

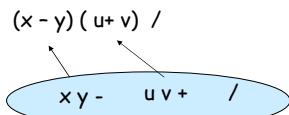
Example: Evaluation of arithmetic expressions (Postfix notation)

Infix **Postfix**

$A + B$

$A B +$

$(x - y) / (u + v)$



INFIX **POSTFIX**

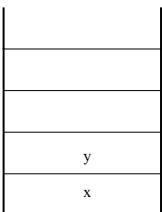
$a+b$	$a b +$
$(x - y + z)$	$x y - z +$
$(x - y - z) / (u + v)$	$x y - z - u v + /$

In general:

$A \underset{\downarrow}{operator} B$
 $A B \underset{\downarrow}{operator}$

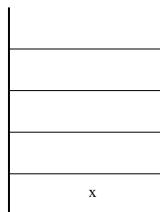
$x y - z - u v + /$

x: push(x)
y: push(y)



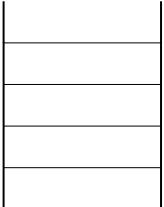
$x y - z - u v + /$

x: push(x)
y: push(y)
- : pop() (we get y)



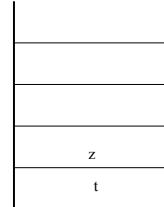
$x\ y - z - u\ v + /$

x: push(x)
y: push(y)
- : pop() (we get y) $\xrightarrow{x-y} [x-y] = t$
pop() (we get x)



$x\ y - z - u\ v + /$

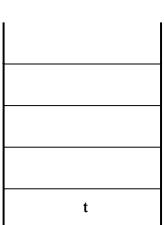
x: push(x)
y: push(y)
- : pop() (we get y) $\xrightarrow{x-y} [x-y] = t$
pop() (we get x) push(t)
z: push(z)



$x\ y - z - u\ v + /$

x: push(x)
y: push(y)
- : pop() (we get y) $\xrightarrow{x-y} [x-y] = t$
pop() (we get x) push(t)

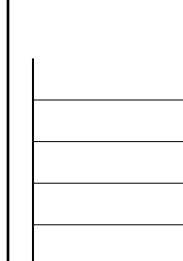
z: push(z)
- : pop() (we get z)



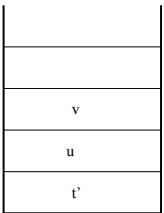
$x\ y - z - u\ v + /$

x: push(x)
y: push(y)
- : pop() (we get y) $\xrightarrow{x-y} [x-y] = t$
pop() (we get x) push(t)

z: push(z)
- : pop() (we get z)
pop() (we get t)

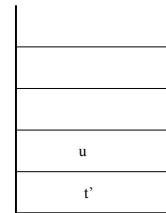


$x \ y - z - u \ v + /$



x: push(x)
y: push(y)
 $- : \text{pop}() \ (\text{we get } y)$ $\xrightarrow{\quad}$ $\boxed{x - y} = t$
 $\text{pop}() \ (\text{we get } x)$ $\xrightarrow{\quad}$ push(t)
z: push(z)
 $- : \text{pop}() \ (\text{we get } z)$ $\xrightarrow{\quad}$ $\boxed{t - z} = t'$
 $\text{pop}() \ (\text{we get } t)$ $\xrightarrow{\quad}$ push(t')
u: push(u)
v: push(v)

$x \ y - z - u \ v + /$



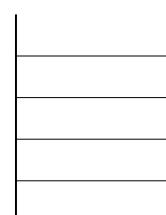
x: push(x)
y: push(y)
 $- : \text{pop}() \ (\text{we get } y)$ $\xrightarrow{\quad}$ $\boxed{x - y} = t$
 $\text{pop}() \ (\text{we get } x)$ $\xrightarrow{\quad}$ push(t)
z: push(z)
 $- : \text{pop}() \ (\text{we get } z)$ $\xrightarrow{\quad}$ $\boxed{t - z} = t'$
 $\text{pop}() \ (\text{we get } t)$ $\xrightarrow{\quad}$ push(t')
u: push(u)
v: push(v)
 $+ : \text{pop}() \ (\text{we get } v)$

$x \ y - z - u \ v + /$



x: push(x)
y: push(y)
 $- : \text{pop}() \ (\text{we get } y)$ $\xrightarrow{\quad}$ $\boxed{x - y} = t$
 $\text{pop}() \ (\text{we get } x)$ $\xrightarrow{\quad}$ push(t)
z: push(z)
 $- : \text{pop}() \ (\text{we get } z)$ $\xrightarrow{\quad}$ $\boxed{t - z} = t'$
 $\text{pop}() \ (\text{we get } t)$ $\xrightarrow{\quad}$ push(t')
v: push(v)
 $+ : \text{pop}() \ (\text{we get } v)$ $\xrightarrow{\quad}$ $\boxed{u + v} = t''$
 $\text{pop}() \ (\text{we get } u)$

$x \ y - z - u \ v + /$



x: push(x)
y: push(y)
 $- : \text{pop}() \ (\text{we get } y)$ $\xrightarrow{\quad}$ $\boxed{x - y} = t$
 $\text{pop}() \ (\text{we get } x)$ $\xrightarrow{\quad}$ push(t)
z: push(z)
 $- : \text{pop}() \ (\text{we get } z)$ $\xrightarrow{\quad}$ $\boxed{t - z} = t'$
 $\text{pop}() \ (\text{we get } t)$ $\xrightarrow{\quad}$ push(t')
v: push(v)
 $+ : \text{pop}() \ (\text{we get } v)$ $\xrightarrow{\quad}$ $\boxed{u + v} = t''$
 $\text{pop}() \ (\text{we get } u)$

$/ : \text{pop}() \ (\text{we get } t')$
Result = t''/t'