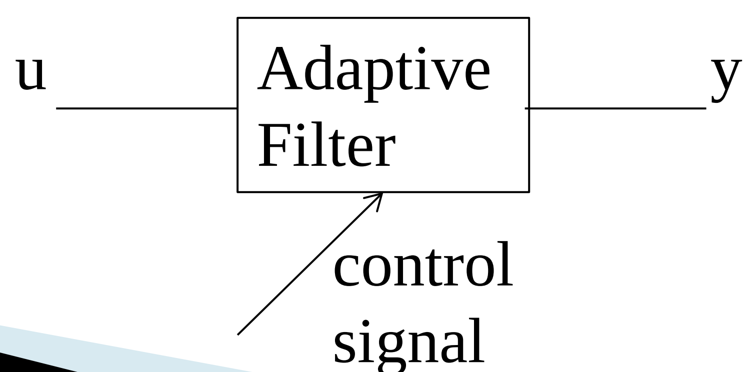
ELG5377 Adaptive Signal Processing

Eric Dubois

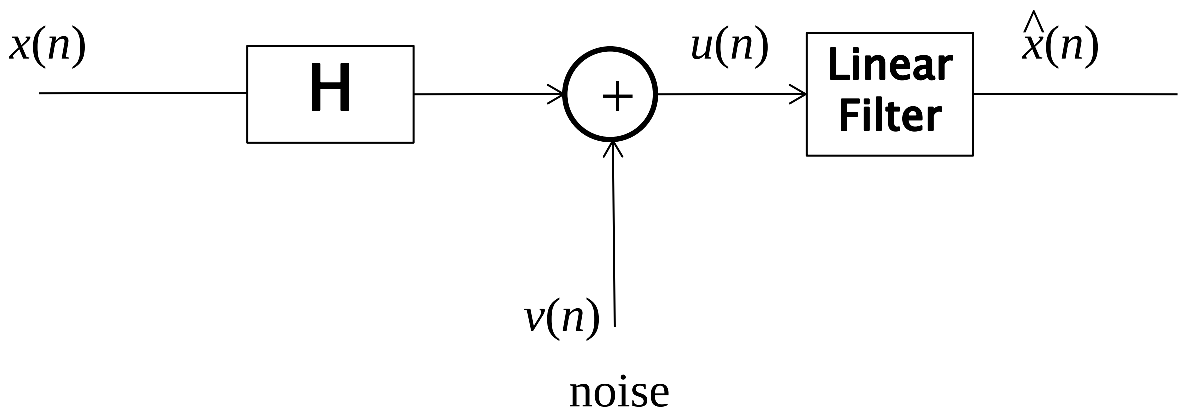
Adaptive Signal Processing

- This course is concerned with adaptive statistical signal processing in discrete time
- The goal is to extract *information* from noisy or corrupted data.
- Usually we want to *estimate* a desired signal from a corrupted version, possibly with the assistance of some auxiliary information.



Basis: Non-adaptive statistical signal processing

- We start by examining non-adaptive statistical signal processing where statistical information is known and fixed



x and v are stochastic processes with known statistics, stationary in time. H is known and fixed.

Statistical Signal Processing

- Under these circumstances, we can find the optimal linear filter to estimate x in the minimum mean squared error (MSE) sense. We will call this the *Wiener Filter*.
- In practice, H is not known and may change over time. Similarly, the statistics of x and v may be unknown and change with time.
- Adaptive filters have been developed for this purpose. They attempt to learn the unknown information and adapt themselves accordingly.

Issues with adaptive filters

- Rate of convergence: if the statistics are fixed, how long does it take to converge to the optimal Wiener filter?
- Misadjustment: Once converged, how does the performance compare with the optimal filter?
- Tracking: If the signal statistics vary over time, how well can the filter track these fluctuations?

