Toward a 6/5 Bound for the Minimum Cost 2-Edge Connected Subgraph Problem [3]

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Abstract

Given a complete graph $K_n = (V, E)$ with non-negative edge costs $c \in \mathbb{R}^E$, the problem *2EC* is that of finding a 2-edge connected spanning multi-subgraph of K_n of minimum cost. The integrality gap α*2EC* of the linear programming relaxation *2EC*$^{\mathrm{LP}}$ for *2EC* has been conjectured to be $\frac{6}{5}$, although currently we only know that $\frac{6}{5} \leq \alpha 2EC \leq \frac{3}{2}$. In this paper, we explore the idea of using the structure of solutions for *2EC*$^{\mathrm{LP}}$ and the concept of convex combination to obtain improved approximation algorithms for *2EC* and bounds for α*2EC*. We focus our efforts on a family J of half-integer solutions that appear to give the largest integrality gap for *2EC*$^{\mathrm{LP}}$. We successfully show that the conjecture $\alpha 2EC = \frac{6}{5}$ is true for any cost functions optimized by some $x^* \in J$. Our methods are constructive and thus also provide a $\frac{6}{5}$-approximation algorithm for *2EC* for these special cases.

Keywords: minimum cost 2-edge connected subgraph problem, approximation algorithm, integrality gap.

1 Introduction

The *2-edge connected subgraph problem* (*2EC*) is that of finding a minimum cost 2-edge connected spanning multi-subgraph of the complete graph $K_n = (V, E)$ with costs $c \in \mathbb{R}^E_{\geq 0}$. This problem has many important applications

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in network design. It is known to be NP-hard even for very special cases [3]. Currently, a $\frac{3}{2}$-approximation algorithm exists for *2EC*. This follows from the fact that for any instance of *2EC*, we can assume, WLOG, that the costs are metric and the solutions do not include multi-edges [1], in which case we can apply the $\frac{3}{2}$-approximation due to Frederickson and Ja'Ja' [4]. For *2EC* where multi-edges are not allowed, a 2-approximation is known [5].

For $e \in E$, letting x_e represent the number of copies of e in the *2EC* solution, *2EC* can be formulated as an integer linear program (ILP) as follows, i.e.:

$$\begin{array}{rlr} \text{Minimize} & cx & \\ \text{Subject to} & \sum(x_{ij} : i \in S, j \notin S) \geq 2 & \text{for all } \emptyset \subset S \subset V, \\ & x_e \geq 0, \text{ and integer} & \text{for all } e \in E. \end{array} \tag{1}$$

The linear programming (LP) relaxation of *2EC*, denoted by $2EC^{\mathrm{LP}}$, is obtained by relaxing the integer requirement in (1). We use OPT(*2EC*) (resp. OPT($2EC^{\mathrm{LP}}$)) to denote the optimal value of *2EC* (resp. $2EC^{\mathrm{LP}}$). Also, given any feasible solution x^* for $2EC^{\mathrm{LP}}$, its *support graph* G_{x^*} is defined to be the subgraph of K_n obtained by taking all edges $e \in E$ for which $x^*_e > 0$.

We are interested in the *integrality gap* $\alpha 2EC$ for $2EC^{\mathrm{LP}}$, which is the worst case ratio between OPT(*2EC*) and OPT($2EC^{\mathrm{LP}}$), i.e.

$$\alpha 2EC = \max_{\substack{c \geq 0 \\ c \neq 0}} \frac{\mathrm{OPT}(2EC)}{\mathrm{OPT}(2EC^{\mathrm{LP}})}.$$

This gives a measure of the quality of the lower bound provided by $2EC^{\mathrm{LP}}$. Moreover, a polynomial-time constructive proof of $\alpha 2EC = k$ would provide a k-approximation algorithm for $\alpha 2EC$.

Even though *2EC* has been intensively studied, little is known about $\alpha 2EC$, except that $\frac{6}{5} \leq \alpha 2EC \leq \frac{3}{2}$ [1]. In [2], Carr and Ravi study $\alpha 2EC$, and conjecture that $\alpha 2EC = \frac{4}{3}$, however no examples are known for which the integrality gap ratio comes close to $\frac{4}{3}$. In [1], Alexander, Boyd and Elliott-Magwood also study $\alpha 2EC$ and make the following stronger conjecture based on their findings:

Conjecture 1.1 *[1] The integrality gap* α2EC *for* $2\mathrm{EC}^{LP}$ *is* $\frac{6}{5}$.

