CEG4311 **Image Processing** **April 22, 1999**

Final exam **Duration: 3 hours**

Professor: Eric Dubois

Closed-book exam: you may not use any books or notes. Answer all questions. You may use a pocket calculator. However, explain all calculations; I am more interested in the reasoning than in precise numerical answers. Unless otherwise specified, you may use the results provided on page 5 without proof if you wish.

Vous pouvez répondre en anglais ou en français.

---

1. A typical sampling raster in digital television consists of an orthogonal grid of 720 samples per horizontal line (picture width or pw) by 480 samples per vertical column (picture height or ph) with an aspect ratio pw/ph of 4:3.

7 (a) Determine numerically in units of ph the horizontal and the vertical sample spacing X and Y respectively, and a sampling matrix V_Λ for the underlying lattice Λ. Compute numerically the determinant of the lattice and give its physical meaning. Determine numerically a sampling matrix for the reciprocal lattice in units of cycles/picture height (c/ph).

9 (b) An image defined on the lattice Λ of (a) is processed with a linear shift-invariant filter with unit sample response

$$h(mX, nY) = \begin{cases} 1 & (m,n) = (-1,0), (0,1), (-1,1) \\ -1 & (m,n) = (1,0), (0,-1), (1,-1) \\ 0 & \text{otherwise} \end{cases}$$

Draw a sketch, *to scale*, to illustrate the unit sample response. Determine the frequency response $H(u,v)$ of this filter, where u and v are in c/ph. Show that the frequency response is purely imaginary and express it in the form $H(u,v) = jH_I(u,v)$ where $H_I(u,v)$ is real. What is the DC response of the filter $H(0,0)$? Sketch to scale the reciprocal lattice and identify on your sketch one period of the frequency response $H(u,v)$.

9 (c) An image $f(mX, nY)$ is processed with this filter to give an output image $g(mX, nY)$. Write the general expression for the filter output g in terms f and h (2D convolution) and simplify the expression for the particular unit sample response of (b). Determine and sketch the response to the two test images

$$f_1(mX, nY) = \begin{cases} 1 & n \geq m \\ 0 & n < m \end{cases}$$

and

$$f_2(mX, nY) = \begin{cases} 1 & n \geq 479 - m \\ 0 & n < 479 - m \end{cases}$$

Assume that the origin (0,0) is in the lower left corner of the image. Ignore the effects at the boundary of the image. Describe qualitatively the effect you would observe if you processed a real image like `barbara` with this filter.

2. A CCD sensor consists of an array of cells as shown in Fig. 1. Each cell measures the integral of the illuminance $f_c(x,y)$ over the cell and associates the result with the corresponding point on a hexagonal lattice. This can be modeled by:

$$f(x,y) = \int_{-X/2}^{X/2} \int_{-X/2}^{X/2} f_c(x+s, y+t)\, ds\, dt, \qquad (x,y) \in \Lambda$$

where Λ is generated by the sampling matrix

$$V_\Lambda = \begin{bmatrix} X & X/2 \\ 0 & X \end{bmatrix}.$$

13 (a) This process can be modeled as the prefiltering the continuous-space input image by a continuous space filter with impulse response $h_a(x,y)$ followed by ideal sampling on Λ. Give the expression for $h_a(x,y)$. Evaluate the frequency response $H_a(u,v)$.

12 (b) Suppose that we want to convert the image sampled on Λ to a rectangularly sampled

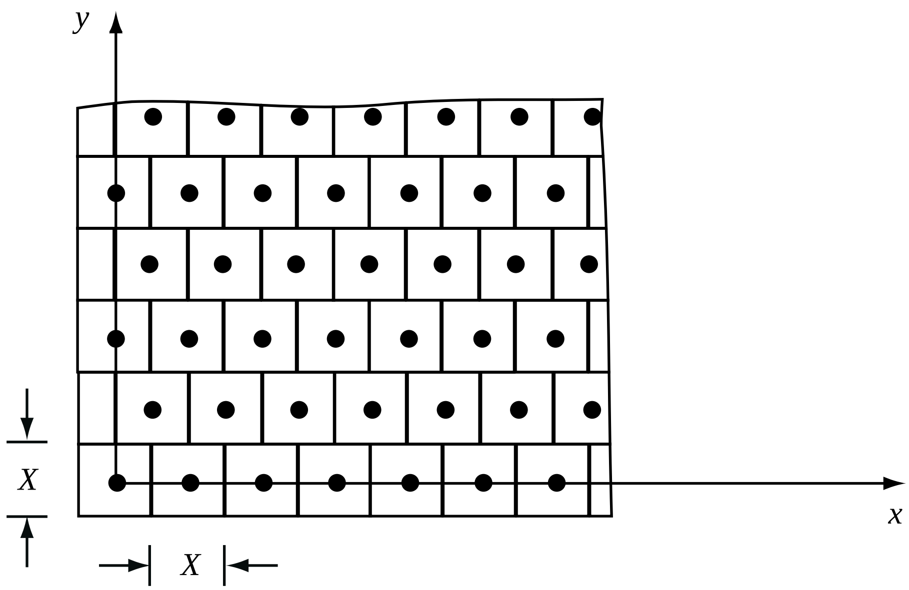

Figure 1: Lower left portion of hexagonal CCD sensor.

