ELG 3175 INTRODUCTION TO COMMUNICATION SYSTEMS

**LABORATORY I**

**Filtering of Signals**

**WARNING!!!** This laboratory is fairly long and so you must come well prepared and organized to the lab session.

**Introduction:**

The most common processing of a signal in a communication system consists of passing the signal through a linear time-invariant system. In this context, such a system is often spoken of as a “*filter*”. These systems are usually applied to reduce some undesirable components in the signal, to compensate for some undesirable distortion of the signal, or to accentuate some characteristic of a signal. In this laboratory, we shall examine the characteristics of some simple filters that might actually be used in practice in situations where we would like to suppress certain unwanted fre­quency components that might be present in a signal and how such systems might be used in a straightforward way to separate a desirable component in a signal from some interfering compo­nents.

**References:**

Sections 6.0–6.4, 9.1, 9.4–9.7 in *Signals and Systems (2nd ed.),* byA.V. Oppenheim, A.S. Willsky & S.H. Nawab.

Sections 4.10.2, 6.0–.3, 6.5, 9.1, 9.4–.7 in *Signals and Systems (1st ed.),* byA.V. Oppenheim, A.S. Willsky & I.T. Young

Chapter 2??? in *Fundamentals of Communication Systems* by J. G, Proakis and M. Saleh, 2nd Edition, 2014.

**Background on Filters:**

The most common use of a filter in many signal processing applications is as a lin­ear time-invariant sys­tem which passes unaltered all frequency components present in a signal in a particular range of frequencies—termed the “*pass band*” of the filter—and totally blocks the frequency com­ponents outside this range—i.e., in the “*stop band*” of the filter. For this to happen, the filter's fre­quency response must be unity in the pass band and zero in the stop band. This abrupt change in the frequency response of the filter characteristic in going from the pass band characteristic to the stop band charac­teristic is impossible to provide with any filter that could ever be constructed; with practi­cal filters, the change in the frequency response must occur more gradually. As a con­sequence, to describe more realistically such filters it is necessary to allow for a narrow range of fre­quen­cies between the pass and stop bands—termed the transition band—over which the fre­quency response of the filter adjusts from the pass band characteristic to the stop band characteris­tic. The figures below show the resulting more realistic (although still somewhat ideal) description of the magnitude frequency response (termed the filter's *gain*) of four basic types of filters in the category we are discussing.









The usual form in which a filter is implemented is as a network of lumped compo­nents (i.e., resis­tors, inductors and capacitors) or their equivalents. The input signal  and output signal  in such a system are related through a linear differential equation of the form



where *N* is referred to as the *order* of the system*.* In discussing such systems in general, the standard tools usually used are not exactly those of the Fourier analysis discussed in ELG 3125 since the signals that appear may be such as not to have Fourier transforms [this is the case in dealing with unstable sys­tems]. Instead, it is usual to make use of parallel results connected with the Laplace transform. Of major significance is the concept of the transfer function of the system defined as the general ratio of the Laplace trans­form of the output over the Laplace transform of the input. From the fact that the Laplace transform of the derivative of a signal is just *s* mul­tiplied by the Laplace transform of the original signal, we can easily show that the trans­fer function of a sys­tem governed by the above differential equation is a real rational function (a function which is the ratio of two polynomials with real-valued coefficients) in *s*:



Whenever the frequency response of such a system can be meaningfully given, it may easily be shown to be given by , a rational function in 



This is the transfer function evaluated along the imaginary axis. The fact that the transfer functions of such filters are ratios of two poly­nomials in *s* represents a very significant restriction on the frequency response that can be realized by such a system. Indeed, this does not permit even the less than ideal characteristics pictured above to be achieved. Any filter with transfer func­tion in this form will have pass band characteristics which deviate from the ideal of unity, and stop band characteristics which deviate from the ideal of zero. The design of such filters is there­fore a pro­cess of trying to minimize these deviations in some way to meet specifications required by a par­ticular application. The items most commonly of concern (other considerations may be of con­cern in some situations) are the pass-band ripple (the maximum deviation of the filter gain from the peak gain) and the stop-band ripple (the minimum attenuation in the stop band).