To investigate $\alpha 2EC$ further, a natural next step is to study $\alpha 2EC$ for some interesting class of cost functions. We investigate $\alpha 2EC$ for the set of cost functions optimized at a particular family of feasible solutions for $2EC^{\mathrm{LP}}$. A feasible solution x^* for $2EC^{\mathrm{LP}}$ is called a *half-integer solution* if $x^*_e \in \{0, \frac{1}{2}, 1\}$

for all $x^*_e \in E$, and it is called *degree-tight* if $\sum_{uv}(x^*_{uv} : u \in V) = 2$ for all $v \in V$. Finally, a degree-tight half-integer solution is called a *half-triangle solution* if the edges in the support graph G_{x^*} corresponding to $x^*_e = \frac{1}{2}$ (called *half-edges*) form disjoint 3-cycles (called *half-triangles*) joined by paths of edges of value 1 (called *1-paths*).

The half-triangle solutions are of interest for studies of $\alpha 2EC$ as there is evidence that $\frac{\text{OPT}(2EC)}{\text{OPT}(2EC^{\text{LP}})}$ is greatest for cost functions optimized at such solutions (see [1], [2]). For example, the largest such ratio known is asymptotically $\frac{6}{5}$ [1], and comes from the infinite family of half-triangle solutions shown in Figure 1, where the numbers shown are the edge costs, edges with cost 0 are not shown, and the "gadget" pattern is repeated k times. Also, in a computational study which found $\alpha 2EC$ exactly for all K_n up to $n = 10$ and all half-integer solutions up to $n = 14$, $\alpha 2EC$ was given by a half-triangle solution for all values of n [1].

The main result of this paper is to show that Conjecture 1.1 is true for any cost function optimized at half-triangles solutions. More specifically, we show that for any half-triangle solution x^* and any cost function $c \geq 0$, there exists a solution of *2EC* of cost at most $\frac{6}{5}cx^*$, which implies that $\alpha 2EC = \frac{6}{5}$ for any cost function optimized at half-triangle solutions (previously, $\frac{4}{3}$ was known [2]). Our methods are constructive and polynomial, so also provide a $\frac{6}{5}$-approximation for *2EC* for such cost functions.

A key idea used in our methods is that of convex combination. In the context of this paper, given a graph $G = (V, E)$, we say that a vector $y \in \mathbb{R}^E$ is a *convex combination* if there exist 2-edge connected spanning multi-subgraphs H_i with multipliers $\lambda_i \in \mathbb{R}_{\geq 0}, i = 1, 2, \ldots, j$ such that $y = \sum_{i=1}^{j} \lambda_i \chi^{E(H_i)}$ and $\sum_{i=1}^{j} \lambda_i = 1$. Here $\chi^{E(H_i)} \in \mathbb{R}^E$ is the *incidence vector* of subgraphs H_i (i.e. $\chi_e^{E(H_i)}$ is the number of copies of edge e in H_i). Our method is essentially an averaging argument, and can be described as follows: let x^* be any feasible solution of $2EC^{\text{LP}}$, and suppose we can show that kx^* is greater than or equal to a convex combination for some value k (in particular $k = \frac{6}{5}$). Then for any non-negative cost vector c we have $kcx^* \geq \sum_{i=1}^{j} \lambda_i c\chi^{E(H_i)}$. This implies that for at least one of the H_i, $c\chi^{E(H_i)} \leq kcx^*$. If c is optimized at x^* for $2EC^{\text{LP}}$, we have $\frac{\text{OPT}(2EC)}{\text{OPT}(2EC^{\text{LP}})} \leq k$.

2 Main Result

Given a graph $G = (V, E)$, we sometimes use $E(G)$ to denote E, and $V(G)$ to denote V. A graph G is called *cubic* if every vertex of G has degree three.