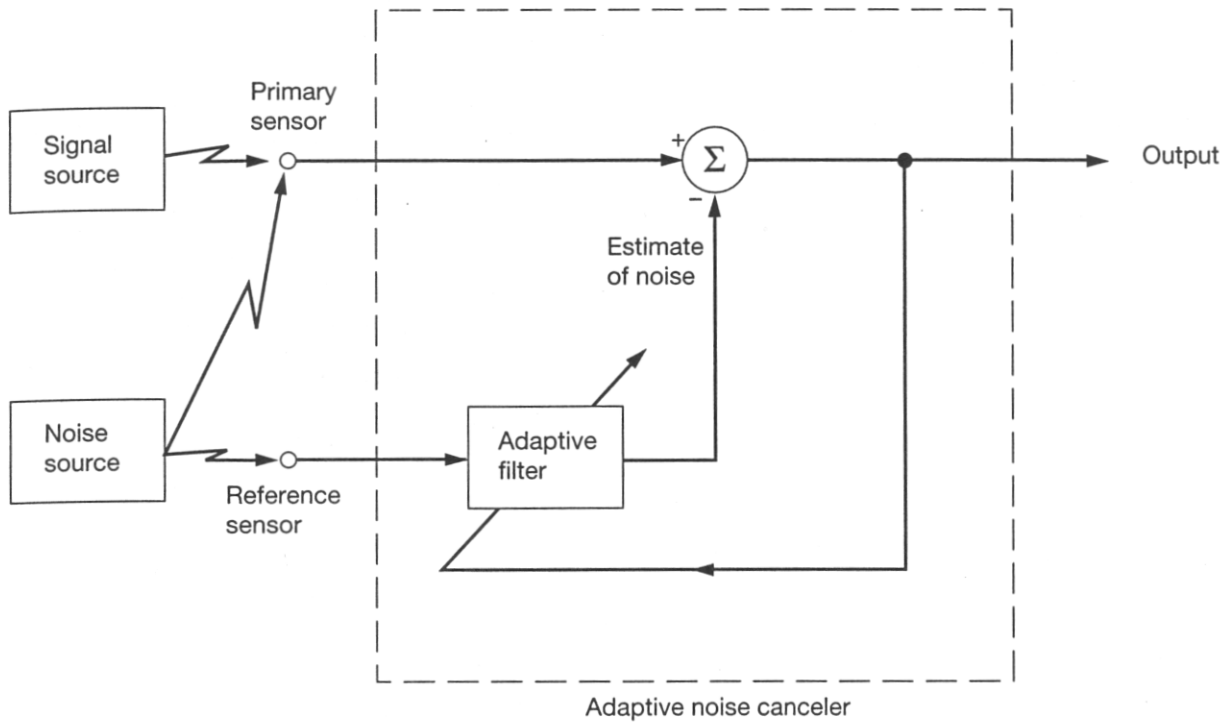
Issues with adaptive filters (2)

- Robustness: Do small disturbances result in small estimation errors?
- Computational complexity: This affects the ability to operate in real time and the cost of the hardware.
- Numerical properties: This comprises the effects of such things as quantization errors on the computations.

