the spectrum of Fig. 1a, sampling spatially along axes at an angle of 120° to each other (hexagonal sampling) can lead to a very efficient packing of the spectrum. However, for the NTSC signal, many sampling schemes of interest are not of this class, and so a different approach has been taken: sampling on a superposition of rectangular grids.

Suppose that $u(x_1,x_2,t)$ is sampled by the superposition of two rectangular grids, offset with respect to each other by $\alpha_1 T_1$, $\alpha_2 T_2$, and $\alpha_3 T_3$:

$$u_S(x_1,x_2,t) = u(x_1,x_2,t) \cdot d(x_1,x_2,t) \quad (5)$$

where

$$d(x_1,x_2,t) = d_R(x_1,x_2,t) + d_R(x_1 - \alpha_1 T_1, x_2 - \alpha_2 T_2, t - \alpha_3 T_3) \quad (6)$$

Using the linearity and shifting properties of the Fourier transform

$$U_S(f_1,f_2,f_3) = U(f_1,f_2,f_3) * D(f_1,f_2,f_3) \quad (7)$$

where

$$D(f_1,f_2,f_3) = D_R(f_1,f_2,f_3) \times (1 + \exp[-j2\pi(\alpha_1 T_1 f_1 + \alpha_2 T_2 f_2 + \alpha_3 T_3 f_3)])$$

which can be written

$$D(f_1,f_2,f_3) = \frac{1}{T_1 T_2 T_3} \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} \sum_{k_3=-\infty}^{\infty} (1 + \exp[-j2\pi(\alpha_1 k_1 + \alpha_2 k_2 + \alpha_3 k_3)])\delta(f_1 - k_1/T_1, f_2 - k_2/T_2, f_3 - k_3/T_3) \quad (8)$$

For arbitrary α_i, the spectrum is replicated with the same period as for rectangular sampling, but the component at $(k_1/T_1, k_2/T_2, k_3/T_3)$ is multiplied by the term $f(k_1,k_2,k_3) = 1 + \exp[-j2\pi(\alpha_1 k_1 + \alpha_2 k_2 + \alpha_3 k_3)]$. When the α_i take on specific rational values, this term will be zero for certain values of k_1, k_2, k_3 (when $2(\alpha_1 k_1 + \alpha_2 k_2 + \alpha_3 k_3)$ is an odd integer) causing those particular replicated versions of the spectrum to be cancelled. This approach is readily extended to a superposition of several rectangular grids, allowing the spectrum arising from a large class of sampling patterns to be derived.

As an example, consider the superposition of two-dimensional rectangular grids, offset by $\alpha_1 = \alpha_2 = \frac{1}{4}$ (Fig. 2a). In this case, the component of the spectrum at $(k_1/T_1, k_2/T_2)$ is multiplied by

$$1 + \exp[-j2\pi(k_1 + k_2)/4] = 1 + j^{k_1+k_2}$$

This term is zero whenever $k_1 + k_2 = 2 \bmod 4$. The resulting spectrum is shown in Fig. 2b. Cancelled components are shown with dashed lines.

If $u(x_1,x_2,t)$ is a sample from a three-dimensional stationary random process, then the above procedure is directly applicable to the power spectrum of u, defined as the Fourier transform of the three-dimensional autocorrelation of u.

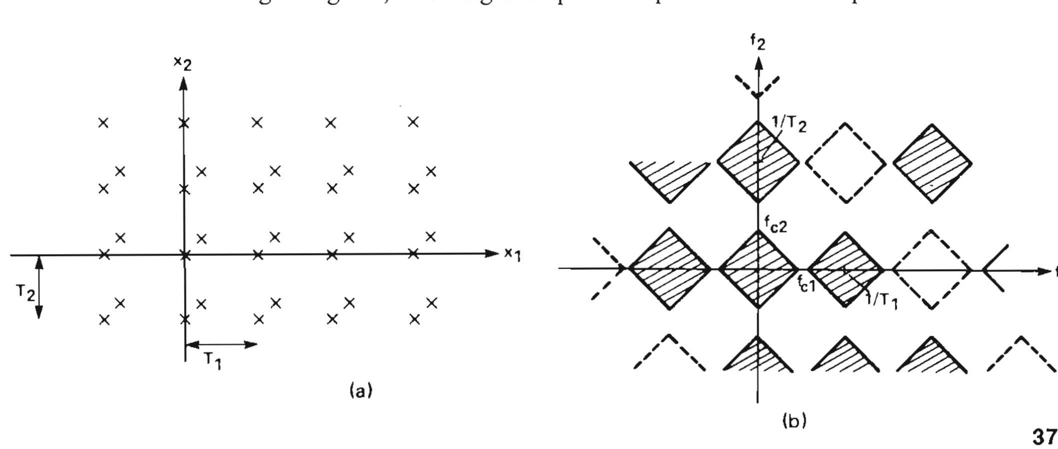
Spectrum of NTSC Signal

The NTSC signal has the form

$$u(t) = y(t) + i(t)\cos(2\pi f_{sc}t + \theta) + q(t)\sin(2\pi f_{sc}t + \theta) \quad (9)$$

where $y(t)$ is the luminance and $i(t)$ and $q(t)$ are chrominance signals. The frequency of the color subcarrier, f_{sc}, is an odd multiple of $f_\ell/2$ and $f_{fr}/2$ (f_ℓ is the line frequency and f_{fr} is the frame frequency), so that the phase of subcarrier changes by π from line to line and from frame to frame. The phase relationships for a line-inter-