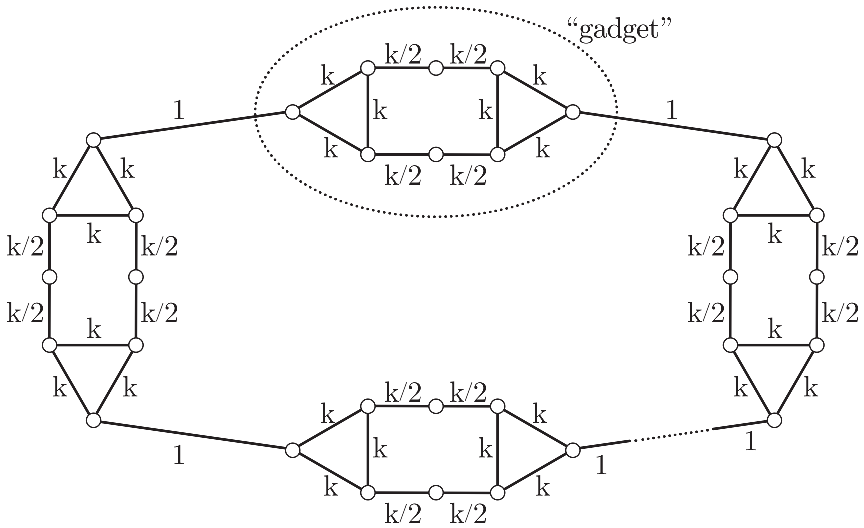

Fig. 1. An example for which $\alpha 2EC = \frac{6}{5}$ [1].

In this section, we prove our main result which is that $\frac{6}{5}x^*$ can be expressed as a convex combination for any half-triangle solution x^*. We do this by first considering the cubic graph we get by shrinking all half-triangles to pseudo-vertices and replacing all 1-paths by singles edges. We obtain a convex combination result for this cubic graph, then show how we can use this result and certain patterns for the half-triangle edges to obtain the result that $\frac{6}{5}x^*$ is a convex combination.

Definition 2.1 $P(G, s) \Leftrightarrow$ Given a cubic 3-edge connected graph $G = (V, E)$ and specified edge $s \in E$, the vector $y^* \in \mathbb{R}^E$ defined by

$$y_e^* = \begin{cases} \frac{4}{7} & \text{if } e = s, \\ \frac{6}{7} & \text{otherwise,} \end{cases}$$

is a convex combination in which none of the 2-edge connected spanning subgraphs use more than one copy of any edge in E.

Lemma 2.2 *$P(G, s)$ holds for all cubic 3-edge connected graphs $G = (V, E)$ with $|V| \geq 4$ and any edge $s \in E$.*

Proof. Suppose the contrary, and let $G = (V, E)$ be the smallest counterexample for which $P(G, s)$ does not hold for some $s \in E$. Since $P(G, s)$ can be shown directly to be true for all G with $|V| = 4$ (see Figure 2, where bold lines indicate edges in H_i and dotted lines indicate edges omitted), we can

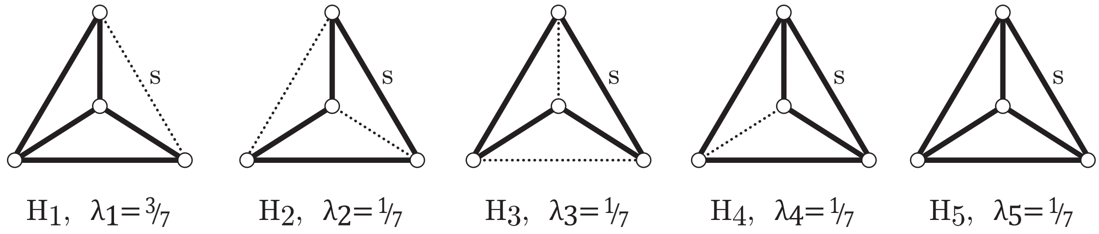

Fig. 2. Proof of Lemma 2.2 for $G = (V, E)$, when $|V| = 4$.

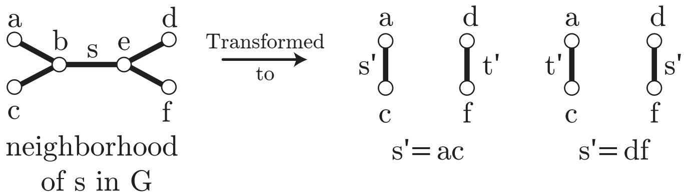

Fig. 3. Inductive step for Case 1.

assume $|V| > 4$.

Case 1 *G has no non-trivial 3-edge cut.*

Let the ends of edge s be b and e, and let the other 2 adjacent vertices at b be a and c, and the other 2 adjacent vertices at e be d and f. G is 3-edge connected, has no non-trivial 3-edge cut, and $|V| > 4$, which implies that a, c, d and f are all distinct. This situation is illustrated on the left of Figure 3, where some edges are not shown for vertices a, c, d and f. Removing b and e and their incident edges, and adding edges ac and df yield a new cubic 3-edge connected graph $G' = (V', E')$ with fewer vertices than G, so $P(G', s')$ holds for all $s' \in E'$. For reasons of symmetry, two cases are considered: one in which $s' = ac$, and one in which $s' = df$, as can be seen in Figure 3.

