Corrigé Devoir 1
Solution Assignment 1

Question 1

Quand le signal est une somme ~~d'un~~ de signaux periodique, la période est le plus petit commun multiple des périodes.
When the signal is a sum of periodic signals, the period is the lowest common multiple of the periods.

(a) $3\sin(2\pi 100t)$ has period $T_1 = \frac{1}{100}$

$4\cos(1000t)$ has period $T_2 = \frac{\pi}{500}$

we need to find two integers, n & m so that $T = nT_1 = mT_2$. In this case it is impossible

(b) $y(t) = e^{-j3t}$

$$y(t+T) = e^{-j3(t+T)} = e^{-j3t}e^{-j3T} = y(t)$$

quand $e^{-j3T} = 1 \Rightarrow$ quand $3T = 2\pi$

alors $T = \frac{2\pi}{3}$

(c) $z(t) \rightarrow$ $\sin 4\pi t$ à $T_1 = \frac{1}{2}$

$\cos 6\pi t$ à $T_2 = \frac{1}{3}$

$9 \sin 8\pi t$ à $T_3 = \frac{1}{4}$

$T = 2 \times \frac{1}{2} = 3 \times \frac{1}{3} = 4 \times \frac{1}{4} = 1$

Question 2

(a) $y_1(t) = x_1(t) - 0.5\, x_1(t-1) + 0.1\, x(t-2)$

$y_2(t) = x_2(t) - 0.5\, x_2(t-1) + 0.1\, x_2(t-2)$

Soit $x_3(t) = a x_1(t) + b x_2(t)$

$y_3(t) = x_3(t) - 0.5\, x_3(t-1) + 0.1\, x_3(t-2)$

$= a x_1(t) + b x_2(t) - 0.5(a x_1(t-1) + b x_2(t-1))$
$+ 0.1(a x_1(t-2) + b x_2(t-2))$

$= a x_1(t) - a\, 0.5\, x_1(t-1) + a\, 0.1\, x_1(t-2)$
$+ b x_2(t) - b\, 0.5\, x_2(t-1) + b\, 0.1\, x_2(t-2)$

$= a y_1(t) + b y_2(t)$

$\Rightarrow$ linear / linéaire

$y(t) = x(t) - 0.5x(t-1) + 0.1x(t-2)$

let $x_1(t) = x(t-\tau)$

$$y_1(t) = x_1(t) - 0.5x_1(t-1) + 0.1x_1(t-2)$$
$$= x(t-\tau) - 0.5x(t-\tau-1) + 0.1x(t-\tau-2)$$

$$y(t-\tau) = x(t-\tau) - 0.5x(t-\tau-1) + 0.1x(t-\tau-2) = y_1(t)$$

time invariant / invariant en temps

$y(t) = h(t)$ si/if $x(t) = \delta(t)$

alors

$$h(t) = \delta(t) - 0.5\,\delta(t-1) + 0.1\,\delta(t-2)$$

(b) $y_1(t) = \cos(x_1(t))$

$y_2(t) = \cos(x_2(t))$

$y_3(t) = \cos(x_3(t))$ où/where $x_3(t) = a x_1(t) + b x_2(t)$

$$y_3(t) = \cos(a x_1(t) + b x_2(t)) \neq a\cos(x_1(t)) + b\cos(x_2(t))$$

$\Rightarrow$ not linear / pas linéaire

$$y(t) = \cos(x(t))$$

$$x_1(t) = x(t-\tau)$$

$$y_1(t) = \cos(x_1(t))$$

$$= \cos(x(t-\tau))$$

$$y(t-\tau) = \cos(x(t-\tau))$$

$$= y_1(t) \quad \text{alors}$$

invariant en temps / time invariant

(c) $y_1(t) = x_1(t)\cos(2\pi 1000 t)$

$$y_2(t) = x_2(t)\cos 2\pi 1000 t$$

$$y_3(t) = x_3(t)\cos 2\pi 1000 t$$

$$= (a x_1(t) + b x_2(t))\cos 2\pi 1000 t$$

$$= a x_1(t)\cos 2\pi 1000 t + b x_2(t)\cos 2\pi 1000 t$$

$$= a y_1(t) + b y_2(t)$$

$\Longrightarrow$ linéaire / linear

$y(t) = x(t)\cos 2\pi 1000t$

$x_1(t) = x(t-\tau)$

$y_1(t) = x_1(t)\cos 2\pi 1000t$

$= x(t-\tau)\cos 2\pi 1000t$

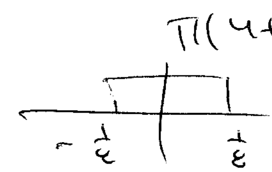
$y(t-\tau) = x(t-\tau)\cos 2\pi 1000(t-\tau)$