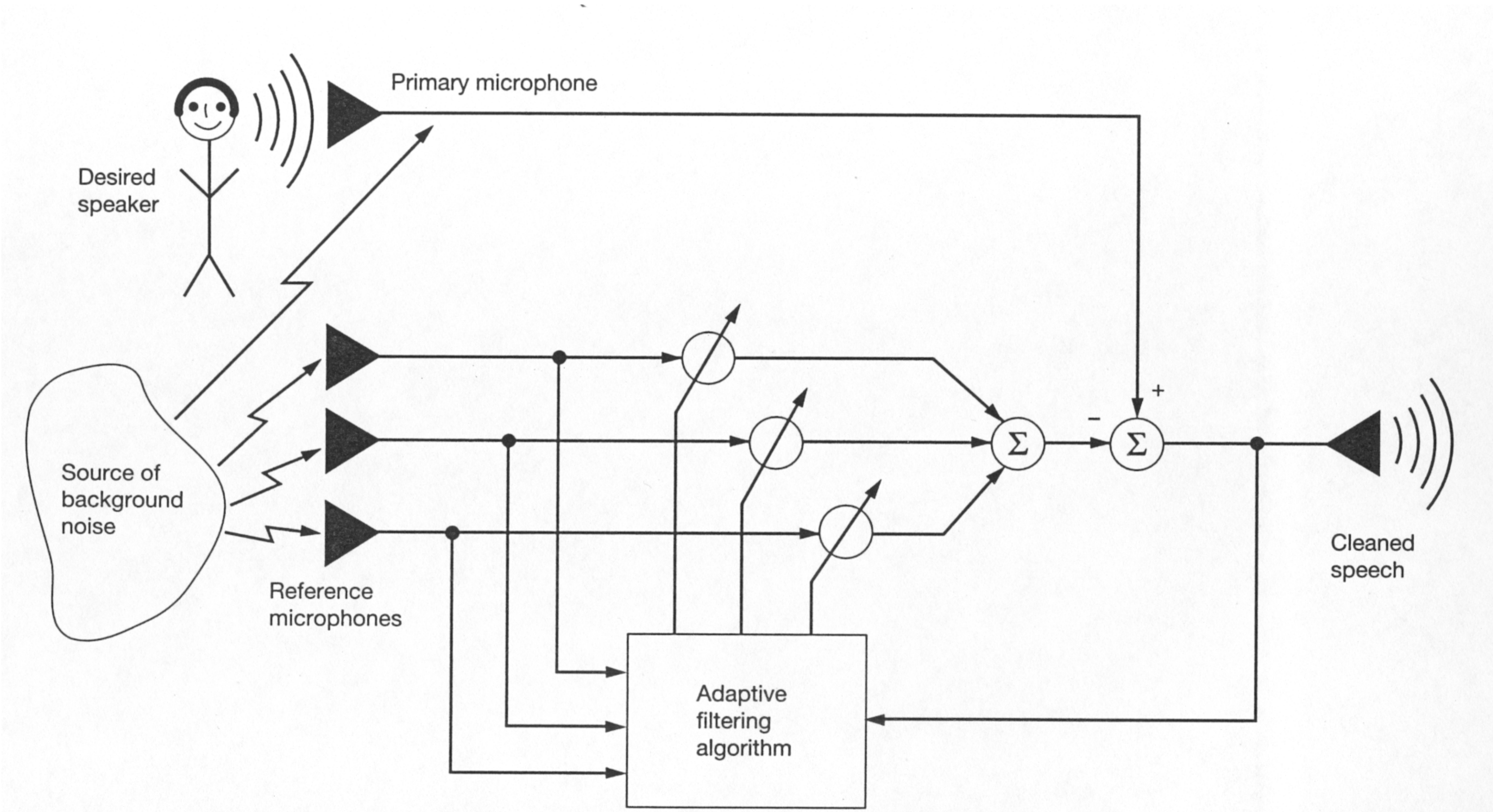
Interference canceling

- Examples:
  - Echos in phone lines; Ghosts in TV pictures
  - Acoustic noise; Room reverberation
- The observed signal is the sum of a desired signal and an undesired interfering signal. We want to remove the undesired signal.
- One way is to estimate the undesired signal and subtract it from the observed signal.
- Sometimes we have additional information, such as training signals, or measurements of the interfering signal alone.