Figure 2. (a) Sampling grid construction as $d_R(x_1,x_2) + d_R(x_1 - T_1/4, x_2 - T_2/4)$. (b) Spectrum of signal sampled using sampling grid shown in (a). Dashed components have been cancelled.



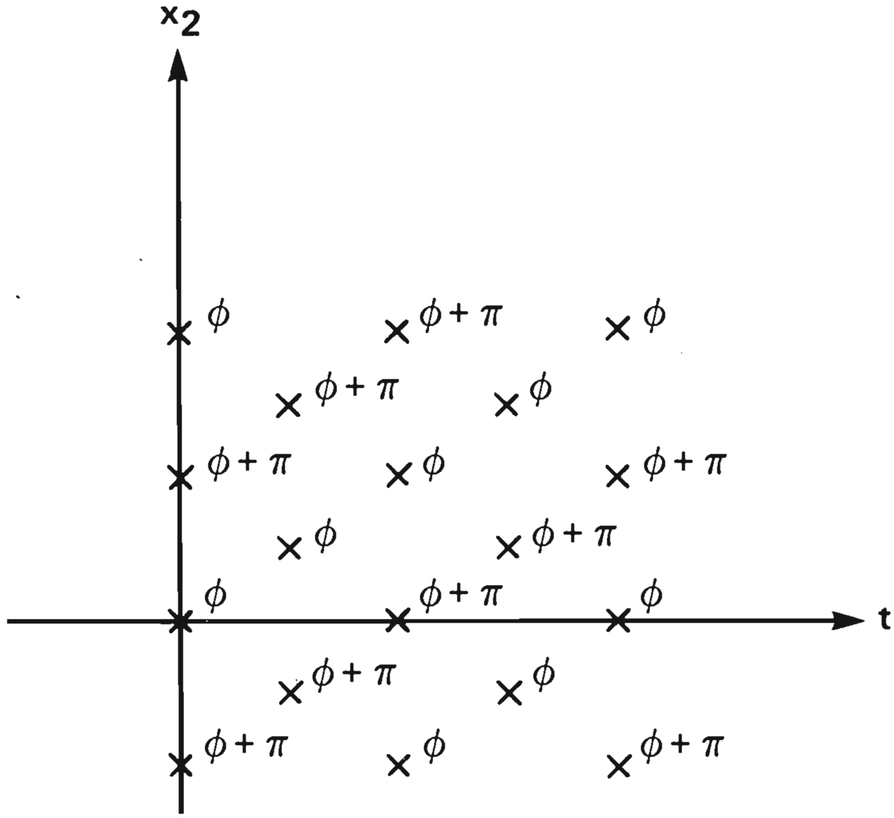
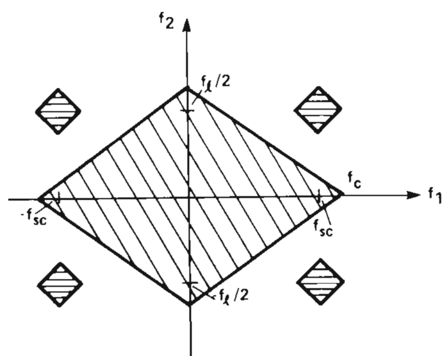


Figure 3. Subcarrier phase relationships for the NTSC color signal.

laced sampling grid are shown in Fig. 3. Based on this, we can write an expression for a fictitious three-dimensional NTSC signal, which when scanned in a line-interlaced fashion, would result in (9).

$$u(x_1,x_2,t) = y(x_1,x_2,t) + i(x_1,x_2,t)\cos(2\pi f_{sc}x_1 + \theta)\cos\pi(f_{fr}t - f_\ell x_2) + q(x_1,x_2,t)\sin(2\pi f_{sc}x_1 + \theta)\cos\pi(f_{fr}t - f_\ell x_2) \quad (10)$$



(a)

If y, i and q are stationary, three-dimensional random processes, with power spectra S_y, S_i and S_q respectively, then the power spectrum of u is easily seen to be

$$S_u(f_1,f_2,f_3) = S_y(f_1,f_2,f_3) + \tfrac{1}{4}[S_c(f_1 - f_{sc}, f_2 - f_\ell/2, f_3 + f_{fr}/2) + S_c(f_1 + f_{sc}, f_2 - f_\ell/2, f_3 + f_{fr}/2) + S_c(f_1 - f_{sc}, f_2 + f_\ell/2, f_3 - f_{fr}/2) + S_c(f_1 + f_{sc}, f_2 + f_\ell/2, f_3 - f_{fr}/2)] \quad (11)$$

where $S_c = S_i + S_q$. It can be seen that chrominance energy is concentrated at the frequencies $(\pm f_{sc}, f_\ell/2, -f_{fr}/2)$ and $(\pm f_{sc}, -f_\ell/2, f_{fr}/2)$.