Details will now be provided for the case with $s' = ac$. Since $P(G', ac)$ holds, there exists a set of 2-edge connected spanning subgraphs H_i with multipliers λ_i, $i = 1, 2, \ldots, k$ such that $s' = ac$ occurs $\frac{4}{7}$ of the time overall in the convex combination, and $t' = df$ occurs $\frac{6}{7}$ of the time overall. There are four patterns possible for the occurrence of edges s' and t' in the subgraphs H_i, which we label λ_A, λ_B, λ_C and λ_D (see Figure 4, where an edge marked in bold indicates an edge which occurs in H_i). For each pattern Z, we let λ_Z represent the total occurrence of pattern Z over all H_i in the convex combination, i.e. $\lambda_Z = \sum(\lambda_i : \text{pattern } Z \text{ occurs in } H_i)$.

Using the fact that s' occurs exactly $\frac{4}{7}$ of the time, t' occurs exactly $\frac{6}{7}$ of

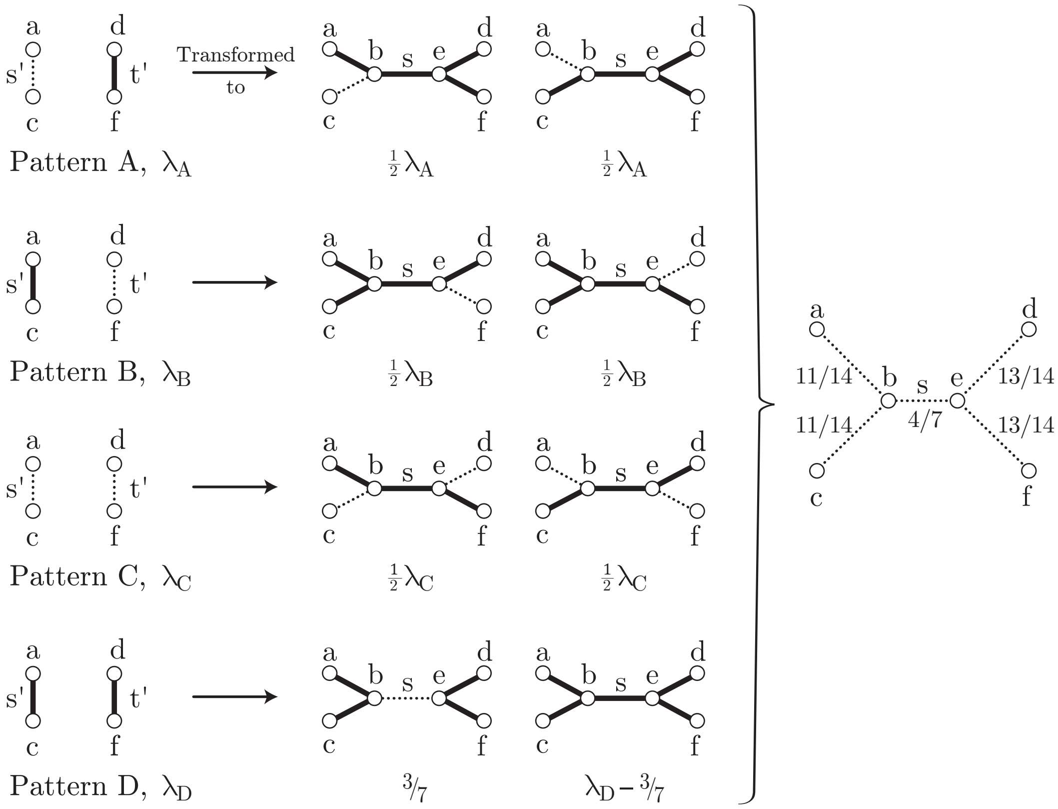

Fig. 4. Patterns for s' and t', and their transformations.

the time and $\lambda_A + \lambda_B + \lambda_C + \lambda_D = 1$, it follows that

$$\lambda_B + \lambda_D = \frac{4}{7},\ \lambda_A + \lambda_D = \frac{6}{7},\ \lambda_A + \lambda_C = \frac{3}{7} \text{ and } \lambda_B + \lambda_C = \frac{1}{7}. \tag{2}$$

Note that from (2) we can deduce that $\lambda_D \geq \frac{3}{7}$.

