

# 1. Section 6 Exercises

Program Memory

## Exercise 6-1 - Sum from 1 to N

Working memory

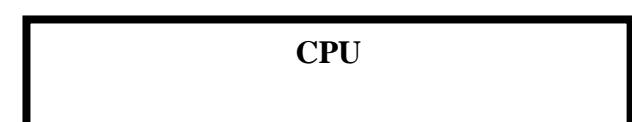
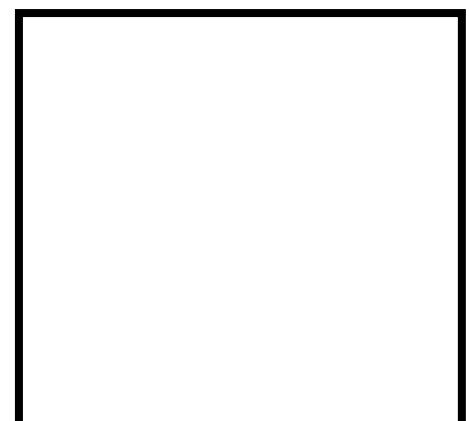
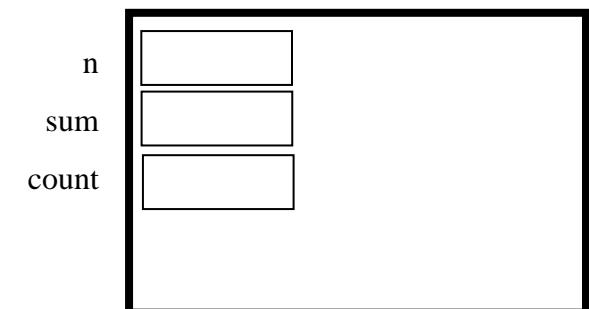
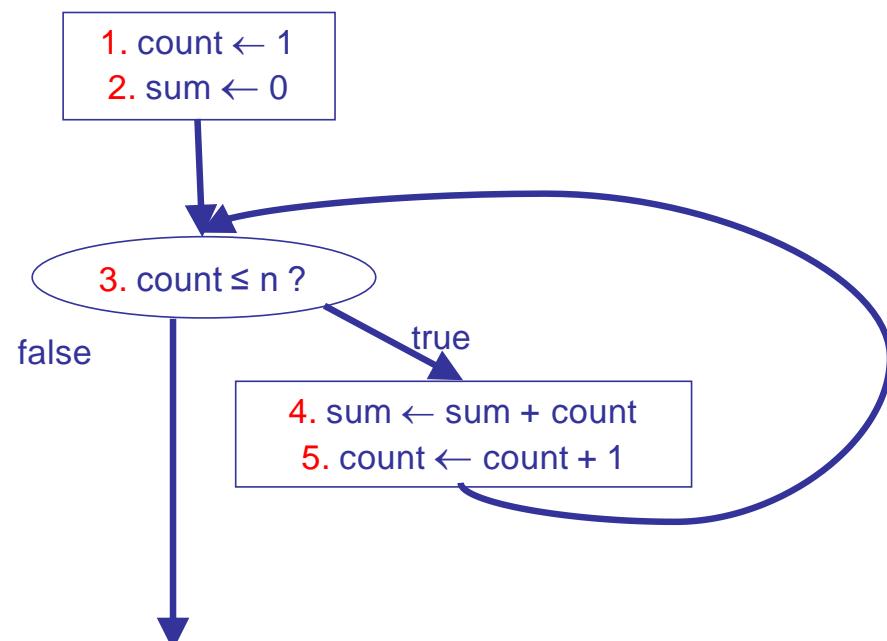
**GIVEN:** n (a positive integer)

**INTERMEDIATE:** count (index going from 1 to n)

**RESULT:** sum (sum of integers 1 to N)

**HEADER:** sum  $\leftarrow$  sum1ToN(n)

**BODY:**



### Exercise 6-1: Trace of sum1toN(3)

	n	count	sum
Init.	<b>3</b>	?	?
1.		<b>1</b>	
2.			<b>0</b>
3. TRUE			
4.			<b>1</b>
5.		<b>2</b>	
3. TRUE			
4.			<b>3</b>
5.		<b>3</b>	
3. TRUE			
4.			<b>6</b>
5.		<b>4</b>	
3. FALSE			

Working Memory

n	3
sum	0ΛΞ6
count	1ΖΞ4

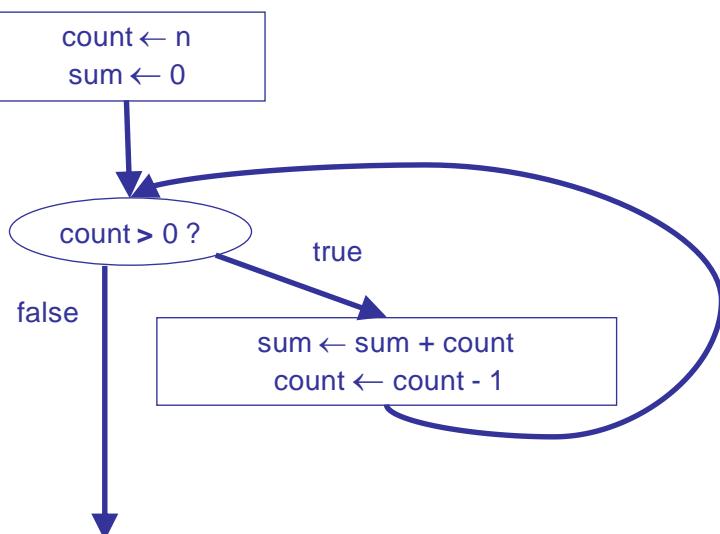
**GIVEN:** n (a positive integer)

