Chapter 19: Turing Machines

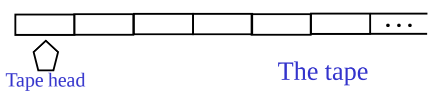
I. Theory of Automata

II. Theory of Formal Languages

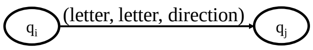
→ III. Theory of Turing Machines …

A Turing machine is:

1. an alphabet Σ of input letters ($\Delta \notin \Sigma$)
2. a tape, finite to the left but infinite to the right, divided into cells. The tape contains the input word starting in the first cell. The rest of the tape is filled with blanks.
3. a tape head initially positioned at the first cell
4. an alphabet Γ of output characters which can be printed on the tape ($\Delta \notin \Gamma$)
5. a finite set of states $q_1, q_2, q_3, \ldots$ with exactly one start state and some (maybe none) halt states
6. a program which is a set of rules that indicate, for a letter on the tape and a state, the transition, the print character, and the direction to move the tape head.



Final state: No transition out of final state.



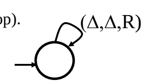
Program Rules

- The first letter is the character the tape head reads from the cell to which it is pointing.
- The second letter is what the tape head prints in the cell before it leaves the cell.
- The direction is R (right) or L (left) and indicates if the tape head should move one cell to the right or one cell to the left.

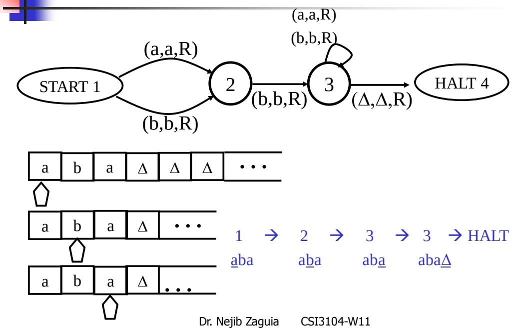
- Turing machines are deterministic. There is at most one transition out of each state labeled with the same first letter.
- The machine crashes if there is no arrow labeled with the letter in the cell pointed to by the tape head .
- The machine crashes if it tries to move left from the first cell.
- A word is accepted when a final state is entered.

Execution of a Turing Machine: 3 possibilities

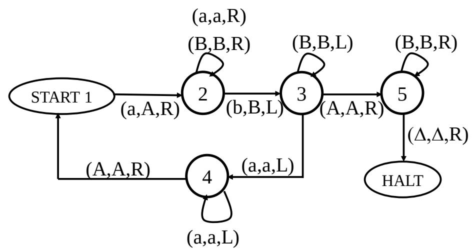
1. The execution terminates successfully if a halt state is entered. We say that execution halts. The word on the tape (before execution started) is accepted. (It is in the language accepted by the machine.)
2. The machine crashes. Either the tape head tries to move left from the first cell on the tape, or the we are in a state and read a letter that offers no choice of path to another state. In this case the execution terminates unsuccessfully.
3. The execution does not halt (infinite loop).
   For example:

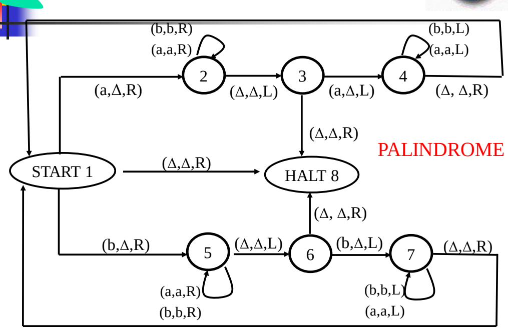


The tape head moves to the right one cell at a time forever.



Example: $\{a^n b^n\}$ $\Sigma=\{a,b\}$ $\Gamma=\{a,A,B\}$

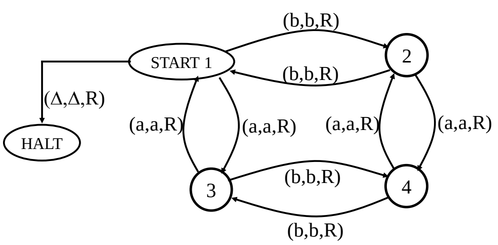


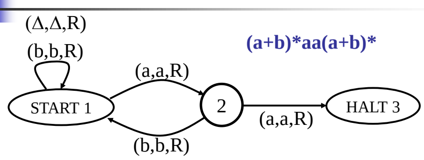


- Theorem. For every regular language, there is a Turing machine that accepts it.

  Proof: We transform a finite automaton to a Turing machine.
  - The start state of the finite automaton is the start state of the Turing machine.
  - Add a halt state to the Turing machine, and one transition form every final state of the finite automaton to the new halt state. (Erase all the + symbols.) Label the new transitions (Δ,Δ,R).
  - Change each label a of the finite automaton to (a,a,R), and similarly for all other letters of Σ.

