CEG4311 Digital Image Processing
Fall 2007
Problem Set 1
Due Oct. 26, 2007 by 3 PM in the CEG4311 assignment box.

1. A *continuous-space* linear shift-invariant (LSI) system has impulse response $h_1(x, y) = c\exp(-(x^2 + y^2)/2r^2)$.

(a) What is the frequency response of this system?

(b) Find the value of the constant c so that the DC gain of the filter is 1.0. What is the significance of the DC gain?

(c) What is the output of this filter if the input is $f(x, y) = \delta(x - r, y - 0.5r)$?

(d) What is the output of the filter if the input is $f(x, y) = 2\cos(2\pi(\frac{x}{4r} + \frac{y}{2r}))$?

(e) Answer (a) to (d) again for the LSI system with impulse response $h_2(x, y) = c\exp(-(\frac{x^2}{4r^2} + \frac{y^2}{r^2}))$?

2. A two-dimensional signal defined on the rectangular lattice Λ with horizontal sample spacing X and vertical sample spacing Y has the Fourier transform

$$F(u, v) = 2 + 4\cos(2\pi uX) + j2\sin(2\pi vY) - 8e^{-j2\pi uX}e^{-j2\pi vY}.$$

Determine an expression for the signal $f[x, y]$, $(x, y) \in \Lambda$.

3. For each of the following pairs of lattices Λ_1 and Λ_2, state whether $\Lambda_1 \subset \Lambda_2$, $\Lambda_2 \subset \Lambda_1$ or neither. If neither, find (by inspection) the least dense lattice Λ_3 such that $\Lambda_1 \subset \Lambda_3$ and $\Lambda_2 \subset \Lambda_3$. For each lattice Λ_1, Λ_2 and Λ_3 (if required), determine and sketch the reciprocal lattice and a unit cell of the reciprocal lattice. For each of these pairs of lattices, specify a sampling structure conversion system to transform a signal $f[\boldsymbol{x}]$ sampled on Λ_1 to a signal $g[\boldsymbol{x}]$ sampled on Λ_2. Assume that ideal lowpass filters are used where filters are required and sketch their passband in the frequency domain and indicate the gain.

(*i.*) $V_{\Lambda_1} = \begin{bmatrix} X & 0 \\ X & 2X \end{bmatrix}$ $\qquad V_{\Lambda_2} = \begin{bmatrix} 2X & 2X \\ 0 & 2X \end{bmatrix}$

(*ii.*) $V_{\Lambda_1} = \begin{bmatrix} 1.5X & 0 \\ 0 & 1.5X \end{bmatrix}$ $\qquad V_{\Lambda_2} = \begin{bmatrix} X & 0 \\ 0 & X \end{bmatrix}$