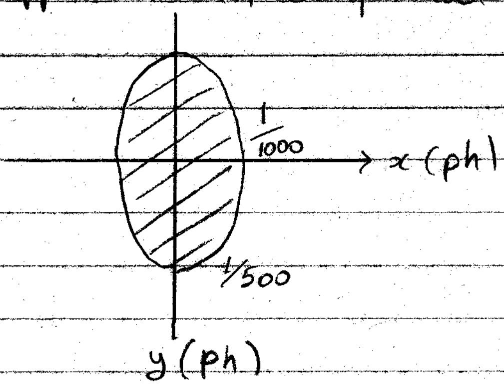
CEG 4311 Mid-term Exam
Oct. 2001
Solutions

1 (a) $h(x,y) = \begin{cases} \frac{1}{2\pi R_1 R_2} & \left(\frac{x}{R_1}\right)^2 + \left(y/R_2\right)^2 \le 1 \\ 0 & \text{otherwise} \end{cases}$

The region of support is an elliptical region



$$h(x,y) = \frac{1}{2\pi R_1 R_2} \operatorname{circ}\left(x/R_1,\ y/R_2\right) = \frac{250\,000}{\pi} \operatorname{circ}(1000x,\ 500y)$$

(b) Use property (i), and (v) with $\underline{A} = \begin{bmatrix} 1000 & 0 \\ 0 & 500 \end{bmatrix}$

$\det \underline{A} = 500{,}000 \qquad \underline{A}^{-T} = \begin{bmatrix} 1/1000 & 0 \\ 0 & 1/500 \end{bmatrix}$

Thus $$H(u,v) = \frac{250{,}000}{\pi} \cdot \frac{1}{500{,}000} \cdot \frac{J_1\left(2\pi\sqrt{\left(\frac{u}{1000}\right)^2 + \left(\frac{v}{500}\right)^2}\right)}{\sqrt{(u/1000)^2 + (v/500)^2}}$$

$$= \frac{1}{2\pi} \frac{1}{\sqrt{(u/1000)^2 + (v/500)^2}} J_1\left(2\pi\sqrt{\left(\frac{u}{1000}\right)^2 + \left(\frac{v}{500}\right)^2}\right)$$

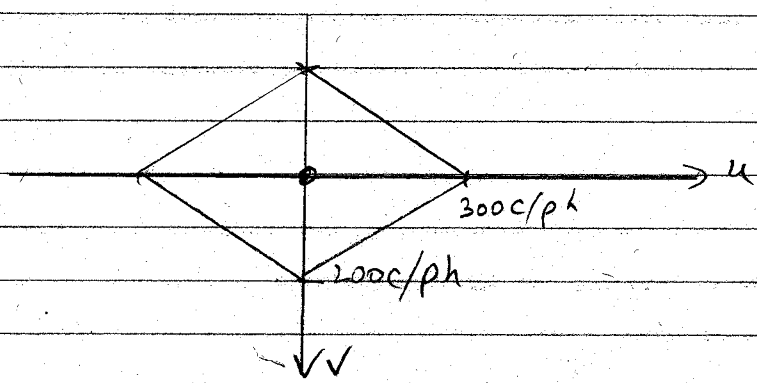
(c) Use property (ii) with $x_0 = (.5, .5)$ and property (v) with $\underline{A} = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$ $\det \underline{A} = 10$ $\underline{A}^{-T} = \begin{bmatrix} 1/5 & 0 \\ 0 & 1/2 \end{bmatrix}$.

$$F(u,v) = \exp(-j2\pi(.5u + .5v)) \cdot \frac{1}{10} \frac{\sin\left(\frac{\pi u}{5}\right) \sin\left(\frac{\pi v}{2}\right)}{\frac{\pi u}{5} \quad \frac{\pi v}{2}}$$

$$= \exp(-j2\pi(.5u + .5v)) \frac{\sin\frac{\pi u}{5} \sin\frac{\pi v}{2}}{\pi^2 uv}$$

From property (vi), $G(u,v) = H(u,v)\, F(u,v)$

2.



(a) The region of support of $F(u,v)$ will tile entire frequency domain with no overlap and no empty space if

$$\Lambda_1^* = \mathrm{LAT}\left(\begin{bmatrix} 300 & 0 \\ 200 & 400 \end{bmatrix} c/ph\right)$$

$$\therefore \Lambda_1 = \mathrm{LAT}\left(\begin{bmatrix} 300 & 0 \\ 200 & 400 \end{bmatrix}^{-T}\right) = \mathrm{LAT}\left(\begin{bmatrix} 1/300 & -1/600 \\ 0 & 1/400 \end{bmatrix} ph\right)$$