To create a convex combination of subgraphs for G, we create two 2-edge connected spanning subgraphs for each subgraph H_i, as shown in Figure 4, each with multiplier $\frac{\lambda_i}{2}$. Moreover, using (2) we have the occurence of edges ab and cb is $\frac{1}{2}(\lambda_A + \lambda_C) + \lambda_B + \lambda_D = \frac{11}{14}$, the occurence of edges ed and ef is $\lambda_A + \lambda_D + \frac{1}{2}(\lambda_B + \lambda_C) = \frac{13}{14}$, the occurence of s is $\frac{4}{7}$, and all the other edges occur $\frac{6}{7}$ of the time (illustrated on the right of Figure 4).

Taking the patterns for $s' = df$ yields the same fractions, but flipped. Therefore, taking a convex combination of the subgraphs for $s' = ac$ and $s' = df$ with a weight of $\frac{1}{2}\lambda_i$ on for each subgraph H_i results in a convex combination for y^* for G. Thus, $P(G, s)$ holds true, contradiction.

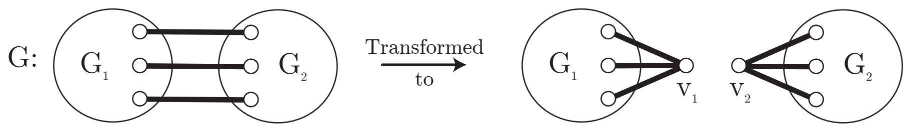

Fig. 5. Contracting both sides of a non-trivial 3-edge cut of G.

Case 2 *G has a non-trivial 3-edge cut.*

In this case we contract each side of the cut to a single vertex, to obtain graphs $G_1 = (V_1, E_1)$ with pseudonode v_1 and $G_2 = (V_2, E_2)$ with pseudonode v_2 (as shown in Figure 5). Both G_1 and G_2 are smaller than G, so $P(G_1, s_1)$ and $P(G_2, s_2)$ hold for any s_1, s_2, moreover the occurrences of the three edges incident to v_1 and v_2 are unique and identical in the subgraphs in the corresponding convex combinations. By appropriate choice of s_1 and s_2, this allows us to "glue" the subgraphs for G_1 and G_2 together to get a convex combination for y^* that shows $P(G, s)$ holds. More specifically, if edge s is in the 3-edge cut, we set $s_1 = s_2 = s$; otherwise, assuming that s is in G_1, we set $s_1 = s$ and s_2 to an arbitrary edge not incident to v_2.

We know $P(G_1, s_1)$ and $P(G_2, s_2)$ hold, moreover the occurrence of the three edges incident to v_1 and v_2 are unique and identical in the subgraphs in the corresponding convex combinations. This allows us to "glue" the subgraphs for G_1 and G_2 together to get a convex combination for G that shows $P(G, s)$ holds, with the possible exception of s_2, which can be added to subgraphs where it does not appear $\frac{2}{7}$ of the time, if necessary. This gives a contradiction. $\square$

We now use Lemma 2.2 to obtain our main result below. We call a graph $G = (V, E)$ a *half-triangle graph* if G is the support graph of a half-triangle solution x^*. If all 1-paths in G consist of a single edge, we call G *simple*.

Definition 2.3 $Q(G, p) \Leftrightarrow$ Given a simple half-triangle graph $G = (V, E)$ and a specified 1-edge $p \in E$, the vector $z^* \in \mathbb{R}^E$ defined by

$$z^*_e = \begin{cases} \frac{3}{5} & \text{if } e \text{ is a half-edge of } G, \\ \frac{4}{5} & \text{if } e = p, \\ \frac{6}{5} & \text{otherwise,} \end{cases}$$

is a convex combination in which none of the 2-edge connected spanning multi-subgraphs use more than one copy of a half-edge or the edge p, and all of them use either one or two copies of a 1-edge.

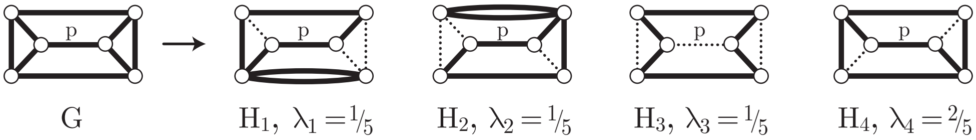

Fig. 6. Convex combination for $Q(G, p)$ when G has only two triangles.

Theorem 2.4 *$Q(G, p)$ holds for all simple half-triangle graphs $G = (V, E)$ and any 1-edge $p \in E$ not in a 2-edge cut in G.*

Proof.