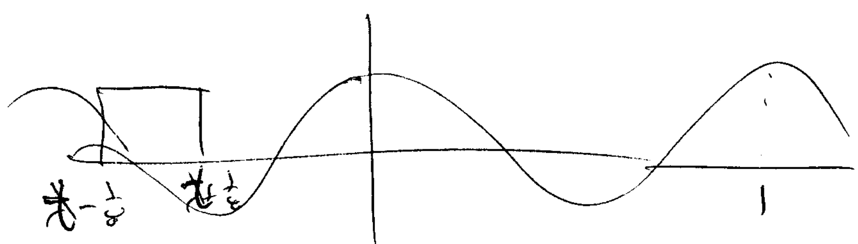
$\neq y_1(t)$

not time invariant / pas invariant en temps

Question 3

(a) $\cos 2\pi t * \Pi(4t)$



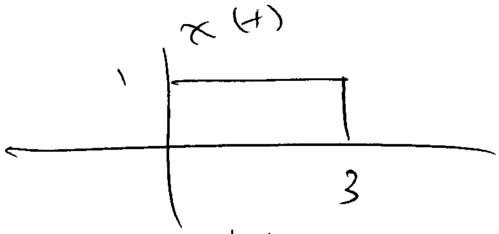


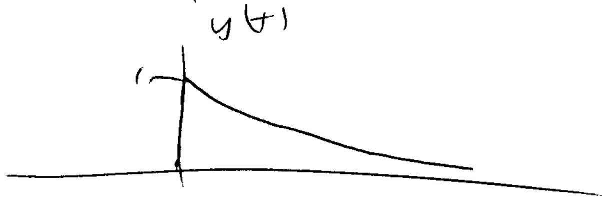
$$\cos 2\pi t * \Pi(4t) = \int_{\cancel{t}-\frac{1}{8}}^{\cancel{t}+\frac{1}{8}} \cos(2\pi\lambda)\, d\lambda$$

$$= \frac{1}{2\pi}\sin 2\pi\lambda \Big|_{t-\frac{1}{8}}^{t+\frac{1}{8}}$$

$$= \frac{1}{2\pi}\left(\sin\left(2\pi t + \frac{\pi}{4}\right) - \sin\left(2\pi t - \frac{\pi}{4}\right)\right)$$

(b) $\underbrace{e^{-t}u(t)}_{y(t)} * \underbrace{(u(t)-u(t-3))}_{x(t)}$

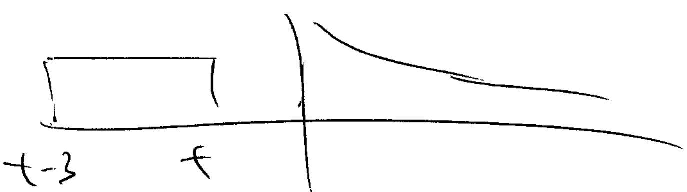




$e^{-t}u(t) * (u(t)-u(t-3))$

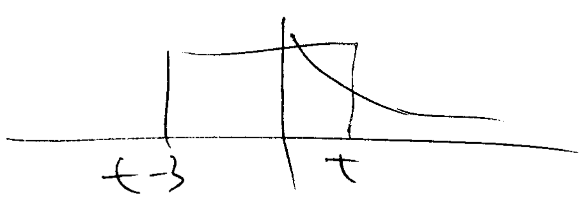
$\cancel{= \int_{-\infty}^{\infty} x(\lambda)\, y(t-\lambda)\, d\lambda}$

$$= \int_{-\infty}^{\infty} x(\lambda)\, y(t-\lambda)\, d\lambda$$



$t < 0$

$x(t) * y(t) = 0$

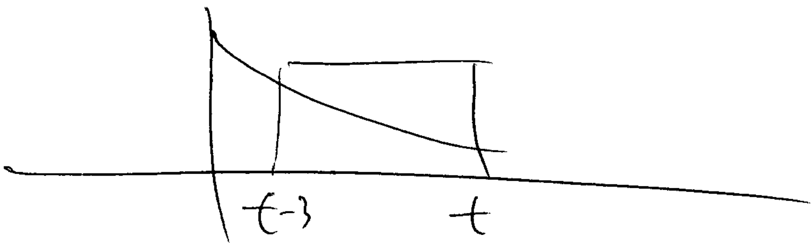


$0 < t < 3$

$$\int_0^t e^{-\lambda}\, d\lambda$$

$$= -e^{-\lambda}\Big|_0^t$$

$$= 1 - e^{-t}$$



$t > 3$

$$\int_{t-3}^{t} e^{-\lambda}\, d\lambda$$

$$= -e^{-\lambda}\Big|_{t-3}^{t}$$

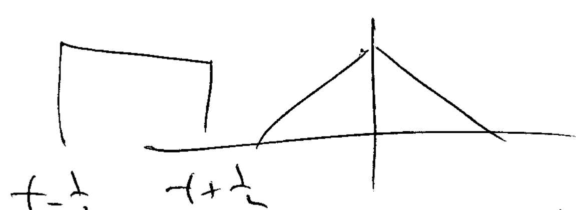
$$= -e^{-t} + e^{-(t-3)}$$

$$= -e^{-t} + e^{-t}e^{3}$$

$$= e^{-t}(e^{3}-1)$$

$$e^{-t}u(t) * (u(t)-u(t-3)) = \begin{cases} 0 & t < 0 \\ 1-e^{-t} & 0 \le t \le 3 \\ e^{-t}(e^{3}-1) & t \geq 3 \end{cases}$$