This is shown in Fig. 4. Figures 4a and 4b show the projections of the spectrum on the spatial, and vertical-temporal frequency axes, while Fig. 4c is a perspective view of the three-dimensional spectrum, where octahedral regions of the support for the components of the spectrum have been assumed. The horizontal cutoff f_c is approximately 4.2 MHz. The vertical and temporal cutoff frequencies are not well defined.

The three-dimensional spectrum of the composite PAL signal has been similarly described, in the context of filtering luminance and chrominance components to eliminate cross-color effects.[2]

Spectrum of Sampled NTSC Signals

As mentioned previously, the vertical-temporal projection of the sampling pattern is predetermined; the horizontal sampling frequency, as well as spatial and horizontal-temporal projections of the sampling pattern remain to be selected. In this section, several sampling structures suitable for use with NTSC signals are examined, and the efficiency with which spectral space is used is compared.

The sampling patterns considered are all temporally aligned, in that corresponding samples in successive frames have the same spatial location. The intra-field sampling pattern may be horizontally aligned or offset between successive fields. Both cases will

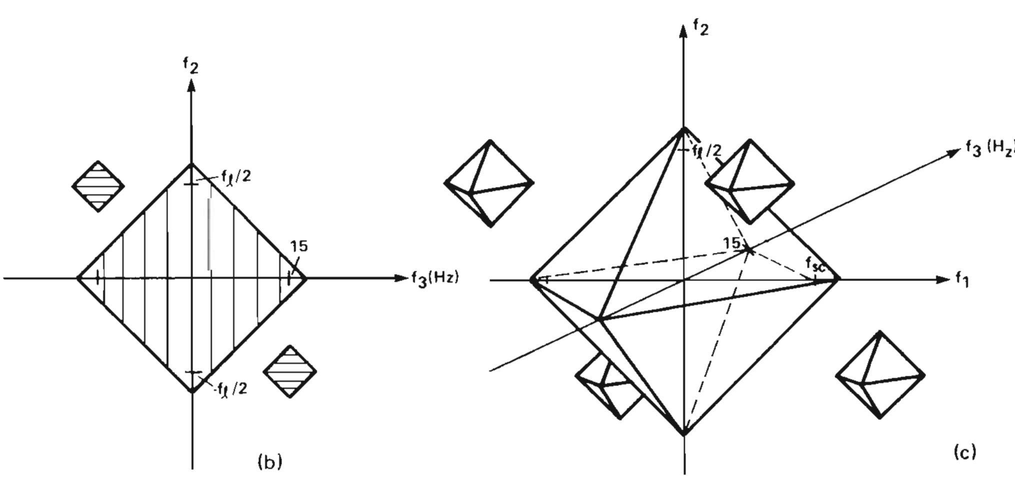


Figure 4. (a) Spatial projection of the basic spectrum of the NTSC color signal. (b) Vertical-temporal projection of the basic spectrum of the NTSC color signal. (c) Perspective view of the basic spectrum of the NTSC color signal.

