Coupled form

Friday, March 24, 2006
1:42 AM

STATE SPACE MODEL

SOURCE BOSE pg. 193

$$v(n+1) = A v(n) + B x(n)$$
$$y(n) = C v(n) + D x(n)$$

P IS A NON-SINGULAR MATRICS

LET'S DEFINE $v(n) = P \cdot \hat{v}(n)$

$$P \hat{v}(n+1) = A P \hat{v}(n) + B x(n)$$
$$y(n) = C P \hat{v}(n) + D x(n)$$

$$\hat{v}(n+1) = P^{-1} A P \hat{v}(n) + P^{-1} B x(n)$$

DEFINE: $\hat{A} = P^{-1} A P$

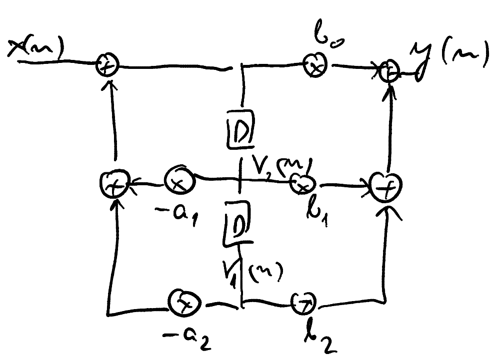
$\hat{B} = P^{-1} B$

$\hat{C} = C P$

$\hat{D} = D$

SOURCE BOSE pg 270

CONSIDER IIR FILTER $H(z) = \dfrac{b_0 + z^{-1} b_1 + z^{-2} b_2}{1 + a_1 z^{-1} + a_2 z^{-2}}$



$$v(n) = \begin{bmatrix} v_1(n) \\ v_2(n) \end{bmatrix}$$

$$v_2(n+1) = -a_1 v_2(n) - a_2 v_1(n) + x(n)$$
$$v_1(n+1) = v_2(n)$$
$$y(n) = \left(-a_1 v_2(n) - a_2 v_1(n) + x(n)\right) \cdot b_0 + v_2(n) b_1 + $$
$$+ v_1(n) \cdot b_2$$

$$y(n) = v_1(n)\left(b_2 - a_2 b_0\right) + v_2(n)\left(b_1 - a_1 b_0\right) + b_0 x(n)$$

$$y(n) = v_1(n)\,(b_2 - a_2 b_0) + v_2(n)\,(b_1 - a_1 b_0) + b_0\, x(n)$$

$$v(n+1) = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} v(n) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x(n)$$

$$y(n) = \begin{bmatrix} b_2 - a_2 b_0 & b_1 - a_1 b_0 \end{bmatrix} v(n) + [b_0]\, x(n)$$

$$P = \frac{1}{\omega}\begin{bmatrix} 0 & 1 \\ -\omega & \sigma \end{bmatrix} \qquad z_{1/2} = \sigma \pm j\omega \qquad P^{-1} = \begin{bmatrix} \sigma & 1 \\ -\omega & 0 \end{bmatrix}$$

$$z^2 + a_1 z + a_2 = 0 \quad \Rightarrow \quad z_{1/2} = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2}$$

$$a_1 = -2\sigma$$

$$\frac{a^2 - 4a_2}{4} = \omega^2 \Rightarrow a_2 = \frac{4\sigma^2 + 4\omega^2}{4}$$

$$a_2 = \sigma^2 + \omega^2$$

$$\hat{A} = P^{-1} A P = \frac{1}{\omega}\begin{bmatrix} -\omega(\sigma + a_1) & a_2 + \sigma(\sigma + a_1) \\ -\omega^2 & \sigma\omega \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix} \qquad \hat{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$v_1(n+1) = \sigma v_1(n) + \omega v_2(n) + x(n)$$
$$v_2(n+1) = -\omega v_1(n) + \sigma v_2(n)$$
$$y(n) = \bar{c}_1 v_1(n) + \bar{c}_2 v_2(n) + d\, x(n)$$