Noise canceler

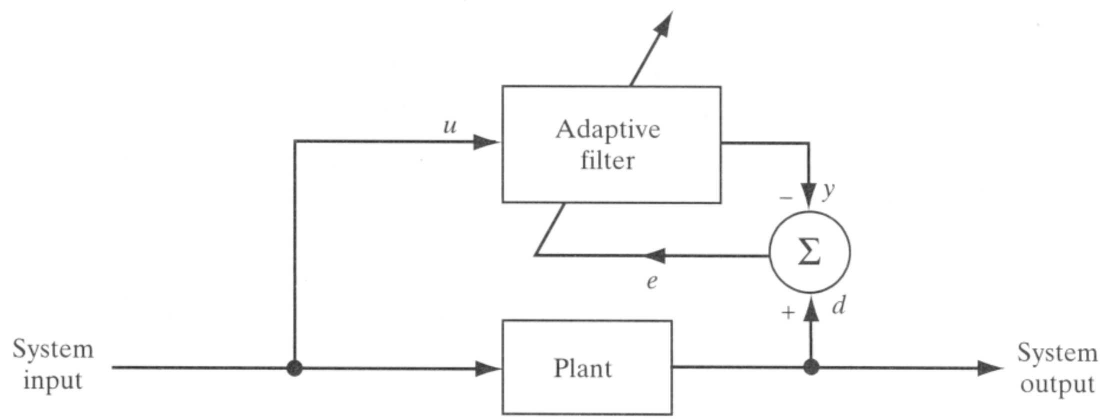


Speech Noise Canceler



Four classes of applications

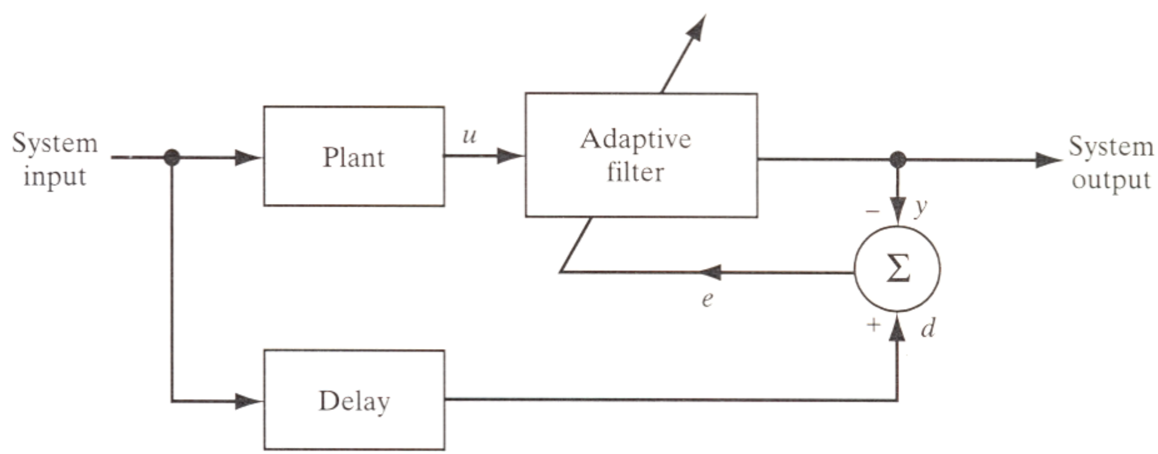
- Identification: find a linear model of an unknown system (plant).



Examples: system identification; layered earth modeling

Four classes of applications (2)