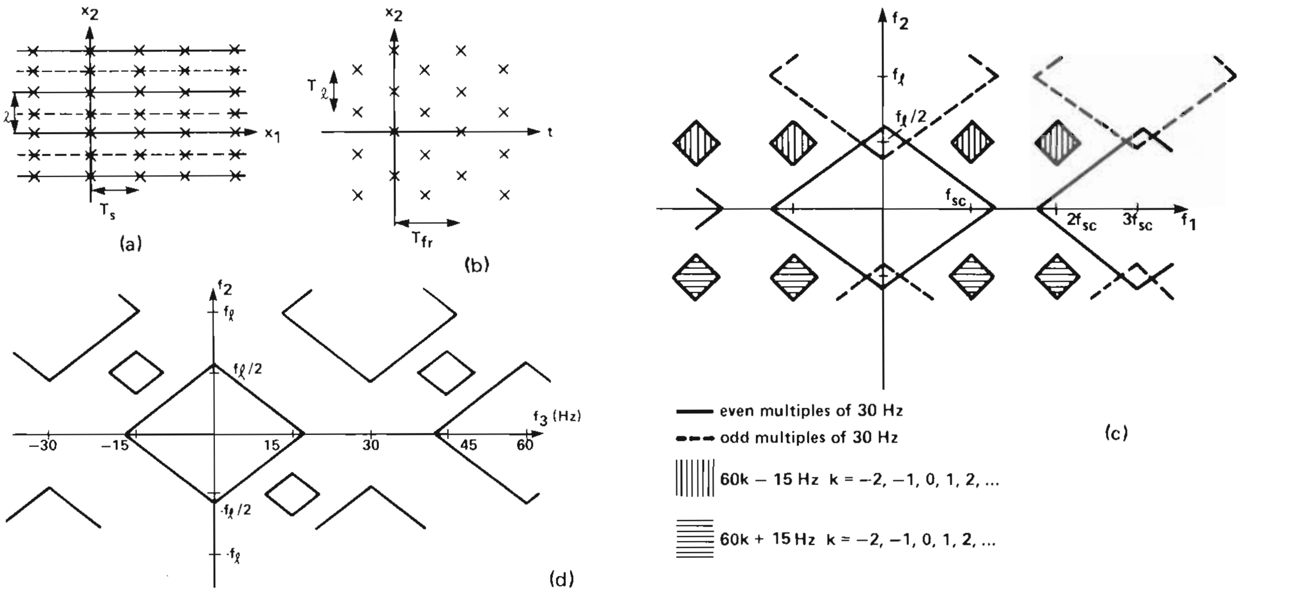


Figure 5. (a) Spatial projection of rectangular field-aligned sampling pattern. (b) Vertical-temporal projection of rectangular field-aligned sampling pattern. (c) Spatial projection of spectrum of NTSC signal sampled at $3f_{sc}$ with rectangular field-aligned sampling pattern. (d) Vertical-temporal projection of spectrum of NTSC signal sampled with rectangular field-aligned sampling pattern.

be considered. The following categories are named based on the intra-field pattern.

Rectangular Sampling

Field aligned. This case is obtained by choosing the spatial and horizontal-temporal projections of the sampling grid to be rectangular. The spatial and vertical-temporal projections are shown in Figs. 5a and 5b. This sampling grid can be expressed as a superposition of two rectangular grids, one for the even fields, one for the odd fields. The basic horizontal, vertical and temporal sample intervals are T_s, the sample spacing, T_ℓ the in-field line spacing, and T_{fr} the frame spacing. Corresponding frequencies are f_s, f_ℓ and f_{fr}.

The sampling grid can be written

$$d(x_1,x_2,t) = d_R(x_1,x_2,t) + d_R(x_1, x_2 - T_\ell/2, t - T_{fr}/2)$$

In this case $f(k_1,k_2,k_3) = 1 + \exp[-j2\pi(k_2/2 + k_3/2)] = 1 + (-1)^{k_2+k_3}$ which is zero when $k_2 + k_3$ is odd. The spatial and vertical-temporal projections of the resulting spectrum are shown in Figs. 5c and 5d. The horizontal sampling frequency must exceed about 8.4 MHz to avoid

