Channel capacity of *n*-antenna BLAST architecture

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The channel capacity of the Bell Labs layered space-time (BLAST) communication architecture is substantially larger than that of traditional systems in certain applications. However, the correlation between individual channels may severely degrade the BLAST performance. An investigation is presented into the BLAST architecture and a simple formula derived for its channel capacity as a function of the correlation coefficient. This gives a simple method of estimating BLAST performance in realistic environments.

Introduction: The Bell Labs layered space-time (BLAST) communication architecture is a very efficient tool for wireless communications in rich multipath environments [1 – 3]. The spectral efficiency of this architecture may be much larger than that of traditional systems in certain applications. The practical value of the BLAST channel capacity depends substantially on the correlation between individual channels (spatial paths) [4, 5]. The performance of the two-antenna BLAST architecture in correlated channels has been analysed in [4]. However, the BLAST architecture provides a substantial advantage over traditional systems when the number of antennas is large. In this Letter, we consider the general case of an *n*-antenna BLAST architecture in correlated channels and derive a simple formula for the BLAST channel capacity when all the correlation coefficients are real and equal. This provides a simple method for estimating the performance of the BLAST architecture in a realistic environment without a detailed electromagnetic analysis.

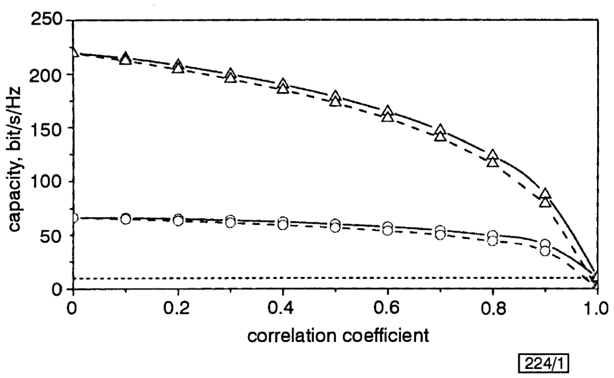

Fig. 1 *BLAST channel capacity against correlation coefficient,* ρ = *30 dB*

—○— eqn. 7, $n = 10$
– –○– – eqn. 8, $n = 10$
—△— eqn. 7, $n = 50$
– –△– – eqn. 8, $n = 50$
- - - - - - Shannon limit

Channel capacity of n-antenna BLAST: In the general case of a fixed linear $n \times n$ matrix channel with additive white Gaussian noise and when the transmitted signal vector is composed of statistically independent equal power components each with a Gaussian distribution, the channel capacity is [2]

$$C = \log_2 \det\Big(\mathbf{I} + \frac{\rho}{n}\mathbf{H}\cdot\mathbf{H}^{+}\Big) \ \ \text{bit/s/Hz} \qquad (1)$$

where n is the number of transmit/receive antennas, ρ is the signal-to-noise ratio, **I** is an $n \times n$ identity matrix, **H** is the normalised channel matrix, and '+' means transpose conjugate. Denoting the matrix under the determinant sign in eqn. 1 by **Z** and after some mathematical development, we arrive at the following expression for det(**Z**):

$$\det(\mathbf{Z}) = \sum_{i_1, i_2, \ldots, i_n} \varepsilon_{i_1 i_2 \ldots i_n} \prod_{k=1}^{n} \Big(\delta_{k i_k} + \rho\sqrt{\beta_{i_k}\beta_k}\cdot r_{k i_k}\Big) \qquad (2)$$

where δ_{ij} is the Kroneker delta, $\varepsilon_{i_1 i_2 \ldots i_n} = 1$ if $[i_1, i_2, \ldots, i_n]$ is an even permutation of $[1, 2, \ldots, n]$, $\varepsilon_{i_1 i_2 \ldots i_n} = -1$ if $[i_1, i_2, \ldots, i_n]$ is an odd permutation of $[1, 2, \ldots, n]$, and $\varepsilon_{i_1 i_2 \ldots i_n} = 0$ otherwise, β_i is the normalised received power in the ith receiver (without the noise contribution):

$$\beta_i = \frac{\langle y_i \cdot y_i^* \rangle}{\sum_k \langle y_k \cdot y_k^* \rangle} = \frac{1}{n}\sum_k |h_{ik}|^2 \qquad (3)$$

where '< >' denotes averaging, '*' denotes complex conjugate and h_{ik} denotes components of **H**. r_{ij} is the correlation coefficient of the received signals, where

$$r_{ij} = \frac{\langle y_i \cdot y_j^* \rangle}{\sqrt{\langle y_i \cdot y_i^* \rangle \langle y_j \cdot y_j^* \rangle}} = \frac{\sum_k h_{ik} h_{jk}^*}{\sqrt{\sum_k |h_{ik}|^2 \sum_k |h_{jk}|^2}} \qquad (4)$$

$\mathbf{X} = (x_1, x_2, \ldots, x_n)^T$ and $\mathbf{Y} = (y_1, y_2, \ldots, y_n)^T$ are transmitted and received signal vectors (complex envelopes), respectively,

$$\mathbf{Y} = \mathbf{H}\cdot\mathbf{X} \qquad (5)$$

and 'T' means transpose. Actually, we consider the received signal in eqn. 5 without a noise component: the noise is already taken into account in eqn. 1. The narrowband assumption is also used (i.e. the channel is considered to be frequency-independent over the signal bandwidth). The following normalisation of **H** is adopted here:

$$\sum_{i,j=1}^{n} |h_{ij}|^2 = n \qquad (6)$$

Hence, when **H** = **I** (completely uncorrelated parallel channels), ρ/n is the signal-to-noise ratio in every channel. Some other kinds of normalisation can also be used, but in this case ρ/n will have a slightly different meaning.