**INTERMEDIATE:** count (index going from n to 1)

**RESULT:** sum (sum of integers 1 to n)

**HEADER:** sum  $\leftarrow$  sum1ToN(n)

**BODY:**



CPU

**GIVEN:** n (a positive integer)

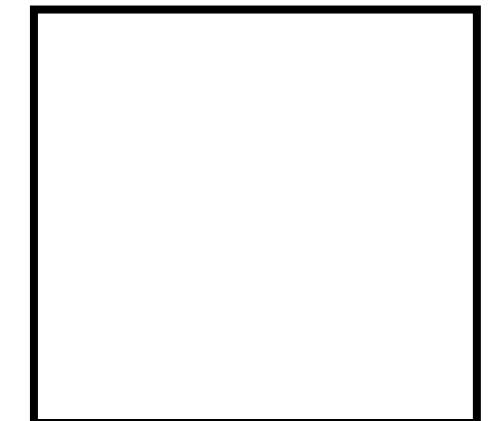
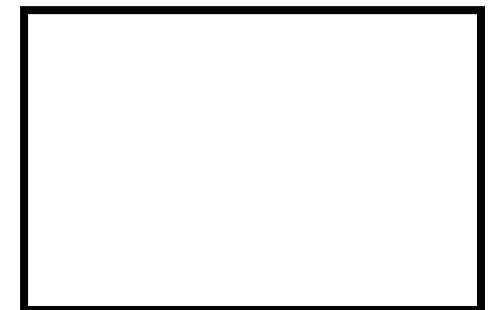
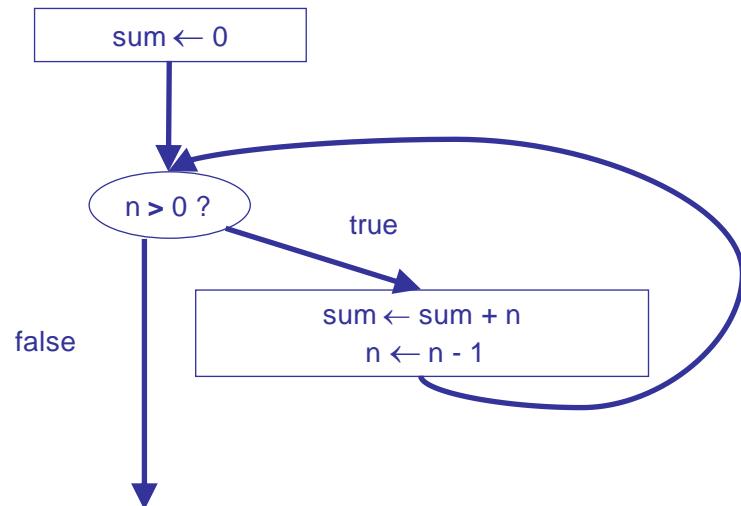
**INTERMEDIATE:** (none)

**RESULT:** sum (sum of integers 1 to N)

**HEADER:** sum  $\leftarrow$  sum1ToN(n)

**BODY:**

**Attention:**  
n (given) is modified  
*locally* only. This approach  
works, but can be  
confusing.



CPU

GIVENS: n (number to compute factorial)

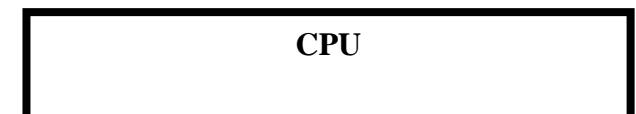
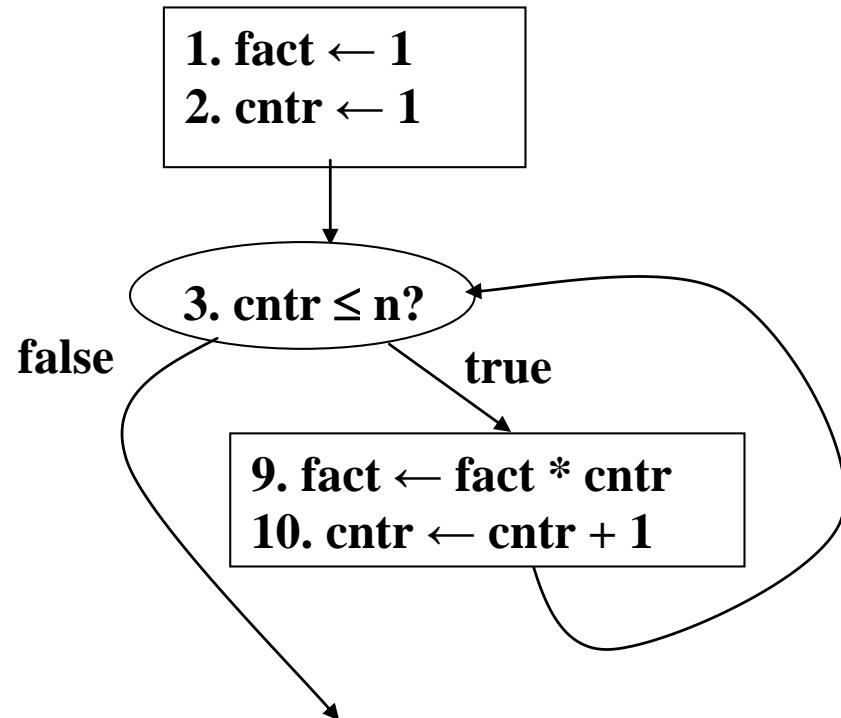
RESULTS: fact (n factorial)

INTERMEDIATES: cntr (to count from 1 to N)

ASSUMPTIONS: n should be positive

HEADER: fact  $\leftarrow$  factorial (n)

BODY:



**GIVENS:**

aNumber      (number)  
divisor      (divisor – serves as log base)

**RESULTS:**

intPart      (integer part of the logarithm, number of times aNumber divisible by divisor)