Case 1 *G has no 2-edge cut.*

If G has only two half-triangles, then $Q(G, p)$ can be shown directly, using the H_i and λ_i shown in Figure 6, where edges represented by dotted lines are omitted, and H_1 and H_2 contain a multi-edge. Otherwise, let $G' = (V', E')$ be the graph obtained from G by shrinking the half-triangles to pseudo-vertices. Let $p = be$ in the new graph G', and let the other edges incident with b and e be ab, bc, ed, and ef. G' is cubic and 3-edge connected and has $|V'| \geq 4$, therefore by Lemma 2.2, $P(G', bc)$, $P(G', ab)$, $P(G', ed)$, $P(G', ef)$ and $P(G', p)$ all hold. Taking a convex combination of all of the resulting subgraphs with a multiplier $\lambda'_i = \frac{1}{5}\lambda_i$ for each subgraph H_i results in an edge occurrence of $\frac{4}{5}$ for edges bc, ab, ed, ef and p. The other edges occur exactly $\frac{6}{7}$ of the time.

The half-triangles (previously contracted to pseudo-vertices) will now be expanded to conclude the proof. Consider any triangle T and let its incident edges be x, y and z. In the new convex combination created for G', we have all three of these edges, or just two of these edges occur in each subgraph H_i. Let

$$\begin{aligned}
\lambda'_{xyz} &= \sum(\lambda'_i : \{x, y, z\} \in H_i), \\
\lambda'_{xy} &= \sum(\lambda'_i : \{x, y\} \in H_i), \\
\lambda'_{yz} &= \sum(\lambda'_i : \{y, z\} \in H_i) \text{ and} \\
\lambda'_{xz} &= \sum(\lambda'_i : \{x, z\} \in H_i).
\end{aligned}$$

Note that

$$\lambda'_{xyz} + \lambda'_{xy} + \lambda'_{yz} + \lambda'_{xz} = 1. \tag{3}$$

First we consider the case where the expanded triangle T is not incident with edge p. For each subgraph H_i in which all three edges x, y and z occur, we include two of the three edges in T $\frac{1}{3}\lambda'_{xyz}$ of the time. These patterns are

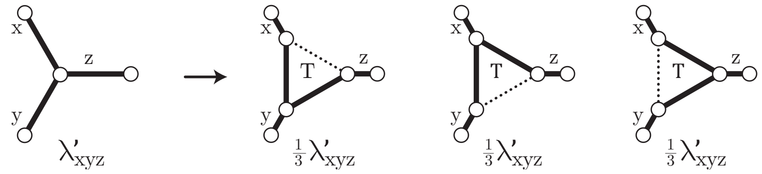

Fig. 7. Patterns used for triangle expansion for subgraphs containing x, y and z.

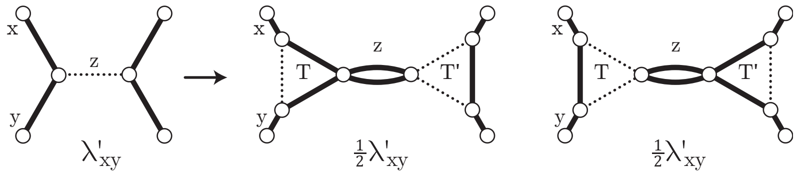

Fig. 8. Patterns used for triangle expansion for an omitted edge z.

illustrated in Figure 7, and result in an occurence of $\frac{2}{3}\lambda'_{xyz}$ for each edge of T. Then for each subgraph H_i in which z is omitted and x and y occur, we consider both triangle T and the other triangle T' incident with z. In this case we include the edges in T incident with z $\frac{1}{2}\lambda'_{xy}$ of the time, and the other edge in T $\frac{1}{2}\lambda'_{xy}$ of the time, and do the opposite in triangle T'. In all cases we also include two copies of edge z. The patterns are illustrated in Figure 8 and result in an occurence of $\frac{1}{2}\lambda'_{xy}$ for each edge in T. We do the same for the cases where x of y are omitted in H_i. The total occurence of each half-edge in T is

$$\frac{2}{3}\lambda'_{xyz} + \frac{1}{2}\lambda'_{xy} + \frac{1}{2}\lambda'_{yz} + \frac{1}{2}\lambda'_{xz},$$

which by (3) is

$$\frac{1}{2} + \frac{1}{6}\lambda'_{xyz}. \tag{4}$$

Since each of the 1-edges x, y and z occur a total of either $\frac{6}{7}$ of $\frac{4}{5}$ of the time in the subgraph H_i, it follows that λ'_{xy}, λ'_{yz} and λ'_{xz} are each $\frac{1}{5}$ or $\frac{1}{7}$. Moreover, at most two of x, y and z are $\frac{1}{5}$. Hence, by (3) we have that

$$\lambda'_{xyz} \leq 1 - \frac{3}{7} = \frac{4}{7}\ ,$$

so by (4) each half-edge in T occurs at most $\frac{25}{42} \leq \frac{3}{5}$ of the time. (For a complete illustration of the case $\lambda'_{xy} = \lambda'_{yz} = \lambda'_{xz} = \frac{1}{7}$, see Figure 9).