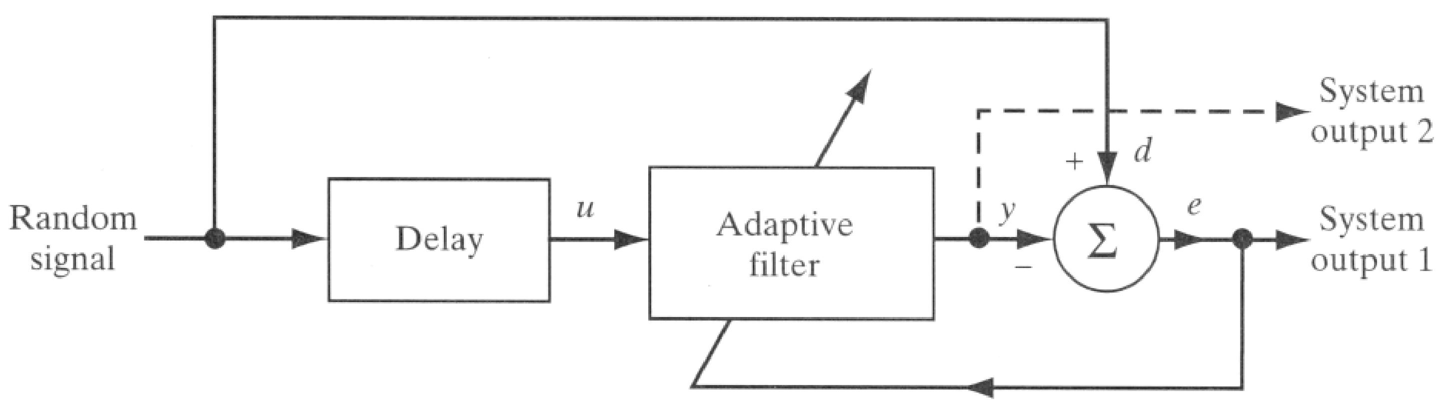
- Inverse modeling: find a linear inverse model of an unknown system (plant).



Example: Communication channel equalization

Four classes of applications (3)

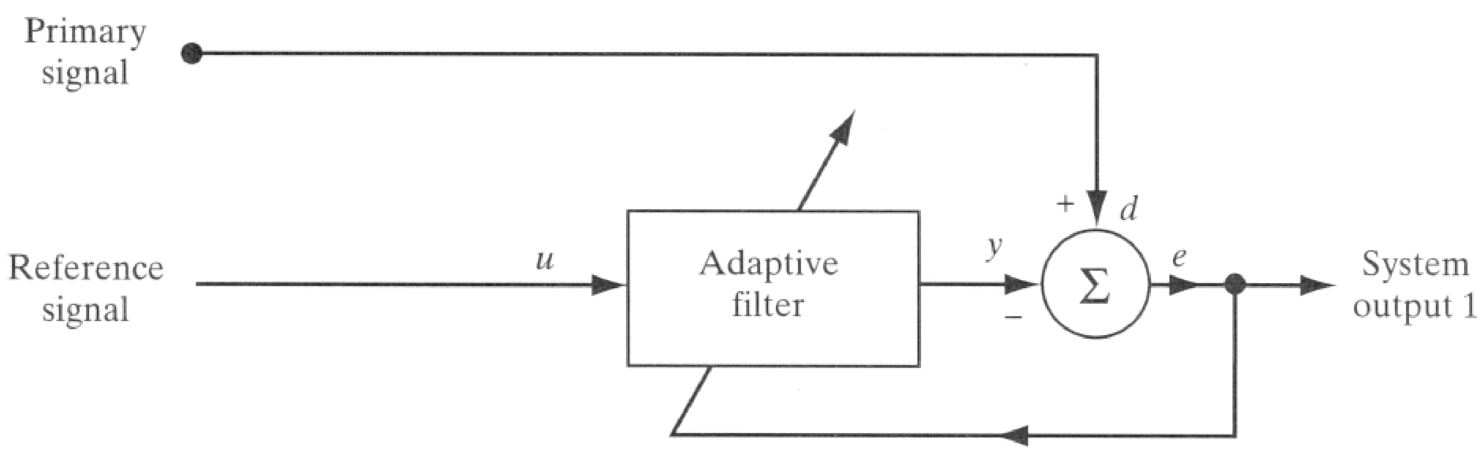
- Prediction: Estimate future values of a random signal.



Example: Speech prediction for compression.

