Assignment #1 - CSI 2111 (Fall 2005)
Solutions

The number of points of each question is indicated between brackets, for a total of 70 points.

Q1. Consider binary words of length N = 8 bits, representing signed numbers.

a) Find the 2's CF representation of the numbers: $(+63)_{10}$, $(-63)_{10}$, $(+115)_{10}$, $(-115)_{10}$. (4)

$(+63)_{10}$ = (00111111)
$(-63)_{10}$ = (11000001)
$(+115)_{10}$ = (01110011)
$(-115)_{10}$ = (10001101)

b) Using 2's CF representation, calculate: $(+115)_{10} + (-63)_{10}$ (2)

```
        01110011   (+115)10
+       11000001   (-63)10
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        00110100   (+52)10
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c) Using 2's CF representation, calculate: $(-115)_{10} - (-63)_{10}$ (2)

```
        10001101   (-115)10
+       00111111   (+63)10
-----------------------
        11001100   (-52)10
```

Q2. Consider signed numbers of length N = 6 bits in 2's CF representation.
Using Booth algorithm, calculate: $(-30)_{10} * (-17)_{10}$. (8)

$(-30)_{10}$ is 100010; $(-17)_{10}$ is 101111 in 6 bits

P	Y	Y_{-1}	Counter = 6
000000	10111**1**	**0**	Initial Values
011110	101111	0	P = P – X
001111	01011**1**	**1**	Arithmetic Shift Right, Counter = 5
000111	10101**1**	**1**	Arithmetic Shift Right, Counter = 4
000011	11010**1**	**1**	Arithmetic Shift Right, Counter = 3
000001	11101**0**	**1**	Arithmetic Shift Right, Counter = 2
100011	111010	1	P = P + X
110001	11110**1**	**0**	Arithmetic Shift Right, Counter = 1
001111	111101	0	P = P – X
000111	**111110**	1	Arithmetic Shift Right, Counter = 0, STOP

Answer: (**000111 111110**) which is $(510)_{10}$

Q3. a) Identify the decimal number in floating point represented in IEEE 754 form by: (4)
(1 10001011 11101000000000000000000)$_{IEEE754}$ = (?)$_{10}$

S = 1 (negative)
E' = (10001011) = 139
E = 139 – 127 = 12

M = (11101)

The decimal number = - (1.11101) X 2^{12}
= -(1111010000000) = -7808

b) Represent the decimal number $(-221.390625)_{10}$ in IEEE 754 form. (4)

$(221.390625)_{10}$ = $(11011101.011001)_2$ = (1.1011101011001) X 2^7
S = 1 (negative number)
E = 7; E' = 127 + 7 = 134 = $(10000110)_2$
M = 1011101011001