Example: EVEN-EVEN

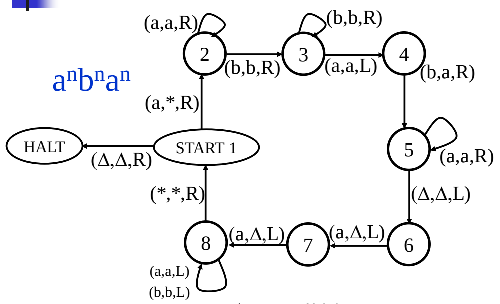


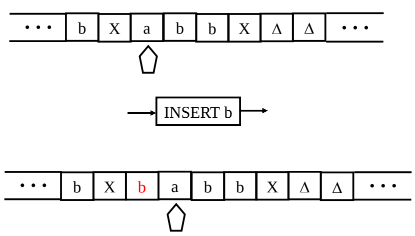


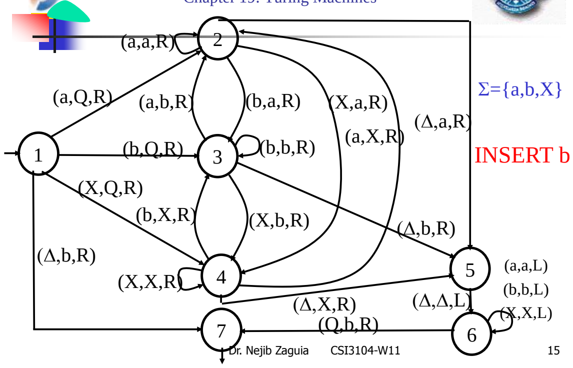
(a+b)*aa(a+b)*

- If we reach the HALT state, execution terminates and the word is accepted. (double a)
- If the machine crashes, the word is rejected. (no aa and ends in a)
- It is also possible to get into an infinite loop. (no aa and ends in b)

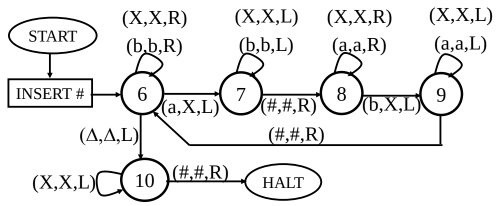
$a^n b^n a^n$

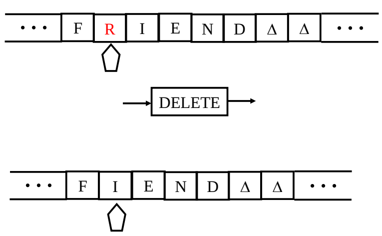

(a,a,R)
2
(b,b,R)
3
(b,b,R)
4
(a,a,L)
(b,a,R)
(a,*,R)
HALT
START 1
(Δ,Δ,R)
5
(a,a,R)
(Δ,Δ,L)
(*,*,R)
8
(a,Δ,L)
7
(a,Δ,L)
6
(a,a,L)
(b,b,L)

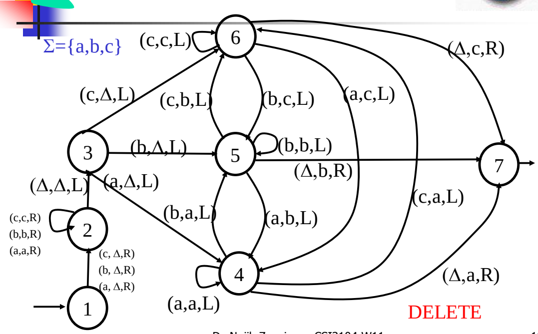

• • • b X a b b X Δ Δ • • •
INSERT b
• • • b X b a b b X Δ Δ • • •


2
(a,a,R)
(a,Q,R)
(a,b,R)
(b,a,R)
(X,a,R)
(a,X,R)
(Δ,a,R)
Σ={a,b,X}
1
(b,Q,R)
3
(b,b,R)
INSERT b
(X,Q,R)
(b,X,R)
(X,b,R)
(Δ,b,R)
(Δ,b,R)
5
(a,a,L)
(b,b,L)
(X,X,L)
(X,X,R)
4
(Δ,X,R)
(Δ,Δ,L)
7
(Q,b,R)
6

EQUAL


START
INSERT #
(X,X,R)
(b,b,R)
(X,X,L)
(b,b,L)
(X,X,R)
(a,a,R)
(X,X,L)
(a,a,L)
6
7
8
9
(a,X,L)
(#,#,R)
(b,X,L)
(#,#,R)
(Δ,Δ,L)
(X,X,L)
10
(#,#,R)
HALT


• • • F R I E N D Δ Δ • • •
DELETE
• • • F I E N D Δ Δ • • •


Σ={a,b,c}
(c,c,L)
6
(Δ,c,R)
(c,Δ,L)
(c,b,L)
(b,c,L)
(a,c,L)
3
(b,Δ,L)
5
(b,b,L)
(Δ,b,R)
7
(Δ,Δ,L)
(a,Δ,L)
(c,c,R)
(b,b,R)
(a,a,R)
2
(b,a,L)
(a,b,L)
(c,a,L)
(c, Δ,R)
(b, Δ,R)
(a, Δ,R)
4
(Δ,a,R)
1
(a,a,L)
DELETE