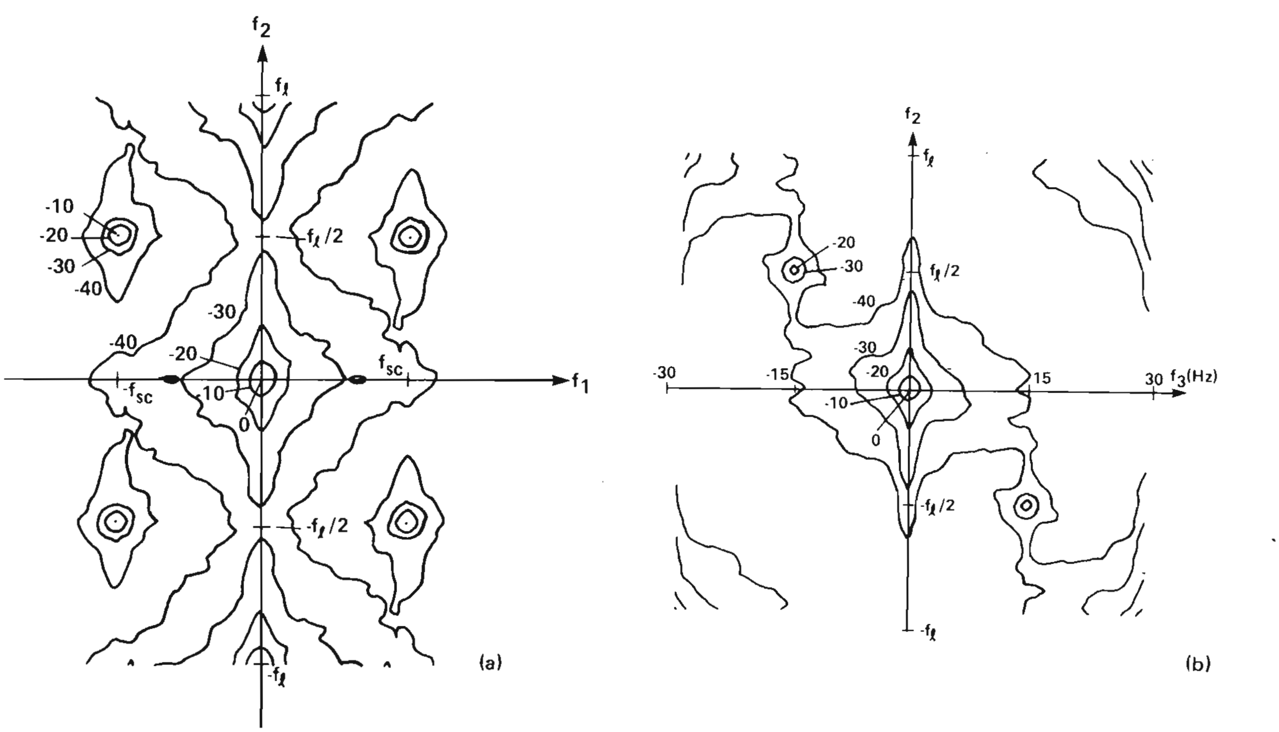


Figure 6. (a) Contours of spatial projection of measured spectrum for video signal sampled at $4f_{sc}$. (b) Contours of vertical-temporal projection of measured spectrum for video signal sampled at $4f_{sc}$. Relative power in decibels shown on selected contours.

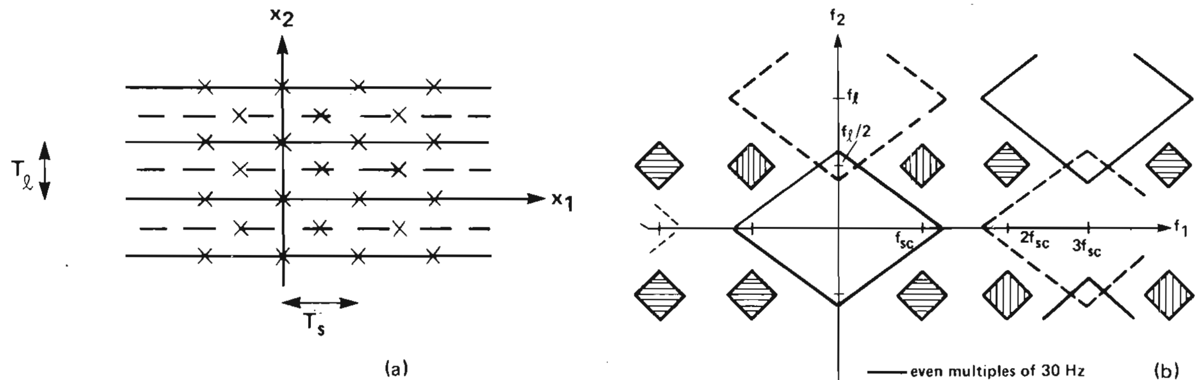


Figure 7. (a) Spatial projection of rectangular field-offset sampling pattern. (b) Spatial projection of spectrum of NTSC signal sampled at $3f_{sc}$ with rectangular field-offset sampling pattern.

aliasing. Since it is desirable for several reasons to sample at a multiple of f_{sc}, the lowest frequencies satisfying this are $3f_{sc} \simeq 10.7$ MHz and $4f_{sc} \simeq 14.4$ MHz. At $2f_{sc} \simeq 7.2$ MHz there is significant aliasing in the luminance component, while the chrominance components overlap completely, rendering this sampling pattern unusable.

Reconstruction of the analog signal is accomplished by a low pass filter in the horizontal direction, with cutoff at 4.25 MHz. Elimination of the dashed components in Fig. 5c is the task of the television receiver.

Figure 6 shows contour plots of measured spectra for an actual video sequence. This sequence consists of 1.2 sec of NTSC signal sampled at $4f_{sc}$ and linearly quantized at 8 bits/pel. The sequence was acquired using the Digital Video Sequence Store[3] (DVS), and is stored on a computer disk, available for software processing and analysis in non-real time. The scene consists of a head and shoulders view of a man in fairly active motion. Figure 6a shows the spatial projection of the spectrum and Fig. 6b shows the vertical-temporal projection. They should be compared with the idealized versions in Figs. 5c and 5d. These spectra were obtained by the method of averaging modified periodograms[4], adapted to two dimensions.

Field offset. For rectangular field-offset sampling, the spatial sampling pattern is as shown in Fig. 7a. In this case

$$d(x_1,x_2,t) = d_R(x_1,x_2,t) + d_R(x_1 - T_s/2, x_2 - T_\ell/2, t - T_{fr}/2)$$

and

$$f(k_1,k_2,k_3) = 1 + \exp[-j2\pi(k_1/2 + k_2/2 + k_3/2)] = 1 + (-1)^{k_1+k_2+k_3}$$

which is zero when $k_1 + k_2 + k_3$ is odd. The spatial spectrum is shown in Fig. 7b for a $3f_{sc}$ sampling rate. For this sampling pattern, a $2f_{sc}$ sampling rate may be possible. The chrominance and luminance components will not overlap because they are offset in the temporal frequency direction. However, temporal processing is required to reconstruct the analog signal. A three-dimensional low-pass filter which passes the desired signal and rejects all replicated versions is required.

