CEG4311 Image Processing
Fall 2007
Problem Set 3
Due 12:00 noon, December 10, 2007 in CEG4311 assignment box

1. Consider the following 16×8 data block from the image alfred_Y used in lab 3, representing two horizontally adjacent blocks $\mathbf{f}^{(1)}$ and $\mathbf{f}^{(2)}$:

```
238 234 222 241 206 155 129  57  77  90 103 116 130 125 150 171
234 232 219 234 201 148 146 121 128 131 123 111  96  91 158 168
229 231 216 242 197 144 127  92  82  78  81  91  99 106 163 169
224 231 226 235 189 150 138  87 114 126 131 134 129 111 150 172
215 220 226 231 190 152 141 119 109  99  85  80  70  87 167 164
212 223 229 235 195 157 125  61  79  94 104 117 122 119 151 168
231 232 229 242 194 150 147 120 127 134 126 112  93  95 164 168
225 233 240 244 202 160 127  87  83  80  79  84  98 103 163 165
```

The MATLAB statements to generate the two blocks are:

```
f = imread('alfred_Y.tif');
f1 = double(f(137:144,457:464));
f2 = double(f(137:144,465:472));
```

In the following questions, you can use MATLAB wherever useful. If you do so, include the MATLAB statements used as part of your solution.

(a) Compute the two-dimensional 8×8 DCTs of the two horizontally adjacent 8×8 blocks (minus 128) and display them to integer precision. Verify for each block that $\sum_{m=0}^{7}\sum_{n=0}^{7}(f^{(i)}[m,n]-128)^2 = \sum_{k=0}^{7}\sum_{l=0}^{7} f_b^{(i)}[k,l]^2$. This holds for the original f_b, not the rounded ones.

(b) We want to quantize this block with a set of uniform quantizers whose step sizes are given by the matrix $qf\,\mathbf{\Delta}_0$, where $qf = 1.2$ and $\mathbf{\Delta}_0$ is given on page 5-8 of the chapter 5 course notes. Display the output of the quantizer encoders $\mathcal{E}_{kl}$ and the output of the quantizer decoders $\mathcal{D}_{kl}$ as matrices for the two blocks of DCT coefficients.

(c) Compute the inverse DCT of the quantized DCT coefficients for the two blocks, and display them to integer precision (don't forget to add 128). Compute the squared error between the original image and the quantized image in the space domain and in the transform domain, and verify that they are equal (as per pg. 4-6 of the notes). Compute the PSNR for each block.

(d) We now want to do the code assignment for the output of the quantizer encoders, as found in (b) for the second block $\mathbf{f}^{(2)}$. Give the list of symbols to be encoded for the block in zigzag order (chapter 5, pg. 5-9). The possible symbol names are: $C_i, i = 0, \ldots, 15$ (the category from table B), $(Z)_i, i = 1, \ldots, 15$ where Z is a signed integer and $2^{i-1} \leq |Z| < 2^i$, and R/C (for Run/Category, called Run/Size in Table K.5), and EOB (Table K.5). For example, the third element in the row of Table A for category 4 is denoted $(-13)_4$.

(e) Give the coded binary stream for the list of symbols you found in (d). Show the codeword boundaries to make it easy to check your result. Explain why the overall code that is used for a DCT block is uniquely decodable. How many bits are needed to represent the coded block. What is the compression ratio for this one block?