The design process for a filter may be described as consisting of the selection of the polyno­mials and  (with some constraints) to arrive at a transfer function and frequency response minimizing the deviation from the desired characteristic, and the selec­tion of a circuit to realize the system. The selection of the polyno­mials is usually dis­cussed in terms of selecting or placing the zeros of the polynomials; the zeros of *,*  are termed the *transmission zeros* or *loss poles*, while the zeros of , are termed the *natu­ral modes* or simply the *poles* of the transfer function.



Both the number of poles and zeros as well as their positions are to be designed (almost always trying to keep the number of poles to a minimum].

The prototype for almost all frequency-selective filter design is the low-pass filter. By appropriate trans­forma­tions it is possible to convert the design of any of the four types of filters pictured above to that of the design of a low-pass filter. For example, if we seek to find a transfer func­tion  of a system of order *N* to meet some high-pass characteristic, then the ratio­nal function  will be the transfer function of some low-pass system of order *N* since . By deter­mining appropriate conditions for the low-pass characteristic  and determining the rational function that best meets it, we can determine the a suitable transfer function for the high-pass sys­tem through the reverse relationship that  . As a result of this and simi­lar approaches, as stated above, it suffices to consider simply the low-pass filter case.

**Classical Filters:**

The design of a low-pass filter traditionally has been based in the main on optimal filter selection based on one of four possible crite­ria primarily concerned with the amplitude responses of the filters. These were cri­teria such as (i) having as smoothly flat an amplitude response in the passband as possible, (ii) having the worst deviations from an ideally flat passband as small as possible, (iii) having the largest amplitude response in the stopband as small as possible, and a combination of (i) and (iii). For each of these criteria the optimal filters are well-known with the results given in the form of filter tables listing the optimum transfer functions, the loca­tions of poles and zeros, and often providing circuits and component values that could realize the filters. These four filters were:

1. *The Butterworth filter.* This filter is based on the requirement of having a mono­tonically decreasing ampli­tude response from DC which has as many derivatives as possible equal to zero as possible at the origin in the hope that this makes the amplitude response over the pass band as flat as possible. This property of this filter is often described as the characteristic that the filter is “maximally-flat”. The *n*th order Butterworth filter transfer function that results from this specifi­cation is found to be specified by

*G*(*s*)=,

which corresponds to a transfer function with no transmission zeros and *n* poles placed uni­formly around a semicircle of radius B in the left half-plane. The parameter B represents an angular frequency in the vicinity of where the pass band characteristic changes to the stop band characteristic. It can be shown then that the magnitude squared amplitude response is given by

|*H*()|2 = .

[See Section 9.7.5 in Oppenheim, Willsky, and Hawab.] The asymptotic decay rate of this in the stopband is 20 dB/decade (6 dB/octave) for each order (e.g., a third order filter response decays at a rate of 60 dB/decade [18 dB/octave]). The figure below shows the amplitude response squared of a Butterworth filter.



2. *The Chebyshev Filter*: This filter arises from the requirement that the maximum deviation from the unit amplitude response in the pass band be minimized. This results in each fluctua­tion in the pass band being of the same amount, a condition termed an “equiripple” pass band. The transfer function has no zeros and its poles are located on an ellipse. The figure below shows the amplitude response of a Chebyshev filter (of some particular order):



The number of peaks over the positive and negative frequency passband corresponds to the order of the filter (as does the asymptotic decay rate); the above picture is therefore the response of a fourth order Chebyshev filter. The variable  is a parameter of design, with smaller epsilon producing a smaller pass­band ripple at the expense of a slower change from passband to stopband characteris­tics.