$(-221.390625)_{10}$ = (1 10000110 10111010110010000000000)$_{IEEE754}$

Q4. By indicating the axioms and theorems used in each step, where applicable:
a) Complement the Boolean function: $f(w, x, y, z) = (w + x + y)(w' + xz) + y'z'$ (4)

f' = ((w + x + y)(w' + xz) + y'z')'
= ((w + x + y)(w' + xz))'.(y'z')'
= ((w + x + y)'+(w' + xz)').(y+z)
= (w'x'y' + w(xz)').(y+z)
= (w'x'y' + w(x' + z')).(y+z)
= (w'x'y' + wx' + wz')(y+z)

b) Prove **algebraically** that: $B + C'D = AB + BC'D' + A'BC + C'D$ (6)

AB + BC'D' + A'BC + C'D

= B(A + A'C) + C'(BD' + D)	Commutative, Associative
= B(A + C) + C'(B + D)	Distributive, Complement
= AB + BC + BC′ + C′D	Distributive
= AB + B(C + C′) + C′D	Distributive
= (AB + B) + C′D	Complement
= B + C′D	Absorption

Alternatively,

B + C'D
= B **. (A + A')** + C'D =AB +A'B + C'D
= AB + A'B **. (C + C')** + C'D = AB + A'BC + A'BC' + C'D
= AB + A'BC + C'D + A'BC**. (D + D')** = AB + A'BC + C'D + A'BC'D + A'BC'D'
= AB + A'BC + C'D **. (1 + A'B)** + A'BC'D' = AB + A'BC + C'D + A'BC'D'
= AB **. (1 + C'D')** + A'BC + C'D + A'BC'D' = AB + ABC'D' + A'BC + C'D + A'BC'D'
= AB + A'BC + C'D + BC'D' **. (A + A')**
= AB + A'BC + C'D + BC'D'

Q5. Let $f(w, x, y, z) = \Pi M (0, 2, 4, 5, 6, 8, 12)$:

a) Determine the minimal SOP form of f. (4)

f = wz+wy+x'z+yz

b) Determine the minimal POS form of f. (4)

f = (y+z)(w+z)(w+x'+y)

Q6. Let $f(w, x, y, z) = \Sigma m(1, 3, 4, 6, 8, 11) + X(0, 5, 14)$:

a) Determine the minimal SOP form of f. (4)

f = w'y' + x'y'z' + x'yz + xyz' , or
f = w'y' + x'y'z' + x'yz + w'xz'

b) Determine the minimal POS form of f. (4)

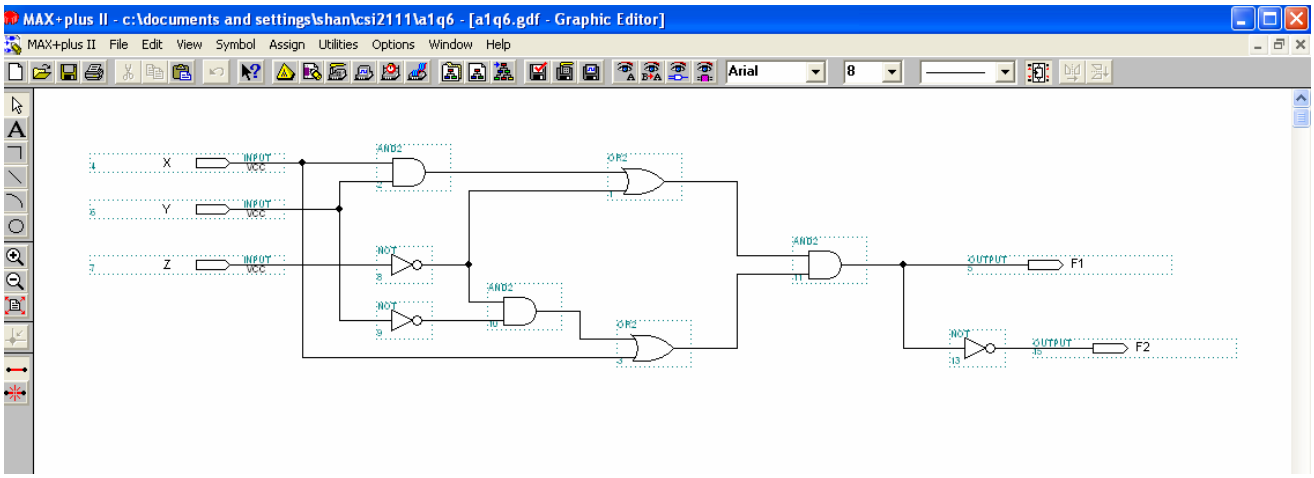
f = (x'+z')(w'+x') (w'+y+z')(x+y'+z)

Q7. Using Max+plus II (without VHDL):

a) Give graphical representation (using the *Graphic Editor*) for the following two functions: (10)

F1$(X,Y,Z) = (XY + Z')(X + Y'Z')$ and
F2$(X,Y,Z) = ((XY + Z')(X + Y'Z'))'$

Use only the components: and2, or2, not, input, output



b) Simulate your circuit using all the combinations of 0's and 1's for X, Y and Z. (10)
Include in your assignment the timing diagram resulting with, (in this order) the input signals *X, Y* and *Z* and the outputs *F1* and *F2*. You can use the total time interval of 800ns (100ns clock period).