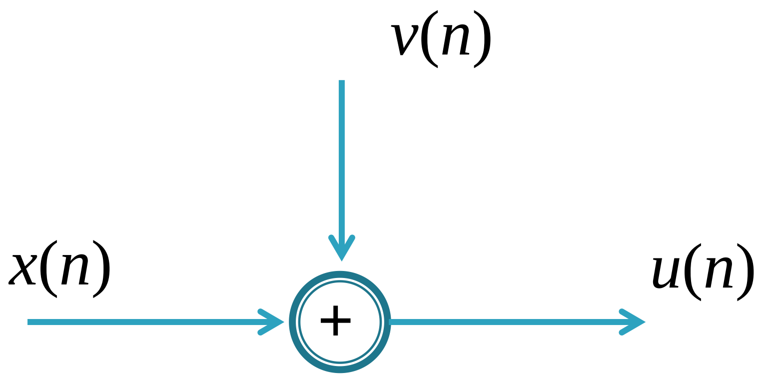
Four classes of applications (4)

- Interference cancellation: cancel an unknown interfering signal.

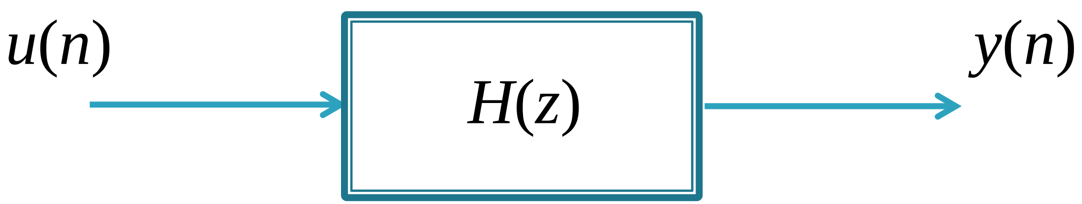


Example: audio noise cancellation

Typical Problem



We wish to estimate the signal $x(n)$ from the noisy observed signal $u(n)$ with a linear FIR filter $H(z)$.



Signal Estimation: Assumptions

1. x and v are complex signals. Complex formulations are widely used in signal processing. It is easy to specialize to real signals.
2. $H(z)$ is a complex FIR filter of order M.

$$H(z) = \sum_{k=0}^{M-1} w_k^* z^{-k}$$

Thus

$$y(n) = \sum_{k=0}^{M-1} w_k^* u(n-k)$$

Signal Estimation: Assumptions

3. The estimation error is $x(n) - y(n)$. We want to choose the filter coefficients w_k to minimize some average measure of error (MSE). We use whatever information we have about x and v to do this.

Signal Estimation: Approaches

1. Assume that x and v are independent stationary random processes. Thus $X(n)$, $V(n)$, $U(n)$, $Y(n)$ are random variables. We choose the w_k to minimize
$$E[|X(n) - Y(n)|^2].$$
The result is known as the Wiener filter.

Signal Estimation: Approaches

2. Assume that the characteristics of the signal and noise don't change over time. We have a set of known representative data $x_t(n)$ that we can pass through the system to measure the output $u_t(n)$, $n = 0, 1, \ldots, N\text{-}1$, where N is quite large. We choose the w_k to minimize

$$\sum_{n=0}^{N-1} |x_t(n) - y_t(n)|^2$$

The result is called the least-squares filter.

Signal Estimation: Approaches

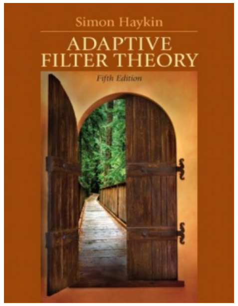
- Both of these approaches (and many others) can be formulated as a *projection* in an inner-product vector space.
- In the first case, the vector space is formed of random variables.
- In the second case, the vector space is formed of sequences of N signal values.
- Thus we will:
  - Review vector spaces, inner products and the projection theorem.
  - Review random process models of signals.