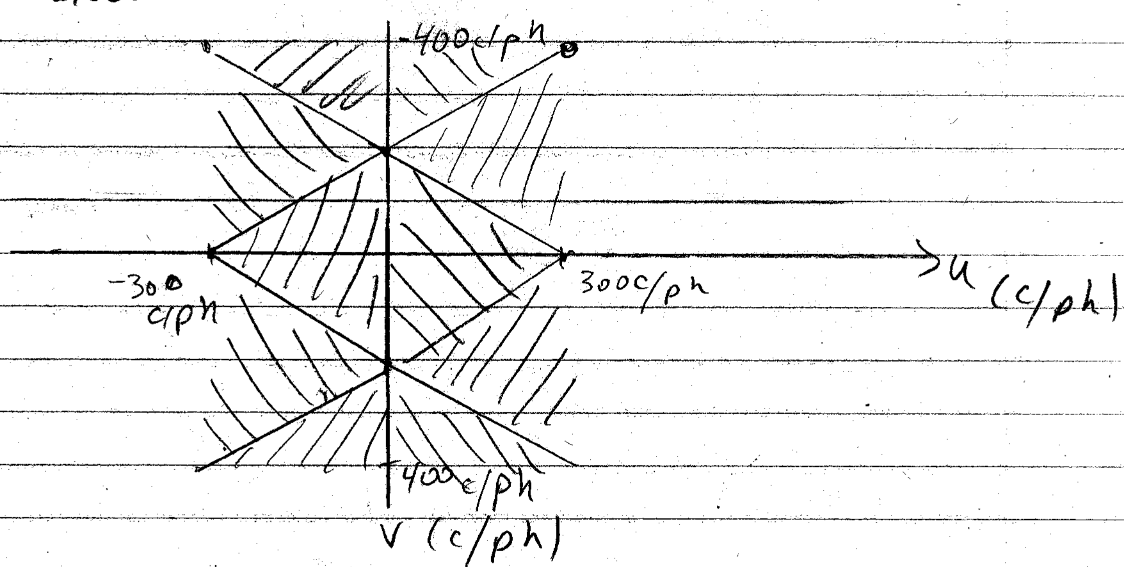
If we use rectangular sampling, we need

$$\Lambda_2^* = \mathrm{LAT}\left(\begin{bmatrix} 600 & 0 \\ 0 & 400 \end{bmatrix} c/ph\right) \therefore \Lambda_2 = \mathrm{LAT}\left(\begin{bmatrix} 1/600 & 0 \\ 0 & 1/400 \end{bmatrix} ph\right)$$

$d(\Lambda_1) = \frac{1}{120000}\ ph^2$ $\therefore$ sampling density is 120,000 $\frac{\text{samples}}{ph^2}$

$d(\Lambda_2) = \frac{1}{240000}\ ph^2$ sampling density is 240,000 sample/ph^2

(b)



(c) The picture area is $pw \times ph = \frac{4}{3}ph \times ph = \frac{4}{3}ph^2$

For hexagonal sampling on Λ_1, number of samples is

$$120000 \frac{\text{samples}}{ph^2} \times \frac{4}{3}ph^2 = 160{,}000 \text{ samples}.$$

For rectangular sampling on Λ_2

$$240{,}000 \frac{\text{samples}}{ph^2} \times \frac{4}{3}ph^2 = 320{,}000 \text{ samples}$$

3. (a) By the linearity of the expression, it is easy to see the the response to αf is αg, and the response to $f_1 + f_2$ is $g_1 + g_2$. Also, it is easy to see that if $f_1[n_1X, n_2X] = f[(n_1-m_1)X, (n_2-m_2)X]$ then $g_1[n_1X, n_2X] = g[(n_1-m_1)X, (n_2-m_2)X]$.

(b) By noting that the expression for $g[n_1X, n_2X]$ is written in the form of a convolution, we can identify

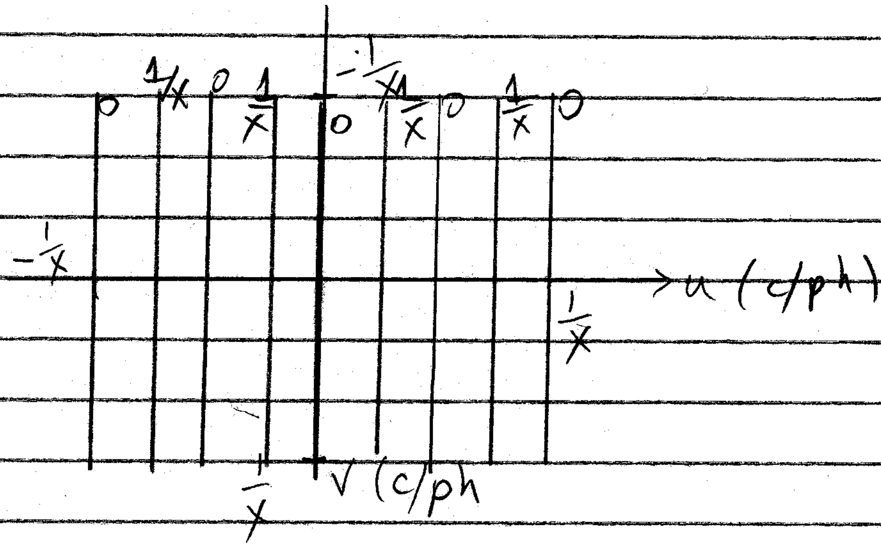
$$h[n_1X, n_2X] = \begin{cases} 1/2X & (n_1, n_2) = (-1, 0) \\ -1/2X & (n_1, n_2) = (+1, 0) \\ 0 & \text{otherwise} \end{cases}$$

Alternatively, by applying $f[n_1X, n_2X] = \delta[n_1X, n_2X]$ we get the same conclusion, $h[n_1X, n_2X] = \frac{1}{2X}\delta[(n_1+1)X, n_2X] - \frac{1}{2X}\delta[(n_1-1)X, n_2X]$

(c) $$H(u,v) = \frac{1}{2X}\exp(-j2\pi u\cdot(-X)) - \frac{1}{2X}\exp(-j2\pi uX)$$

$$= \frac{j}{X}\sin(2\pi uX)$$

(d)



$|H(u,v)| = \frac{1}{X}|\sin 2\pi u X|$ independent of v, so contours are vertical lines.

$|H(-u,v)| = |H(u,v)|$

For a few values of u

u	$\lvert H(u,v)\rvert$
0	0
$1/4X$	$1/X$
$1/2X$	0
$\frac{3}{4X}$	$1/X$
$\frac{1}{X}$	0