**INTERMEDIATES:**

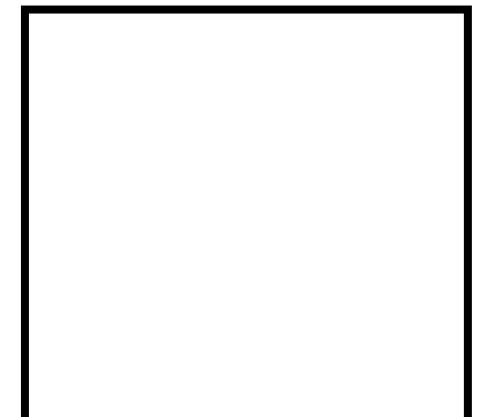
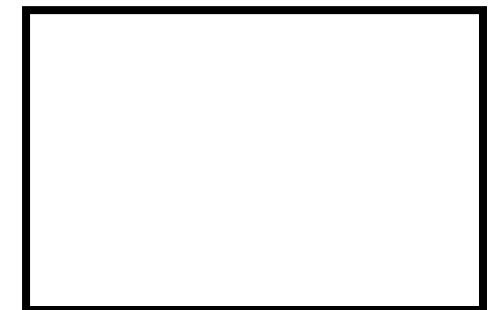
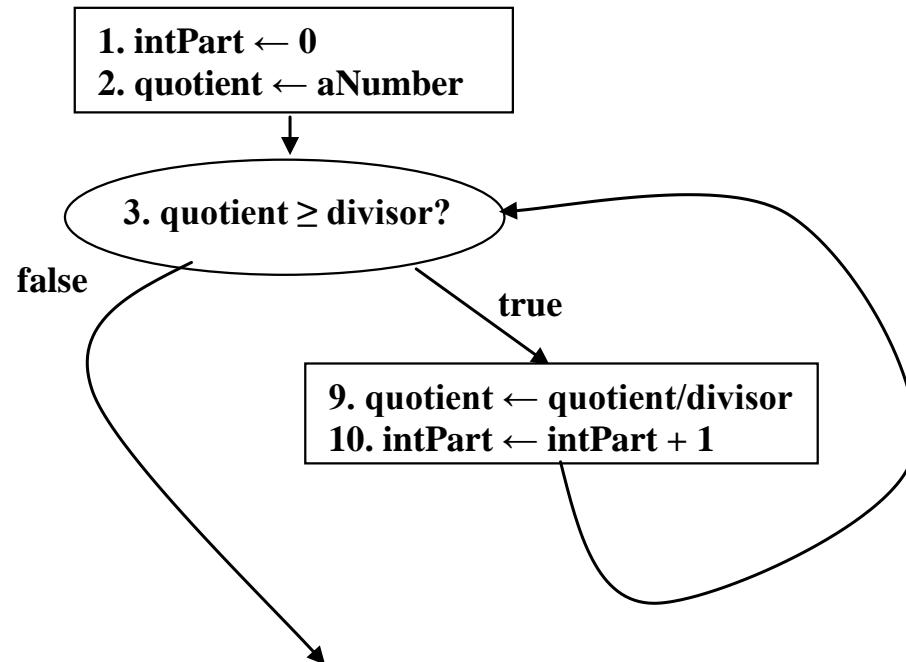
quotient      (quotient from division)

**ASSUMPTIONS:**

aNumber and divisor should be positive

**HEADER:**

intPart  $\leftarrow$  logInt (aNumber, divisor)

**BODY:**

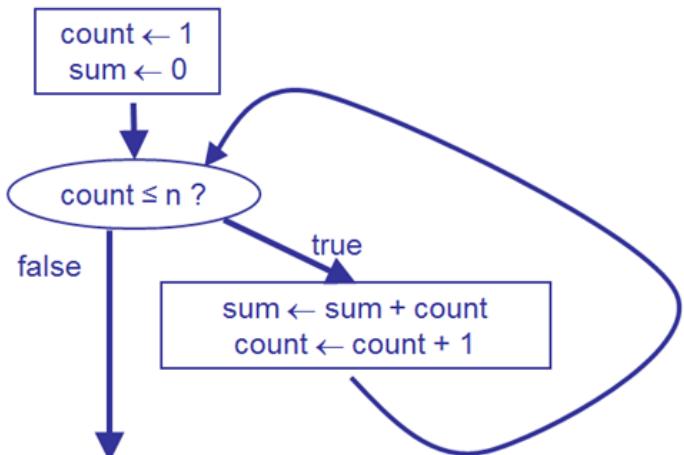
**GIVEN:** n (a positive integer)

**INTERMEDIATE:** count (index going from 1 to N)

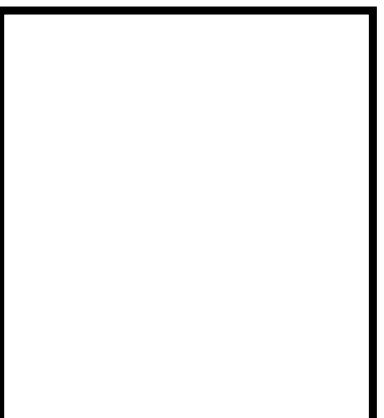
**RESULT:** sum (sum of integers 1 to N)

**HEADER:** sum  $\leftarrow$  sum1ToN(n)

**BODY:**



```
public static int sum1ToN (int n)
{
    // initialize counter
    // and accumulator sum
    int count = 1;
    int sum = 0;
    // loop
    while (count <= n)
    {
        sum = sum + count;
        count = count + 1;
    }
    // return the result
    return sum;
}
```



CPU

**GIVENS:** (none)

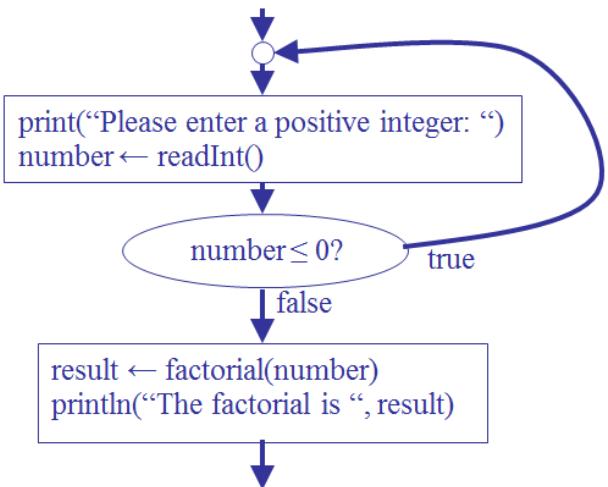
**RESULTS:** (none)

