Devoir 4
Assignment 4

1) (a)
$$x(t) = \delta(t-8)\sin\left(2\pi 10t + \frac{\pi}{2}\right)$$
$$= \delta(t-8)\sin\left(2\pi 80 + \frac{\pi}{2}\right)$$
$$= \delta(t-8)\sin\frac{\pi}{2}$$
$$= \delta(t-8)$$

(b)
$$\int_{-\infty}^{\infty}\left(\delta(t-3)+\delta(t+2)\right)\Lambda(t/10)\,dt$$
$$= \Lambda\left(\frac{3}{10}\right) + \Lambda\left(\frac{-2}{10}\right)$$
$$= 1-\frac{3}{10} + 1-\frac{2}{10}$$
$$= \underline{\underline{1.5}}$$

(c)
$$y(t) = \text{sinc}(8t)\cos 2\pi 15t$$
$$\mathcal{F}\{\text{sinc}(8t)\} = \frac{1}{8}\Pi\left(\frac{f}{8}\right)$$
$$\mathcal{F}\{\text{sinc}(8t)\cos 2\pi 15t\}$$
$$= \frac{1}{16}\Pi\left(\frac{f-15}{8}\right) + \frac{1}{16}\Pi\left(\frac{f+15}{8}\right)$$

2) (a) $\int_{-\infty}^{\infty} \operatorname{sinc}^4(t)\,dt = \int_{-\infty}^{\infty} \operatorname{sinc}^2(t)\operatorname{sinc}^2(t)\,dt$

$$= \int_{-\infty}^{\infty} \Lambda(f)\,\Lambda(f)\,df$$

$$= \int_{-1}^{0} (1+f)^2\,df + \int_{0}^{1} (1-f)^2\,df$$

$$= \left(f + f^2 + \tfrac{1}{3}f^3\right)\Big|_{-1}^{0} + \left(f - f^2 + \tfrac{1}{3}f^3\right)\Big|_{0}^{1}$$

$$= 0 - \left(-1 + 1 - \tfrac{1}{3}\right) + \left(1 - 1 + \tfrac{1}{3}\right)$$

$$= \frac{2}{3}$$

(4) $\int_{-\infty}^{\infty} \operatorname{sinc}(t)\cos 2\pi 10t\,dt$

$$= \int_{-\infty}^{\infty} \Pi(f)\left\{\tfrac{1}{2}\delta(f-10) + \tfrac{1}{2}\delta(f+10)\right\} df$$

$$= \tfrac{1}{2}\Pi(10) + \tfrac{1}{2}\Pi(-10)$$

$$= 0 + 0 = 0$$

(c) $\int_{-\infty}^{\infty} \mathrm{sinc}(10t)\,\sin(2\pi 2t)\,dt$

$$= \int_{-\infty}^{\infty} \frac{1}{10}\Pi\left(\frac{f}{10}\right)\cdot\left[\frac{1}{2j}\,\delta(f-2) - \frac{1}{2j}\,\delta(f+2)\right] df$$

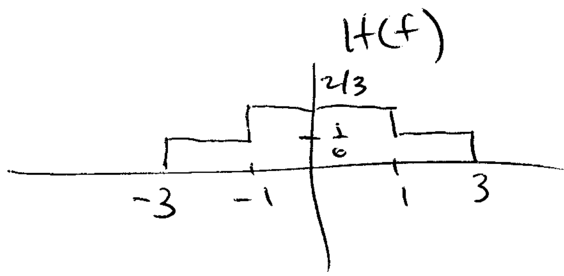
$$= +\frac{1}{20j}\,\Pi\left(\frac{2}{10}\right) - \frac{1}{20j}\,\Pi\left(-\frac{2}{10}\right)$$

$$= \frac{1}{20j}\cdot\frac{8}{10} - \frac{1}{20j}\cdot\frac{8}{10} = 0$$

Question 3

$$h(t) = \mathrm{sinc}(2t) + \mathrm{sinc}(6t)$$

$$H(f) = \frac{1}{2}\Pi\left(\frac{f}{2}\right) + \frac{1}{6}\Pi\left(\frac{f}{6}\right)$$

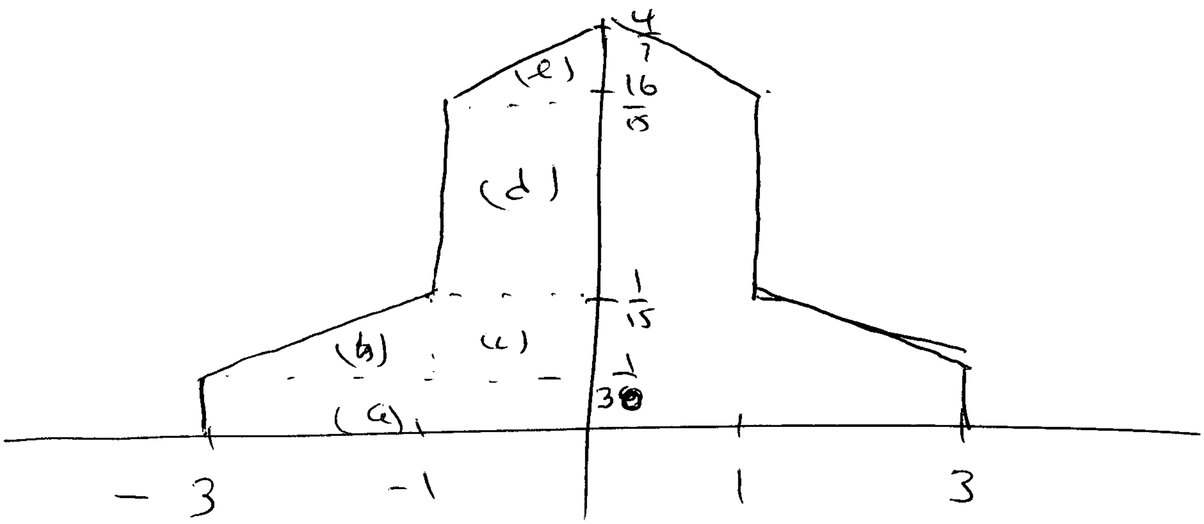


(a) $E_x = \int_{-\infty}^{\infty} G_x(f)\,df$

$$= 10 \times 3 \times \frac{1}{2} = 15$$

(b) $G_Y(f) = G_x(f)\cdot|H(f)|^2$

$$= \begin{cases} \frac{1}{36}G_x(f) & -1 \le f \le 1 \\ \frac{4}{9}G_x(f) & -3 \le f \le -1,\ 1 \le f \le 3 \\ 0 & \text{otherwise} \end{cases}$$



$$E_y = \Big(\overset{(a)}{3 \times \tfrac{1}{30}} + \overset{(b)}{2 \times \tfrac{1}{30} \times \tfrac{1}{2}} + \overset{(c)}{1 \times \tfrac{1}{30}}$$

$$+ \underset{(d)}{1 \times 1} + \underset{(e)}{\left(\tfrac{4}{15} \times 1 \times \tfrac{1}{2} \right)} \Big) \times 2$$

$$= \left(\frac{1}{10} + \frac{1}{30} + \frac{1}{30} + 1 + \frac{4}{30} \right) \times 2$$

$$= \frac{39}{30} \times 2 = \frac{78}{30} = \underline{\underline{2.6}}$$