Assignment #1 Solution - CSI 2111

Q1. Consider binary words of length N = 8 bits, representing signed numbers.
a) Find the 2's CF representation of the numbers: $(+35)_{10}$, $(-35)_{10}$, $(+123)_{10}$, $(-123)_{10}$. (4)

$(+35)_{10} = (00100011)_{2CF}$
$(-35)_{10} = (11011101)_{2CF}$
$(+123)_{10} = (01111011)_{2CF}$
$(-123)_{10} = (10000101)_{2CF}$

b) Using 2's CF representation, calculate: $(+123)_{10} + (-35)_{10}$ (4)

$(+123)_{10} + (-35)_{10} = (01011000)_{2CF} = (+88)_{10}$

c) Using 2's CF representation, calculate: $(-123)_{10} - (-35)_{10}$ (4)

$(-123)_{10} - (-35)_{10} = (-123) + (+35) = (10101000)_{2CF} = (-88)_{10}$

Q2. Consider signed numbers of length N = 6 bits in 2's CF representation.
Using Booth algorithm, calculate: $(-27)_{10} * (+17)_{10}$. (10)

$(+17)_{10} = (010001)_{2CF} = M$; $(-27)_{10} = (100101)_{2CF} = A$

A	Q	Q_1	Comment
000000	100101	0	Initialization, Counter = 6
101111	100101	0	(1.0) : A=A-M
110111	110010	1	Arithmetic Shift Right, Counter =5
001000	110010	1	(0.1) : A=A+M
000100	011001	0	Arithmetic Shift Right, Counter =4
110011	011001	0	(1.0) : A=A-M
111001	101100	1	Arithmetic Shift Right, Counter =3
001010	101100	1	(0.1) : A=A+M
000101	010110	0	Arithmetic Shift Right, Counter =2
000010	101011	0	(0.0) : Arithmetic Shift Right, Counter =1
110001	101011	0	(1.0) : A=A-M

111000	110101	1	Arithmetic Shift Right, Counter =0

So, the product is (A.Q) = $(111000110101)_{2CF} = (-459)_{10}$

Q3. a) Identify the decimal number in floating point represented in IEEE 754 form by: (5)

(1 01111100 10101100000000000000000)$_{IEEE754}$ = (?)$_{10}$

S = 1 : Negative
E' = 124, so Exp = E'-127 = -3
1.M = 1.101011
$-1.101011 * 2^{-3}$ = -0.001101011 = -107/512 = $(-0.208984375)_{10}$

b) Represent the decimal number $(243.09375)_{10}$ in IEEE 754 form. (5)

Positive : S = 0
$(243.09375)_{10}$ = 11110011.00011 = $1.111001100011 * 2^7$
1.M = 1.111001100011
Exp = 7, so E' = Exp + 127 = 134 : 10000110
Thus, we obtain: 0 10000110 1110 0110 0011 0000 0000 000

Q4. By indicating the axioms and theorems used in each step:

a) Prove **algebraically** that: *B + A'C' = AB + A'BC + BC'+ A'C'* (8)

B + A'C = B **(1) (1)** + A'C' *(by identity element, AND, 2 times)*
= B **(A+A')(C+C')** + A'C' *(by complementation, OR, 2 times)*
= **ABC + A'BC + ABC' + A'BC'** + A'C' *(by distributivity, AND, 4 times)*
= ABC + A'BC + ABC' + **ABC'** + A'BC' + A'C' *(by idempotence, OR)*
= **AB(C+C')** + A'BC + ABC'+ A'BC'+ A'C' *(by distributivity, AND)*
= **AB(1)** + A'BC + ABC'+ A'BC'+ A'C' *(by complementation, OR)*
= **AB** + A'BC + ABC'+ A'BC'+ A'C' *(by identity element, AND)*
= AB + A'BC + **BC'(A+A')** + A'C' *(by distributivity, AND)*
= AB + A'BC + **BC'(1)** + A'C' *(by identity element, AND)*
= AB + A'BC + **BC'** + A'C' *(by identity element, AND)*

b) Determine **algebraically** the <u>minimal</u> POS form of ***f*** (x, y, z) = x'y' + xz + x'yz' + yz (8)

= x'y' + xz + x'yz' + **(1).**yz	*(by identity element, AND)*
= x'y' + xz + x'yz' + **(x+x').**yz	*(by complementation, OR)*
= x'y' + xz + x'yz' + **xyz + x'yz**	*(by distributivity, AND)*
= x'y' + xz + x'yz' **+ x'yz + xyz**	*(by commutativity, OR)*
= x'y' + xz + x'y **(z'+ z)** + xyz	*(by distributivity, AND)*
= x'y' + xz + x'y **(1)** + xyz	*(by complementation , OR)*
= x'y' + xz + **x'y** + xyz	*(by identity element , AND)*
= **x'y' + x'y** + xz + xyz	*(by commutativity, OR)*
= **x'(y'+ y)** + xz + xyz	*(by distributivity, AND)*
= **x'(1)** + xz + xyz	*(by complementation , OR)*
= **x'** + xz + xyz	*(by identity element , AND)*
= x' + **xz**	*(by absorption, OR)*
= **x' + x'z** + xz	*(by absorption, OR)*
= x' **+ zx' + zx**	*(by commutativity, AND)*
= x' + **z(x'+x)**	*(by distributivity, AND)*