**INTERMEDIATES:** number (number to compute factorial)  
result (the factorial, i.e. number!)

**CONSTRAINTS:** number should be positive

**HEADER:** main

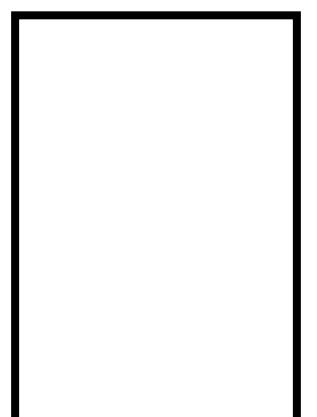
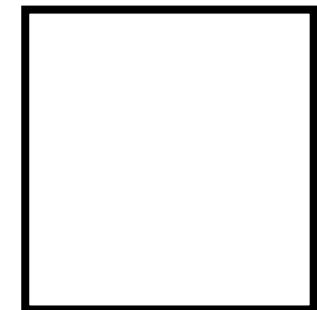
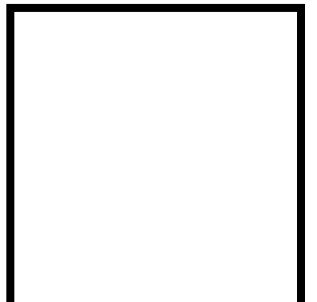
**BODY:**



```

// Translation to Java
public static void main(String args)
{
    // Variables
    int number; // number provided by user
    int results; // factorial, i.e. number!

    // Body
    do // Post-test loop
    {
        System.out.print("Please enter a positive integer: ");
        number = ITI1120.readInt();
    } while(number <= 0);
    reslt = factorial(number);
    System.out.println("The factorial is "+number);
}
  
```

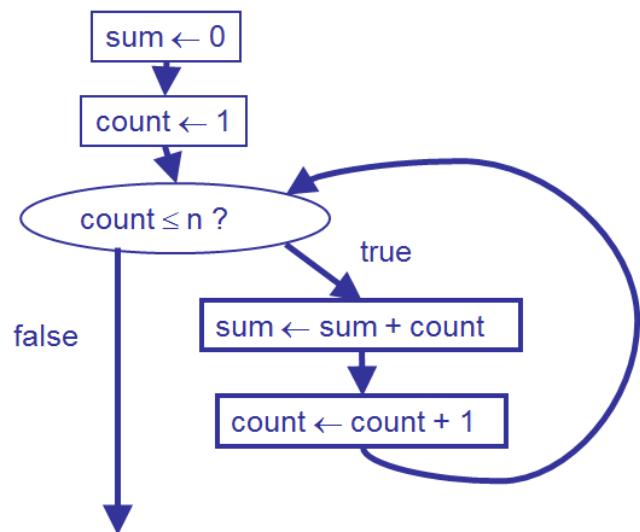


CPU

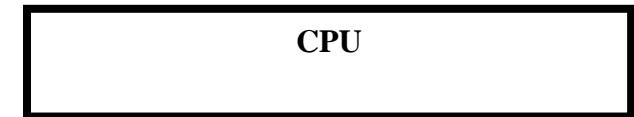
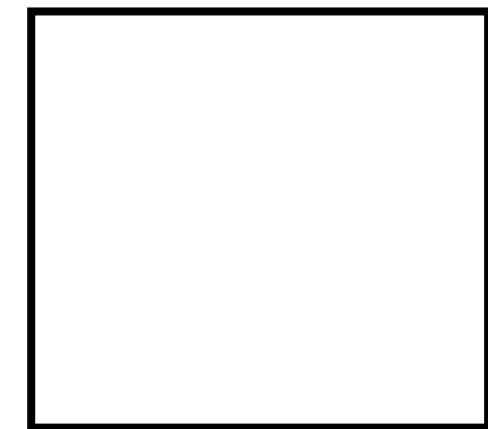
## Program Memory

## Exercise 6-6 - FOR loop to add 1 to N

## Working memory



```
// Translation to Java
sum = 0 ;
for ( count = 1; count <= n; count = count + 1 )
{
    sum = sum + count ;
}
```



### Exercise 6-7 - Array Indexing

- The index (subscript) of an array of length  $l$  may be any integer expression that returns a value in the range 0...( $l-1$ ).
  - Suppose  $k = 2$ , and  $A$  references



$a[2] = 5$   
 $a[k] = 5$   
 $a[2*k-1] = 7$   
 $a[ a[0]+1 ] = 7$

- $A[\text{expression}]$  is just like any ordinary variable and can be used anywhere an ordinary variable can be used.
- Remember  $a$  is a reference variable

## Program Memory

## Exercise 6-8 - Value in Middle of Array

GIVENS:      a      (an array of real numbers)  
                 n      (an odd integer, the length of a)

RESULT:      mid    (the middle member of a)

INTERMEDIATES: (none)

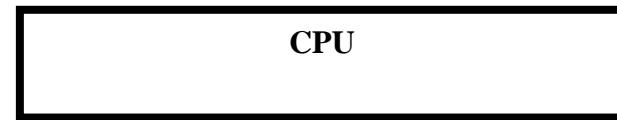
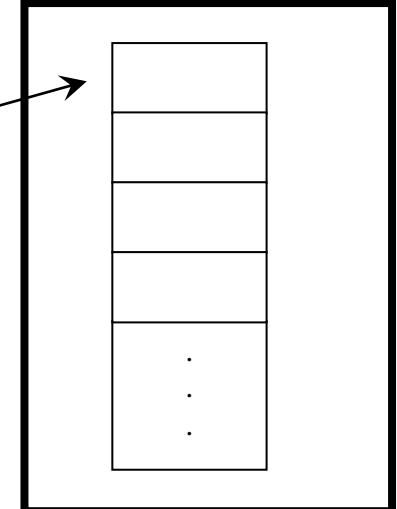
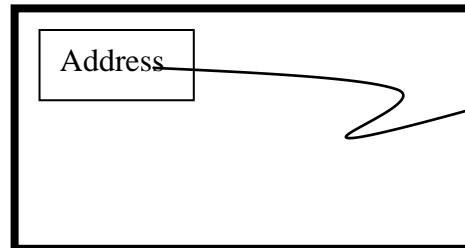
HEADER:      **mid  $\leftarrow$  middle (a, n)**