Checkerboard Sampling

An in-field checkerboard sampling pattern is obtained by offsetting the sampling pattern from one field line to the next by half the horizontal sampling interval. This is shown in Fig. 8a. (This pattern has been referred to as line quincunx[5].) If the pattern in one field is offset horizontally by half the sampling interval, this yields again the same pattern, so that only one case need be considered.

The sampling grid for this pattern is a superposition of four rectangular grids, with basic periods T_s, $T_{2\ell} = 2T_\ell$ and T_{fr}:

$$d(x_1,x_2,t) = d_R(x_1,x_2,t) + d_R(x_1 - T_s/2, x_2 - T_{2\ell}/2, t) + d_R(x_1, x_2 - T_{2\ell}/4, t - T_{fr}/2) + d_R(x_1 - T_s/2, x_2 - 3T_{2\ell}/4, t - T_{fr}/2)$$

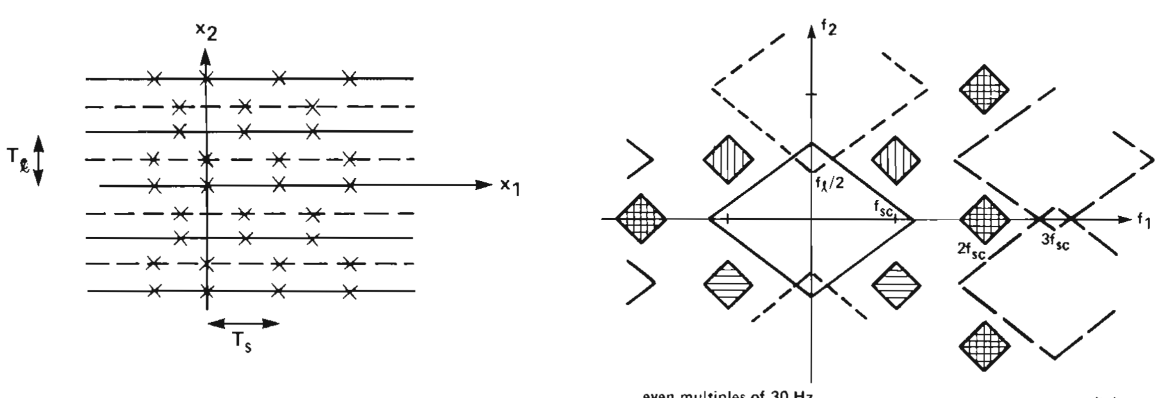


Figure 8. (a) Spatial projection of checkerboard sampling pattern. (b) Spatial projection of spectrum of NTSC signal sampled at $3f_{sc}$ with checkerboard sampling pattern.