The Course Outline

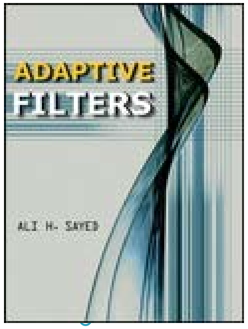
Professor

- Eric Dubois
- CBY A-512
- Tel: 562-5800 X 6400
- edubois@uottawa.ca
- www.eecs.uottawa.ca/~edubois/courses/ELG5377

References

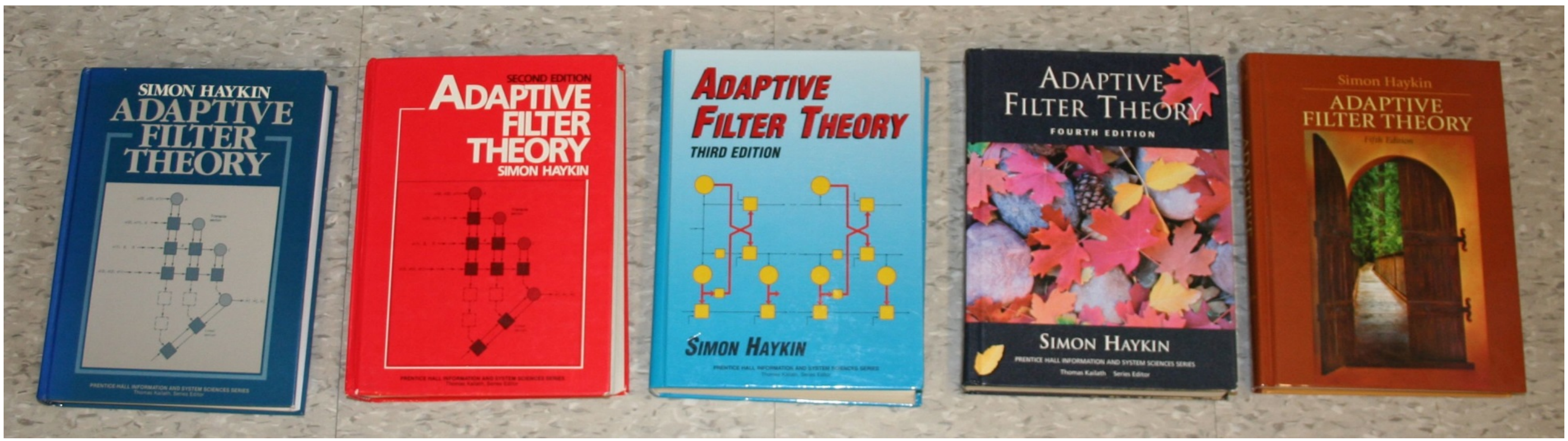


- S. Haykin, *Adaptive Filter Theory*, fifth edition, Prentice Hall, 2013.



- A. Sayed, Adaptive Filters, Wiley-IEEE Press, 2008.
  - E-book at uOttawa
  - On IEEE Xplore

Haykin Editions



1986 1991 1996 2002 2013

Prerequisites

- Basic probability and digital signal processing as typically obtained in an undergraduate Electrical Engineering program
- (e.g., at uOttawa,
  - ELG4177 Digital Signal Processing,
  - ELG3126 Random Signals and Systems
- Linear Algebra

Objective

- The objective of this course is to provide the mathematical framework for an understanding of adaptive statistical signal processing, where the goal is to extract information from noisy or corrupted data.

- The basic tools of vector spaces and discrete-time stochastic processes are reviewed and applied to the methods of Wiener filtering and least-squares filtering.

Objective

- Various types of adaptive filters will be introduced and their properties will be studied, specifically convergence, tracking, robustness and computational complexity.

- Applications will mainly be addressed through student projects.