BODY

**mid  $\leftarrow$  a [ $(n - 1) / 2$  ]**

## Working memory

a



## Program Memory

## Exercise 6-9 - Swap Values in an Array

## Working memory

## Global Memory

GIVENS:       $a$  (an array of integers)  
                  $i, j$       (two indices)

MODIFIEDS:  $a$  (with  $a[i]$  and  $a[j]$  swapped)

INTERMEDIATES:

temp (stores  $a[i]$  temporarily)

RESULT: (None)

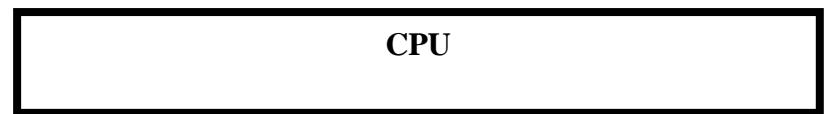
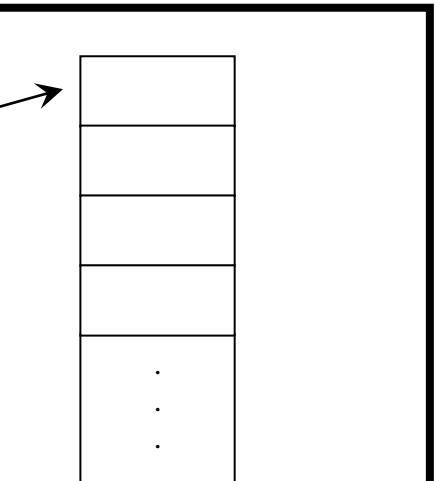
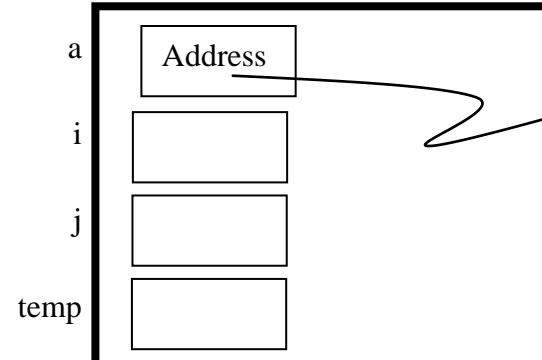
HEADER: swap(  $a, i, j$  )

BODY

$\text{temp} \leftarrow a[i]$

$a[i] \leftarrow a[j]$

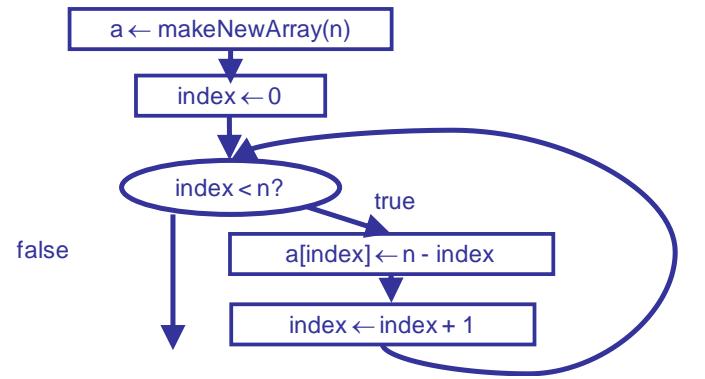
$a[j] \leftarrow \text{temp}$



CPU

## Program Memory

GIVENS: n (a positive integer)  
 RESULTS: a (an reference to an array containing n...1)  
 INTERMEDIATES: index (an index for A)  
 HEADER: a  $\leftarrow$  createNDownTo1(n)  
 BODY:

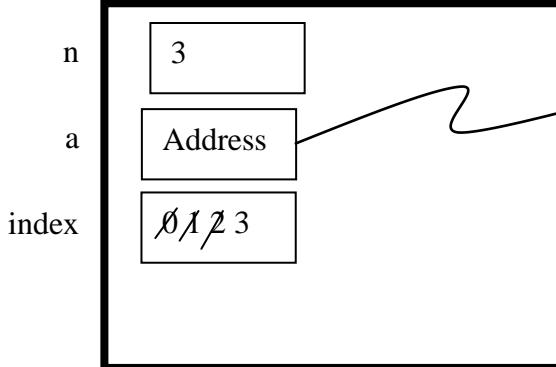


Trace for N=3

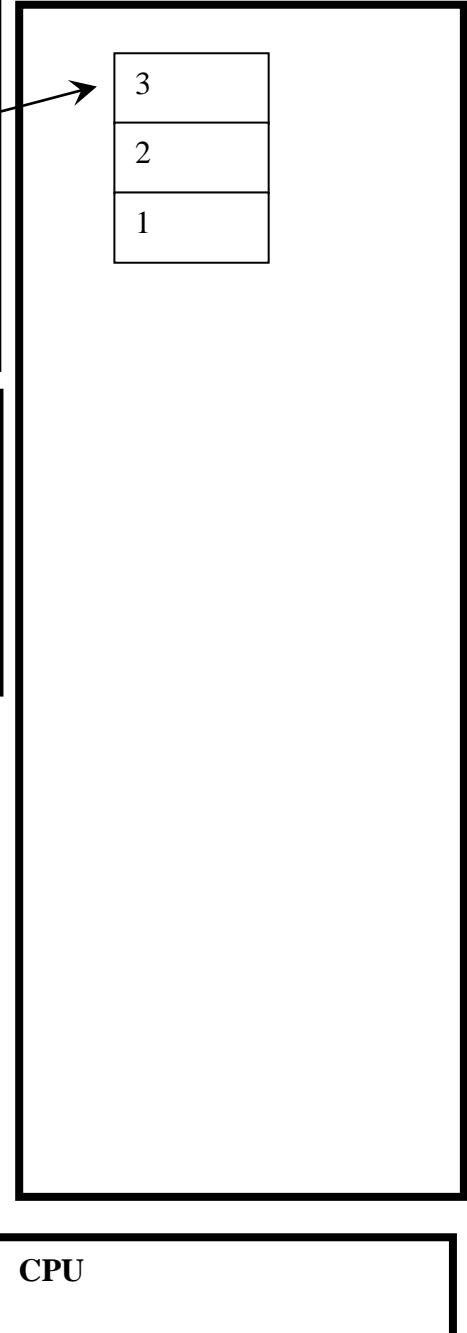
statements	n	index	a
initial values	3	?	?
1. a $\leftarrow$ makeArray(3)			{?, ?, ?}
2. index $\leftarrow$ 0		0	
3. index < n? true			
4. a[index] $\leftarrow$ n - index			{3,?,?}
5. index $\leftarrow$ index + 1		1	
3. index < n? true			
4. a[index] $\leftarrow$ n - index			{3,2,?}
5. index $\leftarrow$ index + 1		2	
3. index < n? true			
4. a[index] $\leftarrow$ n - index			{3,2,1}
5. index $\leftarrow$ index + 1		3	
3. index < n? false			