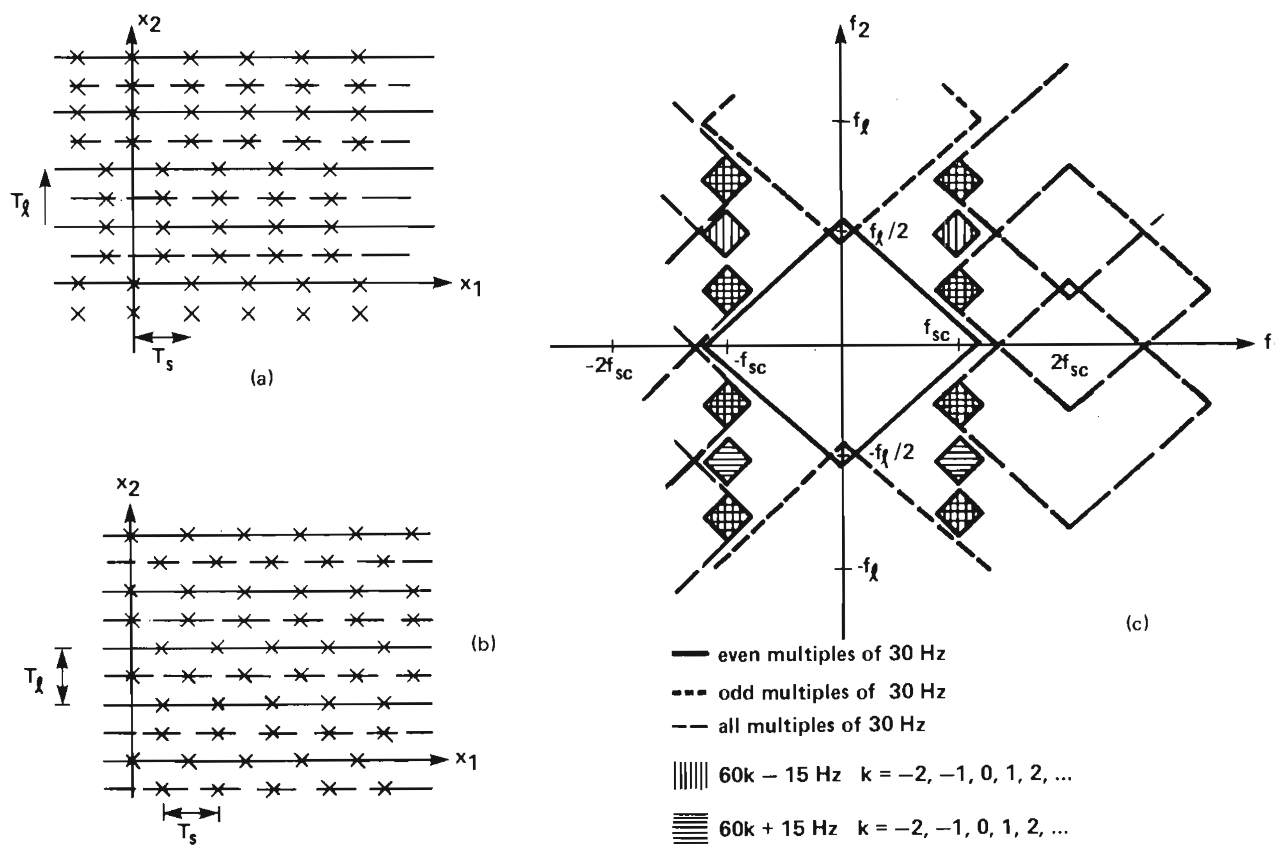


Figure 9. (a) Spatial projection of field-aligned double-checkerboard sampling pattern. (b) Spatial projection of field-offset double-checkerboard sampling pattern. (c) Spatial projection of spectrum of NTSC signal sampled at $2f_{sc}$ with double-checkerboard sampling pattern.

$$f(k_1,k_2,k_3) = 1 + \exp[-j2\pi(k_1/2 + k_2/2)] + \exp[-j2\pi(k_2/4 + k_3/2)] + \exp[-j2\pi(k_1/2 + 3k_2/4 + k_3/2)] = (1 + (-1)^{k_1+k_2}) \cdot (1 + (-j)^{k_2+2k_3})$$

$f(k_1,k_2,k_3) = 0$ if $k_1 + k_2$ is odd, or if $k_2 + 2k_3 = 2 \bmod 4$. The resulting spatial spectrum is shown in Fig. 8b for a $3f_{sc}$ sampling rate.

In this case, it is possible to sample at $2f_{sc}$, since the replicated components are offset in the temporal frequency dimension by 30 Hz, and can thus be removed by temporal filtering. However, it is not possible to remove these components using spatial filtering only, unless the luminance is limited to about 3MHz.

Double Checkboard Sampling

The double-checkerboard sampling grid is shown in Fig. 9. This pattern has been proposed by Rossi for $2f_{sc}$ sampling.[6,7] This grid is constructed as a superposition of eight rectangular grids, of basic periods T_s, $T_{4\ell} = 4T_\ell$ and T_{fr}. Fig. 9a shows the field-aligned pattern and Fig. 9b shows the field-offset pattern. The analysis shows that both patterns lead to the same spectrum. It can be shown that

$$f(k_1,k_2,k_3) = (1 + (-j)^{k_2}) \cdot (1 + (-1)^{k_1+k_2}) \cdot (1 + (e^{-j\pi/4})^{k_2+4k_3})$$

which is zero when $k_2 = 2 \bmod 4$ or $k_1 + k_2$ is odd or $k_2 + 4k_3 = 4 \bmod 8$.

The resulting spatial spectrum is shown in Fig. 9c, for $f_s = 2f_{sc}$. It can be seen that the original spectrum is present unaliased and can be extracted by a spatial filter with pass-band shown in Fig. 10. This is the only pattern we have presented which allows sampling at $2f_{sc}$ and reconstruction with spatial filters only. However, as seen in Fig. 9c, the spectrum is closely packed, and vertical chrominance resolution loss is inevitable. A block diagram of a possible reconstruction