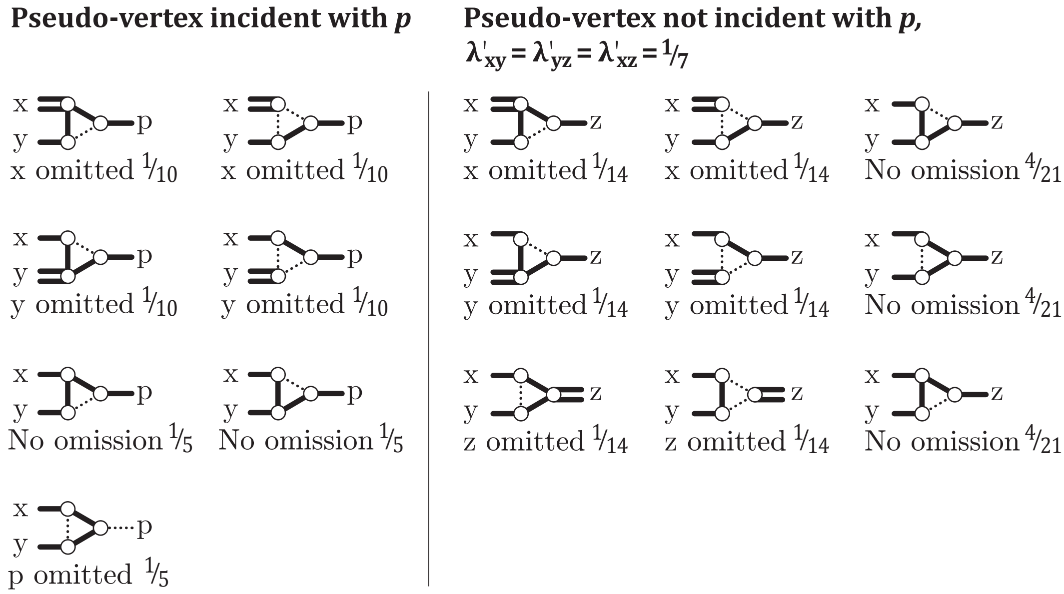

Fig. 9. Examples of edge selection upon expanding the pseudo-vertices of G'.

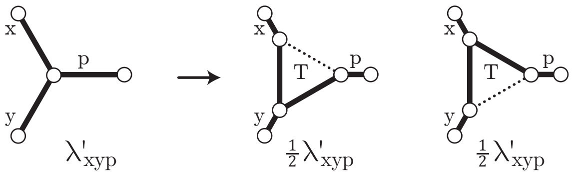

Fig. 10. Patterns used for triangle expansion for subgraphs containing x, y and p.

Each 1-edge which is not p is now doubled whenever it was previously omitted, and thus occurs $\frac{6}{5}$ or $\frac{8}{7} \leq \frac{6}{5}$ of the time. Note also that all patterns used in the expansion of the half-triangles ensure that the new multi-subgraphs created from the subgraphs H_i for G' are also 2-edge connected and spanning in G, as required.

Next we consider the case where triangle T is incident with edge p, and let $p = z$. For each subgraph H_i in which all three edges x, y and p occur, we include two of the three edges in T $\frac{1}{2}\lambda'_{xyp} = \frac{1}{5}$ of the time, using the two patterns illustrated in Figure 10. Then for each subgraph H_i in which p is omitted and x and y occur we include the two edges of T incident with p, and this occurs $\lambda'_{xy} = \frac{1}{5}$ of the time. Note that we do not double edge p. The total occurrence of each edge of T is exactly $\frac{3}{5}$, and p occurs exactly $\frac{4}{5}$ of the time (see Figure 9 for a complete illustration).

We now have, over all cases, the half-edge occurrence is $\leq \frac{3}{5}$, p occurs $\frac{4}{5}$

of the time, and the occurrence of the other 1-edges is $\leq \frac{6}{5}$. Where necessary, we can add back edges arbitrarily where they do not occur to obtain $Q(G, p)$.