## Exercise 6-10 - Creating an Array

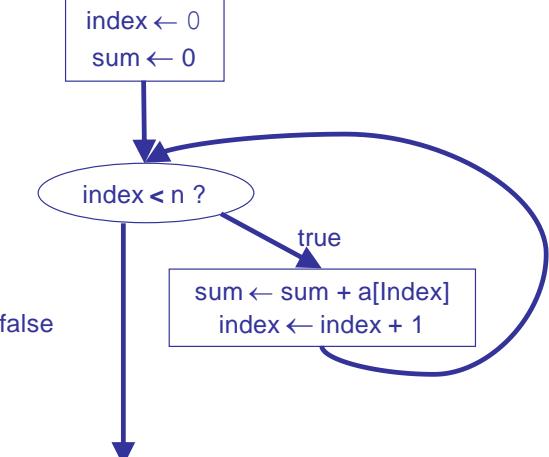
### Working memory



### Global Memory





Algorithm Model	Java
<b>Exercise 6-11 – Find the sum of the values in an array containing N values</b>	
<p>GIVENS: a (An array of numbers) n (Number of array elements)</p> <p>INTERMEDIATE: index (Array index going from 0 to n-1)</p> <p>RESULT: sum (Sum of array contents)</p> <p>HEADER: sum <math>\leftarrow</math> sumArray(a,n)</p> <p>BODY:</p>  <pre> graph TD     Start["index ← 0 sum ← 0"] --&gt; Decision{index &lt; n ?}     Decision -- true --&gt; Process["sum ← sum + a[index] index ← index + 1"]     Process --&gt; Decision     Decision -- false --&gt; Exit   </pre>	<pre> public static int[] sumArray(int[] a) {     // note, given n is a.length     // Results     int sum;     // intermediate     int index;     //     sum = 0; // initialise sum     // loop with for     for(index = 0; index &lt; n; index++)     {         sum = sum + a[index];     }     // return the result     return sum; }   </pre>
<b>Exercise 6-12 (a) – Given a value T and an array X containing N values, check if the sum of X's values exceeds T.</b>	
<p>GIVENS: a (An array of numbers) n (Number of array elements) t (A “threshold” value)</p> <p>INTERMEDIATE: sum (The sum of the array, from example 2)</p> <p>RESULT: exceeds (Boolean: True if sum &gt; t and False otherwise)</p> <p>HEADER: exceeds <math>\leftarrow</math> sumLargerThanT(a,n,t)</p> <p>BODY:</p> <pre> sum ← sumArray(a,n) exceeds ← sum &gt; t   </pre>	<pre> public static boolean sumLargerThanT(int[] a, int t) {     // note, given n is a.length     // Results     int exceeds;     // intermediate     int sum;     //     sum = sumArray(a); // get sum     exceeds = sum &gt; t;     // return the result     return exceeds; }   </pre>

Algorithm Model	Java
<b>Exercise 6-12 (b) – Given a value T and an array X containing N values, check if the sum of X's values exceeds T.</b>	
<p><b>GIVENS:</b> a (An array of numbers) n (Number of array elements) t (A “threshold” value)</p> <p><b>INTERMEDIATE:</b> index (Array index going from 0 to n-1) sum (The sum of the array elements)</p> <p><b>RESULT:</b> exceeds (Boolean: True if sum &gt; t and False otherwise)</p> <p><b>HEADER:</b> exceeds <math>\leftarrow</math> sumLargerThanT(a,n,t)</p> <p><b>BODY:</b></p> <pre> graph TD     Start["index ← 0 sum ← 0"] --&gt; Decision{index &lt; n AND sum ≤ t?}     Decision -- false --&gt; End["exceeds ← sum &gt; t"]     Decision -- true --&gt; Process["sum ← sum + a[index] index ← index + 1"]     Process --&gt; Decision   </pre>	<pre> public static boolean sumLargerThanT(int[] a, int t) {     // note, given n is a.length     // Results     int exceeds;     // intermediate     int index;     int sum;     // loop using while     sum = 0;     index = 0;     while(index &lt; n &amp;&amp; sum &lt;= t)     {         sum = sum+a[index];         index = index+1;     }     exceeds = sum &gt; t;     // return the result     return exceeds; }   </pre>

<b>Exercise 6-13 – Count how many times K occurs in an array containing N values.</b>	
<p><b>GIVENS:</b> a (An array of numbers) n (Number of array elements) k (Value for which to count instances)</p> <p><b>INTERMEDIATES:</b> Index (Array index going from 0 to n-1)</p> <p><b>RESULT:</b> count (Number of times value of k is contained in A)</p> <p><b>HEADER:</b> count <math>\leftarrow</math> countK(a,n,k)</p> <p><b>BODY:</b></p> <pre> graph TD     Start["index ← 0 count ← 0"] --&gt; Decision1{index &lt; n?}     Decision1 -- false --&gt; End[""]     Decision1 -- true --&gt; Decision2{a[index] == k?}     Decision2 -- false --&gt; Process1["count ← count + 1"]     Decision2 -- true --&gt; Process2["index ← index + 1"]     Process1 --&gt; Decision1     Process2 --&gt; Decision1   </pre>	<pre> public static int countK(int[] a, int k) {     // Results     int count;     // intermediate     int index;     // intialias sum and index     count = 0;     for(index = 0; index &lt; a.length; index=index+1) // loop     {         if(a[index] == k)         {             count = count + 1;         }         else { /* do nothing */ }     }     return count; // return the result }   </pre>