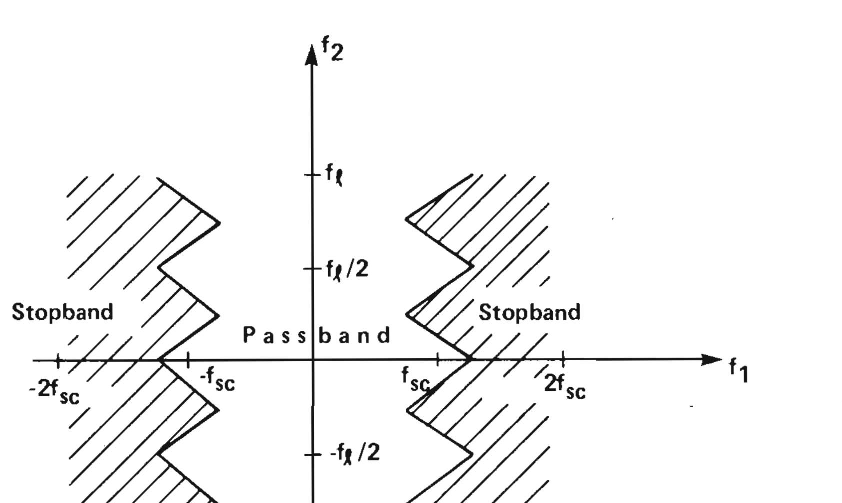


Figure 10. Pass-stop characteristic of ideal two-dimensional filter to reconstruct signal sampled with double-checkerboard sampling pattern.

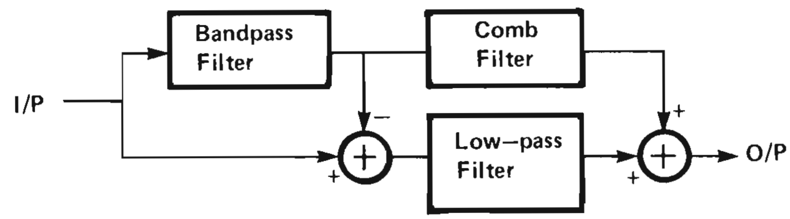


Figure 11. A possible filter structure to approximate the ideal characteristic of Fig. 10.

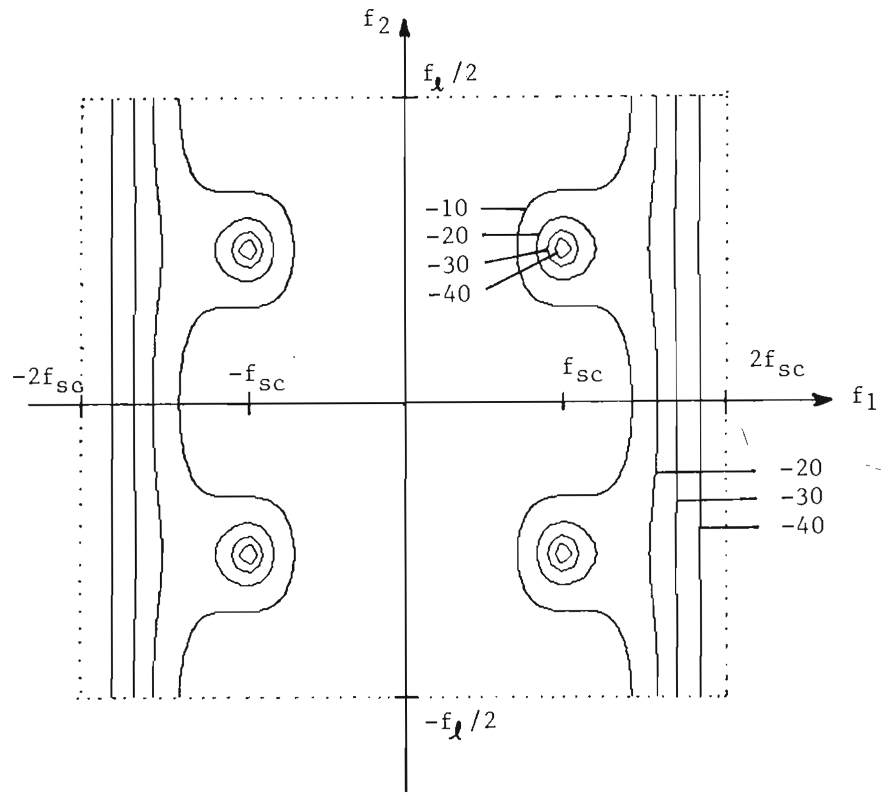


Figure 12. Frequency response of a filter of the type shown in Fig. 11. Attenuation in decibels shown on selected contours.

filter is shown in Fig. 11, and the frequency response of a filter of this type is shown in Fig. 12.

Conclusion

This paper has described the three-dimensional spectrum of NTSC signals, with application to sampling and reconstruction. It has shown that the horizontal sampling rate required depends on the sampling pattern used, and the complexity of reconstruction filters permitted. In particular, the double-checkerboard pattern allows $2f_{sc}$ sampling with reconstruction using spatial filters only, with possible vertical chrominance-resolution loss. The field-offset aligned sampling gives better separation of the components in three-dimensional frequency space, but requires three-dimensional filters for reconstruction. Further work on design and evaluation of such three-dimensional filters is in progress. Other interesting applications include the utilization of three-dimensional filters for demodulation of the composite signal into components. Such three-dimensional approaches are becoming increasingly attractive in view of the declining cost of memory and the advent of high-speed digital processors.

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