3. *The Inverse Chebyshev Filter*: This filter arises from the dual requirement of a maximally flat pass band and a minimum (equiripple) stop band attenuation. The transfer function for this case is provided by *G*ICF(*s*) = 1 - *G*CF(C2/*s*), where *G*CF(*s*) is the transfer function of a Chebyshev filter. The figure below shows the ampli­tude response of one of these Inverse Chebyshev type of filters:



Note that these filters have nulls in the stop band—corresponding to transmission zeros on the imaginary axis. Each null represents an order of two in the filter with a null at infinity adding another order to the filter.

4. *The Elliptic Filter* (or *Cauer*  or *Darlington* filter): This filter is based on the requirement of both an equiripple pass band and stop band. The filter's poles all lie on an ellipse while the zeros lie on the imagi­nary axis. An amplitude response of such a filter is shown below:



The above filters are only the common classical filters that arise in trying to optimally construct lowpass filters for general conditions. In practice one often experiences needs for specific conditions and needs for which a classical filter may not be appropriate either in function or cost. Today it is possible to design filters to meet any specification we might choose to give quite routinely with the use of numerical techniques on a com­puter.. A variety of filter design packages exist which can take a filter frequency response specification and crank out the necessary pole and zero locations to meet the requirement and even provide a circuit design to implement the filter for different implementation technologies.

**Preparation:**

1. In this lab we will be using two variable filters, one of which is the Krohn-Hite 3202B. The manu­facturer claims that the frequency response of the 3202B filter is switch selectable to be either of the Butterworth type, or of an “*RC* -type”. The transfer function of an *n*th order *RC* filter corresponds to a cas­cade of *n* independent *RC* filters:

*H*(*s*) = [ 1 + *s* /*RC*]–*n*

(a) Find and sketch the amplitude and phase responses of the *RC*-type filter for the first few values of *n*.. Compare with the Butterworth characteristics.

(b) Find the relationship between the 3 dB bandwidths of the Butterworth and *RC*-type filters and B, *RC* and *n.*

2. Find the magnitude squared amplitude response of the high-pass Butterworth filter.

[*Hint*: Note that if *H*(*s*) is the transfer function of a system, then the magnitude fre­quency response is given by *G*(*j*) where *G*(*s*) =*H*(*s*) *H*(-*s*). The high pass transfer function is found by substituting C2/*s* for *s* in the low pass filter transfer function.]

3. Suppose *H*(*s*) is a rational function in *s* and is the transfer function of some lowpass filter. What type of filter will the transfer function *G*(*s*) =*H*(*s+*[1*/s*]) represent. Explicitly find *G*(*s*) and the corresponding frequency response *G*(*j*) , when *H*(*s*) = 1/(*s*-1)2. Sketch |*G*(*j*)|.

**Apparatus:**

1 - Spectrum analyzer with a tracking generator (Rohde & Schwarz HMS3010))

1 - true RMS voltmeter (this may have to be shared with another group)

1 - Krohn-Hite 3202 filter

1 - Krohn-Hite 3384 filter

1 - Dual channel oscilloscope (Rohde & Schwarz HMO722)

1 - Frequency counter (Leader LDC-823S, this may have to be shared with another group)

2 - Function generators (Agilent 33500B Series)

1 - Two-input adder or the equivalent (custom lab box)

1 - Power supply (Leader 818-TD or Agilent E3611A)

**CAUTION CAUTION CAUTION**Spectrum analyzers are very expensive, delicate and sensitive pieces of equipment which can be very easily abused. Make sure that at all times the signals you apply to the input does not exceed the maximum allowable input level noted on the front of the unit. If you are unsure of a signal level, measure it on your oscilloscope or with a voltmeter before you apply the signal to the spectrum ana­lyzer.

**Part I: Frequency Response of a Filter**

***Method A*: The basic approach (Totally 5 tasks to do which should be included in your report)**



1. Set the Krohn-Hite 3202 filter to 200 kHz (on its dial) in **the low-pass *RC* mode** (see the switches on the back of the Krohn-Hite 3202 unit).