Algorithm Model	Java
<b>Exercise 6-14 (a) – Given an array X of N values and a number K, see if K occurs in X or not.</b>	
<p>GIVENS: a (An array of numbers) n (Number of array elements) k (Value to find)</p> <p>INTERMEDIATES:</p> <p>count (Number of times K is in array, from algorithm 4)</p> <p>RESULT: found (Boolean : true if K is in array and false otherwise)</p> <p>HEADER: found <math>\leftarrow</math> findK(a,n,k)</p> <p>BODY:</p> <pre>count <math>\leftarrow</math> countK(a,n,k) found <math>\leftarrow</math> count &gt; 0</pre>	<pre>public static boolean findK(int[] a, int k) {     // Results     boolean found;     // intermediates     int count;     // Body     count = countK(a, k);     found = count &gt; 0;     // Return results     return found; }</pre>
<b>Exercise 6-14 (b) – Given an array X of N values and a number K, see if K occurs in X or not.</b>	
<p>GIVENS: a (An array of numbers) n (Number of array elements) k (Value to find)</p> <p>INTERMEDIATES:</p> <p>index (Array index going from 0 to n-1)</p> <p>RESULT: found (Boolean : true if K is in array and false otherwise)</p> <p>HEADER: found <math>\leftarrow</math> findK(a,n,k)</p> <p>BODY:</p> <pre> graph TD     Start["index ← 0 found ← FALSE"] --&gt; Cond{index &lt; n AND NOT found ?}     Cond -- true --&gt; Match{a[index] = k ?}     Match -- true --&gt; FoundTrue["found ← TRUE"]     FoundTrue --&gt; IndexPlusOne["index ← index + 1"]     IndexPlusOne --&gt; Cond     Match -- false --&gt; IndexPlusOne     Cond -- false --&gt; End[ ]   </pre>	<pre>public static boolean findK(int[] a, int k) {     boolean found; // results     int index; // intermediate     // initialize found and index     found = false;     index = 0;     while(index &lt; a.length &amp;&amp; !found) // loop     {         if(a[index] == k)         {             found = true;         }         else { /* do nothing */ }         index=index+1;     }     return found; // return the result }</pre>

Algorithm Model	Java
<p><b>Exercise 6-15</b> – Given an array X of N values and a number K, find the position of the first occurrence of K. (If K does not occur, return -1 as the position.)</p> <p>GIVENS: a (An array of numbers) n (Number of array elements) k (Value to find)</p> <p>INTERMEDIATES:</p> <ul style="list-style-type: none"> <li>index (Array index going from 0 to n-1)</li> </ul> <p>RESULT: position (Position of K in array, or -1 if K is not contained in array)</p> <p>HEADER: position <math>\leftarrow</math> whereIsK(a,n,k)</p> <p>BODY:</p> <pre> graph TD     Start["index ← 0 position ← -1"] --&gt; Cond1{index &lt; n AND position = -1 ?}     Cond1 -- true --&gt; Cond2{a[index] = k ?}     Cond2 -- true --&gt; SetPos["position ← index"]     SetPos --&gt; IncIndex["index ← index + 1"]     Cond2 -- false --&gt; IncIndex     IncIndex --&gt; Cond1     Cond1 -- false --&gt; End["return position"]   </pre>	<p style="text-align: center;"><b>Java</b></p> <pre> public static int whereIsK(int a[], int n, int k) {     int position; // result     int index; // intermediate     // Body     index = 0;     position = -1;     while( (index &lt; n) &amp;&amp; (position == -1) )     {         if(a[index] == k)         {             position = index;         }         else /* do nothing */         index = index + 1;     }     // Return results     return position; }   </pre>

Algorithm Model	Java
<b>Exercise 6-16 – Find the maximum value in an array containing N values.</b>	
<p>GIVENS: n (a positive integer) a (array containing N values)</p> <p>INTERMEDIATE: index (indices for a)</p> <p>RESULT: max (maximum member of a)</p> <p>HEADER: max <math>\leftarrow</math> maxInArray( a, n )</p> <p>BODY</p> <pre> graph TD     Start["index ← 1 max ← A[0]"] --&gt; Cond1{index &lt; n?}     Cond1 -- true --&gt; Cond2{a[index] &gt; max?}     Cond2 -- true --&gt; Update["max ← a[index]"]     Update --&gt; IndexInc["index ← index + 1"]     IndexInc --&gt; Cond1     Cond1 -- false --&gt; End["∅"]   </pre>	<pre> public static int maxInArray(int [] a) {     // DATA DICTIONARY     // GIVEN: a An array of int's     // RESULT:     int max; // The maximum in the array     // INTERMEDIATES:     int n; // The length of array a     int index; //Index into array     // Initialise n     n = a.length;     // ALGORITHM BODY     max = a[0];     index = 1;     while (index &lt; n)     {         if (a[index] &gt; max)         {             max = a[index]; //update         }         else         {             /* do nothing */         }         index = index + 1;     }      // Return results     return max; }   </pre>

