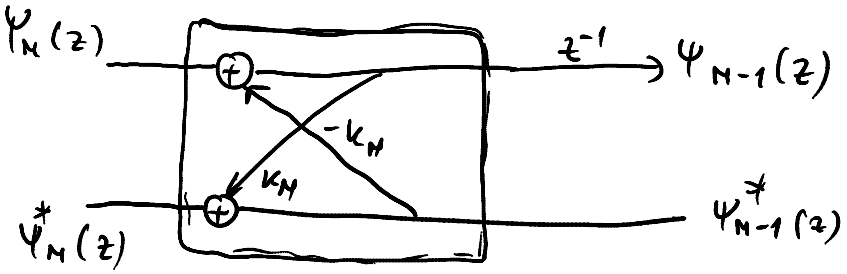
$$H_N(z) = \frac{N_N(z)}{D_N(z)}$$

1° GOAL: FIND A SET OF ORTO NORMAL FUNCTIONS

$\Psi_N(z), \Psi_{N-1}(z), \ldots, \Psi_0(z)$

$$\Psi_N(z) = D_N(z)$$

2° IMPLEMENT A FILTER




$$\Psi_N^*(z) = z^N \Psi_N(z^{-1})$$

3° USING ORTONORMAL FUNCTIONS, REPRESENT $N_N(z)$

$$N_N(z) = \sum_{i=0}^{N} c_i \Psi_i(z)$$

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HOW DO WE COMPUTE $\Psi_i(z)$ $i=0,\ldots,N$?

USING SCHUR POLINOMIALS

$$\Psi_{N-1}(z) = \frac{z^{-1}\left[\Psi_N(z) - K_N \Psi_N^*(z)\right]}{1 - K_N^2}$$

$$\Psi_{N-1}^*(z) = z^{N-1}\Psi_{N-1}(z^{-1}) = \frac{z^{+1}\left[z^{N-1}\Psi_N(z^{-1}) - K_N z^{N-1}\Psi_N(z^{-1})\right]}{1 - K_N^2}$$

$$\Psi^*_{N-1}(z) = z^{N-1}\Psi_{N-1}(z^{-1}) = \frac{z^{+1}\left[z\Psi_N(z^{-1}) - K_N z\Psi'_N(z^{-1})\right]}{1-K_N^2}$$

$$\Psi^*_{N-1}(z) = \frac{\Psi^*_N(z) - K_N\Psi_N(z)}{1-K_N^2}$$

$$\Psi^*_{N-1}(z^{-1}) = (z^{-1})^{(N)}\ \Psi_N(z)$$

$$\boxed{\Psi^*_N(z) = (1-K_N^2)\,\Psi^*_{N-1}(z) + K_N\Psi_N(z)}$$

$$\Psi_{N-1}(z) = \frac{z^{-1}\left(\Psi_N(z) - K_N(1-K_N^2)\Psi^*_{N-1}(z) - K_N^2\Psi_N(z)\right)}{1-K_N^2}$$

$$\boxed{\Psi_{N-1}(z) = z^{-1}\left(\Psi_N(z) - K_N\Psi^*_{N-1}(z)\right)}$$

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HOW DO WE DEAL WITH THE NOMINATOR?

$$N_N(z) = \sum_{i=0}^{N} c_i\,\Psi_i(z)$$

FOR $i = m$ to $0$

1° $c_i = \dfrac{Q^*(0)}{\Psi_i^*(0)}$

2° $Q(z) = Q(z) - c_i\Psi_i(z)$

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PARMI 12.4

$$H(z) = \frac{1 + \frac{5}{12}z^{-1} + \frac{1}{24}z^{-2}}{1 - \frac{9}{8}z^{-1} + \frac{5}{16}z^{-2}}$$

$$\Psi_2(z) = z^2 - 1.125z + 0.3125$$

$$\Psi_1(z) = \frac{z^{-1}\left(\Psi_2(z) - k_N \Psi_2^*(z)\right)}{1 - k_2^2}$$

$$\Psi_2^*(z) = z^2 \cdot \Psi(z^{-1}) = 1 - 1.125z + 0.3125z^2$$

$$k_2 = \frac{\Psi_2(0)}{\Psi_2^*(0)} = \frac{0.3125}{1} = 0.3125$$

$$\Psi_1(z) = \frac{z^{-1}\left(z^2 - 1.125z + 0.3125 - 0.3125 \cdot (1 - 1.125z + 0.3125z^2)\right)}{1 - 0.3125^2}$$

$$\Psi_1(z) = \frac{z(1 - 0.3125^2) + 1.125\ (0.3125 - 1)}{1 - 0.3125^2}$$

$$= z - \frac{1.125}{1 + 0.3125} = z - 0.8571$$

$$\Psi_0(z) = \frac{z^{-1}\left(z - 0.8571 + 0.8571\,(1 - 0.8571z)\right)}{1 - 0.8571z}$$

$$k_1 = -0.8571$$

$$\Psi_0(z) \simeq 1$$

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$$N_N(z) = z^2 + \frac{5}{12}z + \frac{1}{24}$$

$$C_2 = \frac{Q^*(0)}{\Psi_2^*(0)} = \frac{1}{1} = 0$$

$$Q(z) = Q(z) - C_2 \cdot \Psi_2(z) = z^2 + \frac{5}{12}z + \frac{1}{24} - \left(z^2 - 1.125z + 0.3125\right)$$

$$Q(z) = \frac{5}{12}z + 1.125z + \frac{1}{24} - 0.3125$$

$$Q(z) = \frac{5}{12} z + 1.125\, z + \frac{1}{24} - 0.3125$$

$$Q(z) = 1.5417\, z - 0.2708$$

$$C_1 = \frac{1.5417}{1} = 1.5417$$

$$\vdots$$

$$C_0 = 1.0506$$