2. Apply a 5k Hz (approximately) sinusoidal signal of approximately 1 V amplitude to the input of the filter. Measure precisely the input and output signals’ amplitudes using an RMS volt­meter and their frequency. Use these measurements to provide the amplitude frequency response at the frequency measured. Using the oscilloscope, measure the relative phase of the input and output signals, either by direct time shift measure­ments or through the technique of Lissajous figure.

3. Repeat step 2 above for several frequencies in the ranges 10-50 kHz, 50-100 kHz, and 100-200k, 200-500 kHz (at least five frequencies in each range) to determine the overall fre­quency response and particularly the decay rate of the amplitude response in the stop band. (To speed measure­ments, you may assume that the frequency generator out­put's amplitude is constant as the fre­quency is adjusted. You should quickly sweep through these frequencies displaying the fre­quency generator output on the oscillo­scope to verify that this indeed approximately so.) Quickly **plot the amplitude and phase responses as a Bode diagram (Task 1)** to make sure you have enough measurements to accurately sketch the response and determine the decay rate of the response. Determine precisely the frequency at which the filter output is half of its maximum output in power (i.e., the 3 dB point in the frequency response).

4. Repeat the above for the Krohn-Hite filter switched to **“max. flat” mode (task 2)**, and again with the Krohn-Hite 3384 filter **(task 3)** set also to a 200 KHz low pass operation in "Bu" mode (program the filter to so). The instructions for setting the controls on the Krohn-Hite 3384 filter are attached as an appendix.

5. Repeat the above again but with the **Krohn-Hite filter set to 500 kHz in the high-pass max. flat mode** or Krohn-Hite 3384 filter set also to a 500 KHz high pass operation **(task 4)**.

**(Task 5:)**Use the above measurements to plot the amplitude and phase responses of the four filters consid­ered. Deter­mine the order and type of each filter. Also, put an extra paragraph to discuss the difference between the results of Task 2 and Task 3 (both are the results under Butterworth filter, but any difference you observe?)

***Method B*: Using a tracking generator**



Some spectrum analyzers have an output which is a constant amplitude sinusoid at the same frequency as is being observed by the analyzer (sometimes this is generated by a special piece of equipment connected to the spectrum analyzer through other outputs). This is referred to as the “tracking generator output”. By applying this signal to the spectrum analyzer, the display directly provides the amplitude response squared. This then provides a rapid means of determining the amplitude response of a filter and having it graphically displayed.

1. Apply the track­ing generator output to the spectrum analyzer input (set the tracking generator level to its mini­mum level first). Adjust the tracking generator level so that the dis­played signal level is one division below the top of the display. This level represents the unit amplitude response level.

2. Without changing the tracking generator output level, apply the output to the Krohn-Hite 3202 and Krohn-Hite 3384for each of the settings used in Part A (i.e., the settings from Task 1 to Task 4), with the filter output applied to the spectrum analyzer in­put. The spectrum analyzer should be set to cover at least the 100k-800 kHz range using a resolution band­width as small as possible. Record the dis­play and compare the results to those from Method A.

**Part II**: Frequency Domain Signal Separation



1. Apply two sinusoids of equal amplitude but different frequencies *f*1 = 200 kHz and *f*2 = 400 kHz to the input of the adder. The signal of frequency *f*2 is taken as an interfering signal to the desired signal of frequency *f*1. Determine the signal-to-inter­ference ratio at the adder out­put.

2. Apply the adder output to the input of a Krohn-Hite 3384 filter in "Bu" mode and adjust the filter to improve the signal-to-inter­ference ratio as much as possible. What is the maximum signal-to-interference power ratio that can be achieved (in dB)?

3. Repeat part 2 by trying to how close together you could put the frequencies together and use to successfully sepa­rate the frequencies. (Try separating 200 kHz from 220 kHz, 500 kHz from 520 kHz, and 2000 kHz from 2020 kHz. All are separated by 20 kHz, but are they equally well separated by the filter?)