Algorithm Model	Java
<b>Exercise 6-17 (a) – Find the position of the first occurrence of the maximum value in an array containing N values.</b>	
<p>GIVENS: a (An array of numbers) n (Number of array elements)</p> <p>INTERMEDIATES:</p> <ul style="list-style-type: none"> <li>index (Array index going from 0 to n-1)</li> <li>max (Maximum value contained in a)</li> </ul> <p>RESULTS: position (Position of Maximum value contained in a)</p> <p>HEADER: position <math>\leftarrow</math> maxPosInArray(a,n)</p> <p>BODY:</p> <pre> graph TD     Init["max ← maxInArray(a,n) index ← 0 position ← -1"] --&gt; Cond{index &lt; n AND position = -1 ?}     Cond -- true --&gt; SubCond{"a[index] = max ?"}     SubCond -- true --&gt; SetPos["position ← index"]     SubCond -- false --&gt; IncIndex["index ← index + 1"]     IncIndex --&gt; Cond     Cond -- false --&gt; Exit   </pre>	<pre> public static int maxPosInArray(int [] a) {     // GIVEN: a An array of int's     int position; //RESULT: Position of max in the array     // INTERMEDIATES:     int n;// The length of array a     int index; //Index into array     int max; // maximum value     // Body     max = maxInArray(a); // get the max value     index = 0; // search for max value in the array     position = -1;     while(index &lt; a.length &amp;&amp; position == -1)     {         if(a[index] == max)             position = index;         else             index = index + 1;     }     return position; // Return the results }   </pre>
<b>Exercise 6-17 (b) – Find the position of the first occurrence of the maximum value in an array containing N values.</b>	
<p>GIVENS: a (An array of numbers) n (Number of array elements)</p> <p>INTERMEDIATES:</p> <ul style="list-style-type: none"> <li>max (Maximum in array from example 7)</li> </ul> <p>RESULTS: position (Position of Maximum value contained in A)</p> <p>HEADER: position <math>\leftarrow</math> maxPosInArray(a,n)</p> <p>BODY:</p> <pre> max ← maxInArray(a,n) position ← whereIsK(a,n,max)   </pre>	<pre> public static int maxPosInArray(int [] a) {     // DATA DICTIONARY     // GIVEN: a An array of int's     int position; // // RESULT: Position of max in array     // INTERMEDIATES:     int n;// The length of array a     int index; //Index into array     int max; // maximum value     // Body     max = maxInArray(a); // get the max value     position = whereIsK(a, max); // get the position     return position; // Return the results }   </pre>

Algorithm Model	Java
<b>Exercise 6-17 (c) – Find the position of the first occurrence of the maximum value in an array containing N values.</b>	
<p><b>GIVENS:</b> a (An array of numbers) n (Number of array elements)</p> <p><b>INTERMEDIATES:</b> index (Array index going from 0 to n-1) max (Maximum value contained in a)</p> <p><b>RESULTS:</b> position (Position of Maximum value contained in a)</p> <p><b>HEADER:</b> position <math>\leftarrow</math> maxPosInArray(a,n)</p> <p><b>BODY:</b></p> <pre> graph TD     Init["index ← 1 max ← a[0] position ← 0"] --&gt; Cond1((index &lt; n?))     Cond1 -- true --&gt; Cond2((a[index] &gt; max?))     Cond2 -- true --&gt; Update["max ← a[index] position ← index"]     Update --&gt; IndexInc["index ← index + 1"]     IndexInc --&gt; Cond1     Cond1 -- false --&gt; Result["∅"]     Result --&gt; Exit   </pre>	<pre> public static int maxPosInArray(int [] a) {     // GIVEN: a An array of int's     int position; // // RESULT: Position of max in array     // INTERMEDIATES:     int index; //Index into array     int max; // maximum value     // Body     index = 1;     max = a[0];     position = 0;     while(index &lt; a.length)     {         if(a[index] &gt; max)         {             max = a[index];             position = index;         }         else index = index + 1;     }     return position; // Return the results }   </pre>

Algorithm Model	Java
<b>Exercise 6-18 – Check if an array of N values contains any duplicates.</b>	
<p><b>GIVENS:</b> a (An array of numbers) n (Number of array elements)</p> <p><b>INTERMEDIATES:</b> checkIndex (Array index for current element) dupIndex (Array index for duplicate               checking of current element)</p> <p><b>RESULT:</b> duplicates (True if there are duplicates in               A and false otherwise)</p> <p><b>HEADER:</b> duplicates <math>\leftarrow</math> hasDuplicates(a,n)</p> <p><b>BODY:</b></p> <pre> graph TD     Start(( )) --&gt; Init1[duplicates ← False]     Init1 --&gt; Init2[checkIndex ← 0]     Init2 --&gt; Cond{checkIndex &lt; n-1 AND Not duplicates ?}     Cond -- true --&gt; DupIndex1[dupIndex ← checkIndex + 1]     DupIndex1 --&gt; Cond2{dupIndex &lt; n AND Not duplicates ?}     Cond2 -- true --&gt; Comp{a[checkIndex] = a[dupIndex] ?}     Comp -- true --&gt; Done[duplicates ← True]     Done --&gt; End(( ))     Comp -- false --&gt; IncCheck[checkIndex ← checkIndex + 1]     IncCheck --&gt; Cond     Cond -- false --&gt; Result(( ))     Result --&gt; End   </pre>	<pre> public static boolean hasDuplicates(int [] a) {     // GIVEN: a An array of int's, a.length used for n     boolean duplicates; // RESULT: true if dupls found     int checkIndex; // INTERMEDIATE: Index of current element     int dupIndex; // INTMEDT: checking of dupls for current     // ALGORITHM BODY     duplicates = false;    checkIndex = 0;     while (checkIndex &lt; a.length-1 &amp;&amp; !duplicates)     {         dupIndex = checkIndex + 1;         while(dupIndex &lt; a.length &amp;&amp; !duplicates)         {             if (a[checkIndex] == a[dupIndex])             {                 duplicates = true; // found a duplicate             }             else { /*do nothing*/ }             dupIndex = dupIndex + 1;         }         checkIndex = checkIndex + 1;     }     return duplicates; // Return results }   </pre>

### Exercise 6-19: Comparing Strings

- What is the value of `result` for these examples?

- Example 1:

```
String str1 = "abcde" ;
String str2 = "abcfg" ;
int result = str1.compareTo(str2) ;
```

**result is less than zero**

- Example 2:

```
String str1 = "abcde" ;
String str2 = "ab" ;
int result = str1.compareTo(str2) ;
```

**result is greater than zero**