image to simplify further processing. Specifically, we will upsample the image $f(x, y)$ to the lattice Γ specified by the sampling matrix

$$V_\Gamma = \begin{bmatrix} X/2 & 0 \\ 0 & X/2 \end{bmatrix}.$$

This can be achieved by zero-insertion followed by a lowpass filter. Describe these operations in detail, and give the possible frequency response of an ideal lowpass filter for this purpose. Illustrate the lattices Λ and Γ as well as the reciprocal lattices Λ^* and Γ^*.

3. Fig. 2 illustrates 4 basis functions that could be used for the representation of 2 by 2 images.

9 (a) Is this basis orthogonal? Is this basis orthonormal? Is the basis separable? If the basis is not orthonormal, convert it to an orthonormal basis.

7 (b) Sketch (in a similar style to Fig. 2 and with an appropriate legend) the 16 vectors of a disjoint-block orthonormal basis derived from the basis of Fig. 2 for the space of 4 by 4 images.

9 (c) Compute (and carefully identify) the 16 coefficients in the expansion of the following

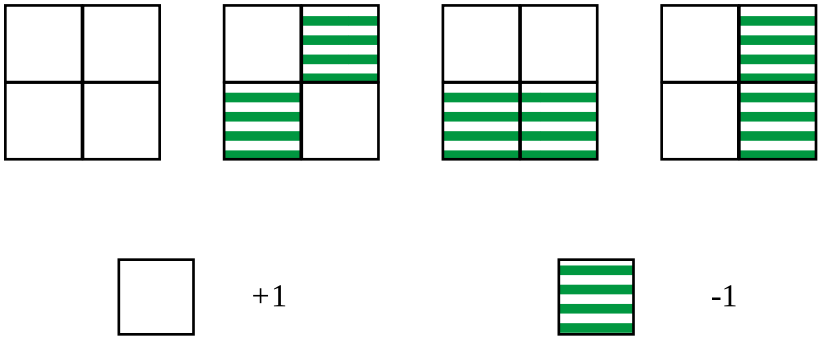

Figure 2: Four basis vectors for space of 2 by 2 images.

4 by 4 image f with respect to your orthonormal basis. What image is reconstructed using 25% of the coefficients, namely one coefficient per 2 by 2 block? Compute the mean-squared error between this reconstructed image and the original.

130	133	70	51
179	151	70	73
192	135	95	144
156	102	128	193

4. JPEG is currently the most widespread system for compression of still images. You have studied various aspects of this coding algorithm in class, in the last problem set and in the last computer assignment.

15 (a) Give a complete block diagram of a JPEG encoder that converts an RGB image to a binary file. Assume that the transform encoder operates on luminance and chrominance, where the two chrominance components are subsampled by two in both the horizontal and vertical dimension. Describe clearly the purpose and operation of each component in your system. Of course you are not expected to know the actual parameters, codes, tables etc. that are used, but you should illustrate what they are like in as much detail as you can.

10 (b) Give a complete block diagram of a JPEG decoder that converts a binary file generated with the encoder of (a) to an RGB image. Again, describe clearly the purpose and operation of each component in your system.

CEG4311 Image Processing

Final exam

Formulas

Continuous-space two-dimensional Fourier transform:

$$F_a(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_a(x,y)\exp(-j2\pi(ux+vy)\,dx\,dy$$

$$f_a(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_a(u,v)\exp(j2\pi(ux+vy)\,du\,dv$$

Discrete-space Fourier transform on a lattice Λ:

$$F(\boldsymbol{u}) = \sum_{\boldsymbol{x}\in\Lambda} f(\boldsymbol{x})\exp(-j2\pi\boldsymbol{u}\cdot\boldsymbol{x})$$

$$f(\boldsymbol{x}) = d(\Lambda)\int_{\mathcal{P}^*} F(\boldsymbol{u})\exp(j2\pi\boldsymbol{u}\cdot\boldsymbol{x})\,\boldsymbol{du}$$