Thus, eqn. 2 gives the explicit dependence of the BLAST channel capacity on the individual channel correlation. To obtain some insight, we now consider a simple case when all the received powers are equal ($\beta_i = 1/n$) and all the correlation coefficients are also equal and real ($r_{ij} = r$, $\mathrm{Im}(r) = 0$). This case is somewhat artificial because it is expected that the correlation of neighbouring channels is larger than that of distant channels. However, the case of equal correlation coefficients provides a worst-case estimation and some insight into the operation of the BLAST architecture in correlated channels, so it deserves to be considered (besides, r may be interpreted as an 'average' correlation coefficient). In this case, after some mathematical transformations that do not change the determinant, and using eqn. 2, we present eqn. 1 in the following form:

$$C = n\cdot\log_2\Big(1+\frac{\rho}{n}(1-r)\Big) + \log_2\Big(1+\rho\cdot r\cdot\Big(1+\frac{\rho}{n}(1-r)\Big)^{-1}\Big) \qquad (7)$$

For $n = 2$, eqn. 7 reduces to eqn. 8 in [4]. As a detailed analysis of eqn. 7 shows (see Fig. 1), the channel capacity decreases substantially for $|r| \geq 0.5 - 0.8$, which agrees well with known results on spatial diversity techniques and also with the case of a two-antenna BLAST architecture. For $r = 1$, eqn. 7 reduces to the famous Shannon formula. It should be noted that the second term in eqn. 7 is essential only when the correlation coefficient is close to 1. However, in this case the advantage of the BLAST architecture over traditional techniques is very small (the channel capacity is close to the Shannon limit) and it is not reasonable to use it. Thus, when the BLAST architecture provides a substantial advantage, its channel capacity can be estimated to be (for $0 \leq r < 1$)

$$C \simeq n\cdot\log_2\Big(1+\frac{\rho}{n}(1-r)\Big) \qquad (8)$$

Fig. 1 compares the BLAST channel capacity computed using eqns. 7 and 8. Obviously, eqn. 8 provides quite a good approximation for eqn. 7. In the limiting case of $n \to \infty$, we obtain from eqn. 8

$$C_\infty \simeq \frac{\rho(1-r)}{\ln 2} \qquad (9)$$

When $r = 0$, the last two equations reduce to the well-known formulas (in this case, **H** = **I**) [2]

$$C = n\cdot\log_2\Big(1+\frac{\rho}{n}\Big) \qquad (10)$$

$$C_\infty = \frac{\rho}{\ln 2} \qquad (11)$$

Comparison of eqn. 8 with eqn. 10, and eqn. 9 with eqn. 11, clearly indicates that the effect of the channel correlation is equivalent to a decrease in the signal-to-noise ratio. Hence, for example, $r = 0.5$ is equivalent to a 3dB reduction in the signal-to-noise ratio. Another interpretation of eqns. 8 and 9 is that the correlation of individual channels gives an increase in the noise level because for each particular channel all the other channels are just the sources of interference.

Thus, eqns. 7 – 9 enable us to estimate the $n \times n$ BLAST channel capacity in correlated channels in a simple way, without requiring a detailed electromagnetic analysis but using well-known results on spatial diversity techniques [6, 7].

Conclusion: In this Letter, we have developed a simple mathematical model of the spatial channel in order to investigate the communications theory aspect of the problem. The correlation of individual channels is a fundamental limitation to increasing the BLAST channel capacity by employing more and more antennas. A simple formula for estimating the channel capacity of an n-antenna BLAST architecture in correlated channels is proposed. This model provides the worst-case estimation and enables us to apply the known results on spatial diversity techniques to the present problem. A more accurate analysis of the BLAST architecture would require more precise electromagnetic models of the spatial channel and, in particular, of the electromagnetic environment, the antennas and their coupling.

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References

1 FOSCHINI, G.J.: 'Layered space-time architecture for wireless communication in a fading environment when using multiple antennas', *Bell Lab. Tech. J.*, 1996, **1**, (2), pp. 41–59

2 FOSCHINI, G.J., and GANS, M.J.: 'On limits of wireless communications in a fading environment when using multiple antennas', *Wireless Personal Commun.*, 1998, **6**, (3), pp. 311–335

3 GOLDEN, G.D., FOSCHINI, C.J., VALENZUELA, R.A., and WOLNIANSKY, P.W.: 'Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture', *Electron. Lett.*, 1999, **35**, (1), pp. 14–16

4 LOYKA, S.L.: 'Channel capacity of two-antenna BLAST architecture', *Electron. Lett.*, 1999, **35**, (17), pp. 1421–1422

5 LOYKA, S.L., and MOSIG, J.R.: 'Spatial channel properties and spectral efficiency of BLAST architecture'. AP2000 Millennium Conf. Antennas & Propagation, Davos, Switzerland, 2000

6 LEE, W.C.Y.: 'Mobile communications engineering' (McGraw Hill, New York, 1998)

7 JAKES, W.C., JR.: 'Microwave mobile communications' (John Wiley and Sons, New York, 1974)