Case 2 *G has a 2-edge cut $C = \{hi, jk\}$.*

Suppose the contrary, and let G be the smallest counter-example for which $Q(G, p)$ does not hold. Let G_1, G_2 be the two sides of the cut C in G, with h and j in G_1 and i and k in G_2, and WLOG choose C such that $G_1 + hj$ is 3-edge connected and does not contain p. By smaller example and Case 1, $Q(G_1 + hj, hj)$ and $Q(G_2 + ik, p)$ hold. We now "glue" together in the obvious way, the subgraphs in the convex combination for $G_1 + hj$ where hj is omitted with the subgraphs in the convex combination for $G_2 + ik$ which have ik doubled (both patterns occur $\frac{1}{5}$ of the time) by removing the double edge ik and adding two copies of edges hi and jk. Similarly, we glue the subgraphs for $G_1 + hj$ and $G_2 + ik$ where hj and ik occur as single edges in the subgraphs (both patterns occur $\frac{4}{5}$ of the time) by removing hi and ik and adding edges hi and jk. We obtain $Q(G, p)$, contradiction. □

By replacing 1-edges by 1-paths in the convex combinations for $Q(G, p)$, and doubling the path for p wherever p was omitted, we can obtain $\frac{6}{5}x^*$ as a convex combination for any half-triangle solution x^*, i.e. there exist 2-edge connected spanning multi-subgraphs H_i with multipliers $\lambda_i \in \mathbb{R}_{\geq 0}$, $i = 1, 2, \ldots, j$ such that $\sum_{i=1}^{j} \lambda_i = 1$ and

$$\frac{6}{5}x^* = \sum_{i=1}^{j} \lambda_i \chi^{E(H_i)}. \tag{5}$$

Now consider any non-negative cost vector $c \in \mathbb{R}^E$ which is optimized at x^* for $\mathit{2EC}^{\mathrm{LP}}$, i.e. $cx^* = \mathrm{OPT}(\mathit{2EC}^{\mathrm{LP}})$. By multiplying both sides of (5) by c, we obtain

$$\frac{6}{5}\mathrm{OPT}(\mathit{2EC}^{\mathrm{LP}}) = \sum_{i=1}^{j} \lambda_i c \chi^{E(H_i)}$$

and thus, for at least one subgraph H_i in the convex combination,

$$c\chi^{E(H_i)} \leq \frac{6}{5}\mathrm{OPT}(\mathit{2EC}^{\mathrm{LP}}). \tag{6}$$

Since $\mathrm{OPT}(\mathit{2EC}) \leq c\chi^{E(H_i)}$, it follows that $\frac{\mathrm{OPT}(\mathit{2EC})}{\mathrm{OPT}(\mathit{2EC}^{\mathrm{LP}})} \leq \frac{6}{5}$ for such cost functions. As there exist examples of half-triangle solutions which show $\alpha \mathit{2EC} \geq \frac{6}{5}$ [1], we obtain the following corollary to Theorem 2.4.

Corollary 2.5 *The integrality gap* $\alpha 2\mathrm{EC} = \frac{6}{5}$ *when restricted to cost functions optimized at half-triangle solutions.*

Note that $\mathrm{OPT}(\mathit{2EC}^{\mathrm{LP}}) \leq \mathrm{OPT}(\mathit{2EC})$, thus by (6), we have obtained a 2-edge connected multi-subgraph H with cost at most $\frac{6}{5}\mathrm{OPT}(\mathit{2EC})$. As our proofs are constructive and the steps taken to obtain H can be performed in polynomial time, we also obtain a $\frac{6}{5}$-approximation algorithm for *2EC* for any cost functions optimized at a half-triangle solution of $\mathit{2EC}^{\mathrm{LP}}$.

References

[1] Alexander, A., Boyd, S., Elliott-Magwood, P., On the Integrality Gap of the 2-Edge Connected Subgraph Problem, *Technical Report TR-2006-04*, SITE, University of Ottawa, Ottawa, Canada.

[2] Carr R., Ravi R., A new bound for the 2-edge connected subgraph problem, *Proceedings of Integer Programming and Combinatorial Optimization (IPCO)*, Lecture Notes in Computer Science, Springer, 112-125, 1998.

[3] Csaba, B., Karpinski, M., Krysta, P., Approximability of dense and sparse instances of minimum 2-connectivity, tsp and path problems, *D. Eppstein (Ed.), SODA*, ACM/SIAM, 74-83, 2002.

[4] Frederickson, G. N., Ja'Ja', J. On the relationship between the biconnectivity augmentation and travelling salesman problems, *Theoretical Computer Science*, 19, 189-201, 1982.

[5] Jain, K., A factor 2 approximation algorithm for the Generalized Steiner Network Problem, *Combinatorica 21 No 1*, 39-60, 2001.