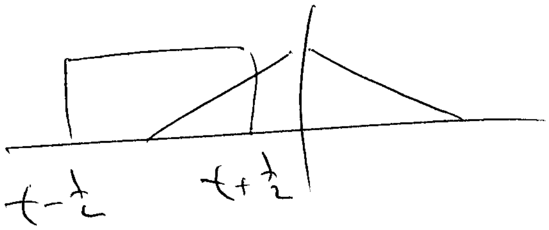
(c) $\Lambda(t) * \Pi(t)$



$\Lambda(t) * \Pi(t) = 0$

$t + \frac{1}{2} < -1$

$\underline{t < -1.5}$



$-1.5 < t < -0.5$

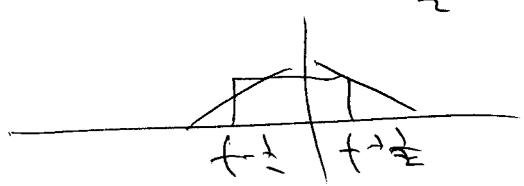
$$\Delta(t) * \Pi(t) = \int_{-1}^{t+\frac{1}{2}} (1+\lambda)\,d\lambda$$

$$= \left(\lambda + \frac{1}{2}\lambda^2\right)\Big|_{-1}^{t+\frac{1}{2}}$$

$$= t + \frac{1}{2} + \frac{1}{2}\left(t+\frac{1}{2}\right)^2 + 1 - \frac{1}{2}$$

$$= t + 1 + \frac{1}{2}t^2 + \frac{1}{2}t + \frac{1}{8}$$

$$= \frac{3}{2}t + \frac{1}{2}t^2 + \frac{9}{8}$$



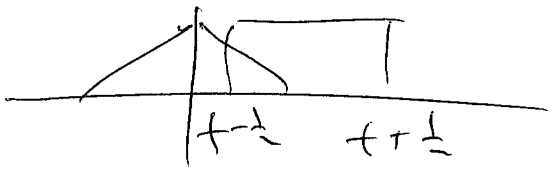
$-0.5 < t < 0.5$

$$\Delta(t) * \Pi(t) = \int_{t-\frac{1}{2}}^{0} (1+\lambda)\,d\lambda + \int_{0}^{t+\frac{1}{2}} (1-\lambda)\,d\lambda$$

$$= \left(\lambda + \frac{1}{2}\lambda^2\right)\Big|_{t-\frac{1}{2}}^{0} + \left(\lambda - \frac{1}{2}\lambda^2\right)\Big|_{0}^{t+\frac{1}{2}}$$

$$= -\left(t-\frac{1}{2}\right) - \frac{1}{2}\left(t-\frac{1}{2}\right)^2 + \left(t+\frac{1}{2}\right) - \frac{1}{2}\left(t+\frac{1}{2}\right)^2$$

$$= \frac{3}{4} - t^2$$



$$\frac{1}{2} < t < 1.5$$

$$\Lambda(t) * \Pi(t) = \int_{t-\frac{1}{2}}^{1} (1-\lambda)\,d\lambda$$

$$= \lambda - \frac{1}{2}\lambda^2 \Big|_{t-\frac{1}{2}}^{1}$$

$$= 1 - \frac{1}{2} - \left(t - \frac{1}{2} - \frac{1}{2}\left(t - \frac{1}{2}\right)^2\right)$$

$$= \cancel{1}\,\frac{1}{2} - \left(t - \frac{1}{2} - \frac{1}{2}t^2 + \frac{1}{2}t - \frac{1}{8}\right)$$

$$= \frac{1}{2} - \left(\frac{3}{2}t - \frac{1}{2}t^2 - \frac{5}{8}\right)$$

$$= \frac{9}{8} - \frac{3}{2}t + \frac{1}{2}t^2$$

$$t > 1.5 \qquad \Lambda(t) * \Pi(t) = 0$$

$$\Lambda(t) * \Pi(t) = \begin{cases} 0 & t < -1.5 \\ \frac{3}{2}t + \frac{1}{2}t^2 + \frac{9}{8} & -1.5 \le t < -0.5 \\ \frac{3}{4} - t^2 & -0.5 \le t < 0.5 \\ \frac{9}{8} - \frac{3}{2}t + \frac{1}{2}t^2 & 0.5 \le t < 1.5 \\ 0 & t \ge 1.5 \end{cases}$$