Course Outline

1. Introduction
   - scope and objectives of the course
   - overview of issues in adaptive filtering
   - survey of a few applications

2. Signal spaces
   - vector spaces
   - inner product
   - Projection theorem

3. Random processes and signal modeling
   - discrete-time random processes
   - correlation and power spectrum
   - models: linear processes, harmonic processes, AR, MA, ARMA processes

Course Outline (2)

4. Wiener filters and linear prediction
    - optimal linear filtering
    - forward and backward prediction
    - Levinson-Durbin algorithm
    - lattice predictors
    - joint-process estimation

5. LMS adaptive filtering
    - method of steepest descent
    - LMS algorithm
    - stability and performance analysis

6. Method of least squares
    - least-squares solution
    - Properties
    - singular value decomposition and pseudo-inverse
    - recursive least-squares method

Grading

- 20% **Assignments**: Several assignments, to be handed in during class on the due-date specified. There will be a 5% penalty for each day late, and no assignment will be accepted after one week.

- 30% **Project**: An individual project on an **application** of adaptive signal processing involving some experimental work. A project report and presentation at the end of the course will be required. More details will follow early in the course.

- 20% **Midterm exam**: Closed-book exam, 80 minutes in length.

- 30% **Final exam**: Closed-book exam, 3 hours in length, covering the whole course.

Blackboard Learn

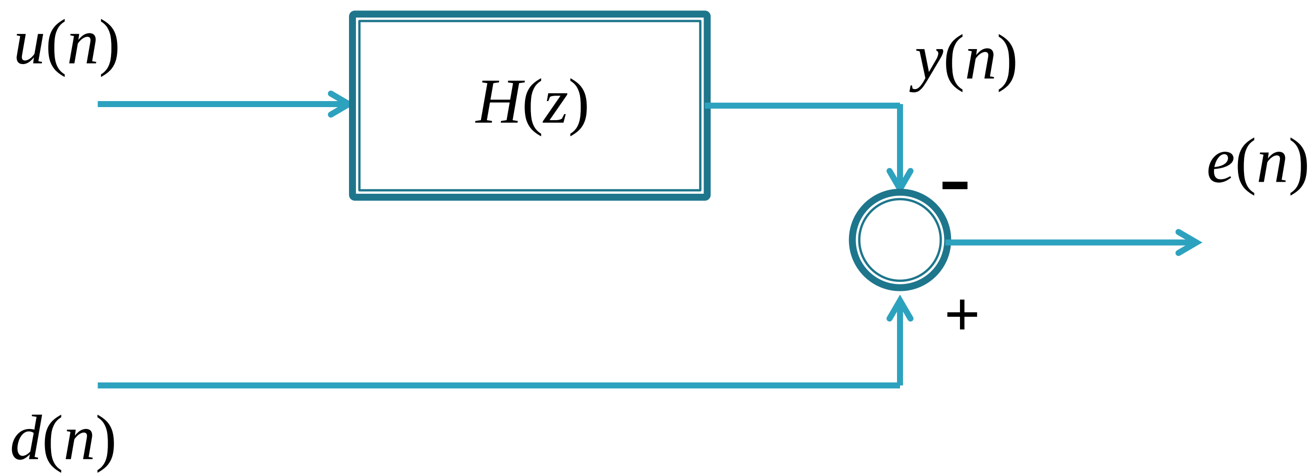
- Will be used for exercises and exercise solutions
- Assignment submission
- Project file submission
- Grades are posted there

- Link to Blackboard Learn

- Carleton students, please see me to arrange access to Blackboard Learn site for this course.

Signal Estimation Problem

- We wish to estimate a desired signal *d* based on an observed signal *u* using FIR filtering.



$$H(z) = \sum_{k=0}^{M-1} w_k^* z^{-k}$$

Signal Estimation Problem

$$y(n) = \sum_{k=0}^{M-1} w_k^* u(n-k)$$

This can also be expressed in terms of signals and the delay operator $\mathcal{T}_k$

$$y = \sum_{k=0}^{M-1} w_k^* (\mathcal{T}_k u)$$

The estimate y is a *linear combination* of the M shifted versions of u. These are assumed to belong to a *vector space*.

Next lectures – review of vector spaces

- We will review the definition of a vector space and some basic properties.
- We introduce the inner product and the concept of orthogonality.
- We prove the projection theorem which is the basis of most of this course.