= x’ + z**(1)** *(by complementation , OR)*
= x’ + **z** *(by identity element , AND)*

Q5. Let ***f*** (w, x, y, z) = ΠM (1, 3, 5, 6, 9, 11) :

a) Determine the <u>minimal</u> SOP form of ***f***. (8)

f = y'z' + wx + x'z' + xyz

b) Determine the <u>minimal</u> POS form of ***f***. (8)

f' = x'z + w’yz' + w’xyz’, so
f = (x+z') (w+y'+z) (w+x’+y’+z)

Q6. Let ***f*** (w, x, y, z) = Σm (2, 3, 5, 7, 8, 11, 12) X (0, 9, 14) :

a) Determine the <u>minimal</u> SOP form of ***f***. (8)

3 possible solutions:

f = wy'z' + w'xz + wx'z + w'x'y
f = wy'z' + w'xz + x'yz + w'x'y
f = wy'z' + w'xz + x'yz + w'x'z'

b) Determine the <u>minimal</u> POS form of ***f***. (8)

2 possible solutions:

f' = wxz + w'xz' + wyz' + w'x'y', so **f = (w'+x'+z').(w+y'+z).(w'+y'+z).(w+x+y)**
f' = wxz + w'xz' + wyz' + x'y'z, so **f = (w'+x'+z').(w+y'+z).(w'+y'+z).(x+y+z')**

Q7. Let ***f*** *(x, y, z) = ((x + y' + z') y) + xy'z*

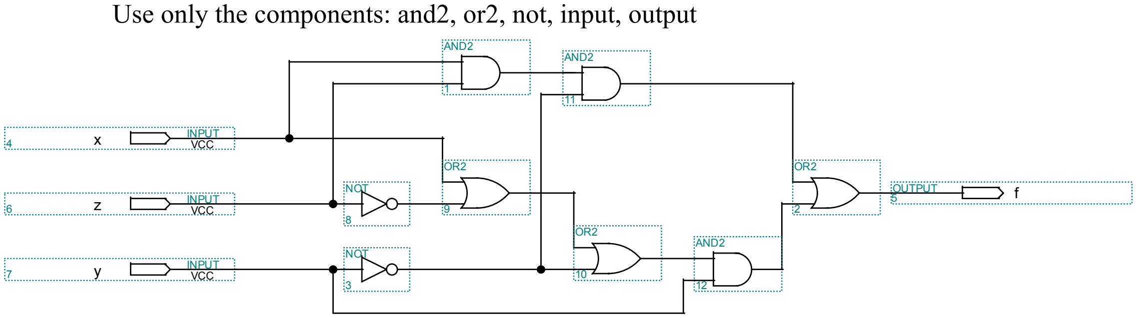
For this question, use Max+plus II (without VHDL) in *Functional SNF Extractor* mode.

- In the *Max+plus II* menu, select *Compiler*
- Click on the *Compiler* window
- In the *Processing* menu, select *Functional SNF Extractor*

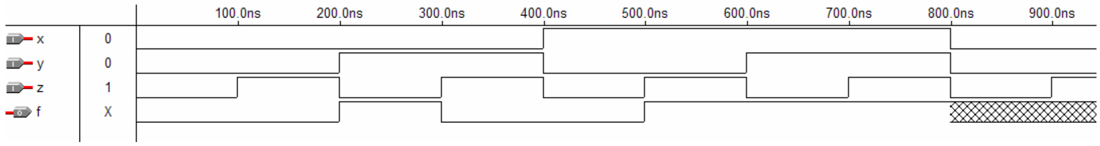
This mode eliminates signal transmission delays.

Please simply include screen captures (or cut/paste to a Word document) of your diagram and of the resulting timing diagram (signals). They must however be readable on paper.

a) Give graphical representation of ***f*** *(x, y, z)* (10)
Use only the components: and2, or2, not, input, output



b) Simulate your circuit using all the combinations of 0’s and 1’s for *x, y* and *z.* (10)
Include in your assignment the timing diagram resulting with, in this order, input signals *x, y* and *z* and output ***f***. You can set the total time interval to 800ns (100ns clock period).



Note : f (w, x, y, z) = Σm(2, 5, 6, 7)