Performance Analysis of Coded V-BLAST with Optimum Power and Rate Allocation

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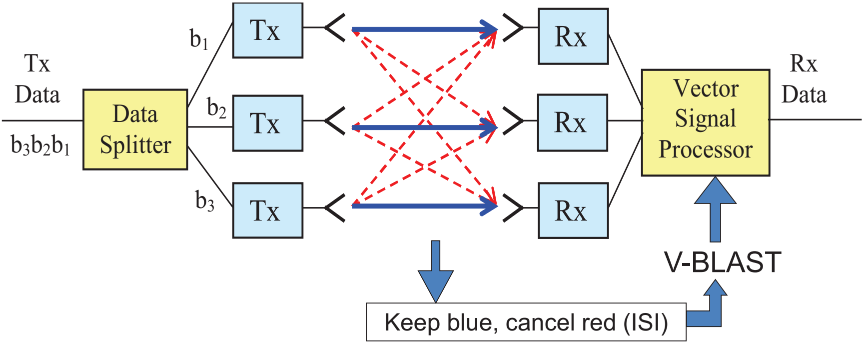
Motivation

- Why V-BLAST?
  - MIMO: high spectral efficiency
  - V-BLAST is a practical (not too complex) approach
  - MMSE BLAST achieves full MIMO capacity
  - SIC implements the chain rule of MI
- V-BLAST challenges
  - Error propagation effect degrades performance
  - Ordering: high computational complexity
- V-BLAST improvements
  - Optimum power/rate allocation
- Coded V-BLAST
  - Most of prior work: uncoded BLAST
  - Uncoded systems are rare; powerful (capacity-approaching) codes exist
- Performance analysis: challenging but insights

V-BLAST: launch multiple bit streams



Interference cancellation: cancel out the interference from already detected symbols.

Interference nulling (ZF): project out the interference from yet to be detected symbols.

Optimal ordering procedure: symbol with highest after-processing SNR is detected first - *excluded*

System Model

- Generic multi-stream transmission or ZF V-BLAST (no ordering)
- Instantaneous power and/or rate allocation
- Capacity-achieving temporal codes
- Channel model

$$\mathbf{r} = \mathbf{H}\mathbf{\Lambda}\mathbf{s} + \boldsymbol{\xi} = \sum_{i=1}^{m} \mathbf{h}_i \sqrt{\alpha_i} s_i + \boldsymbol{\xi}$$

- System capacity: capacity of the extended channel (with the V-BLAST transmission/processing architecture)
- Outage probability: probability that the system cannot support a target rate mR,

$$P_{out} = \mathbb{P}\left[C < mR\right]$$

Instantaneous vs. Average Optimization

- Total power constraint: $\sum_{i=1}^{m} \alpha_i = m$
- Target rate: mR

Power/rate allocation strategies:

$$
\begin{aligned}
\overline{\boldsymbol{\alpha}}_C &= \arg\max_{\boldsymbol{\alpha}(\gamma_0)} \mathbb{E}\left[C(\boldsymbol{\alpha})\right] \\
\overline{\boldsymbol{\alpha}}_{out} &= \arg\min_{\boldsymbol{\alpha}(\gamma_0)} \mathbb{P}\left[C(\boldsymbol{\alpha}) < mR\right] \\
\boldsymbol{\alpha}_C &= \arg\max_{\boldsymbol{\alpha}(\gamma_0,\mathbf{H})} C(\boldsymbol{\alpha}) \\
\boldsymbol{\alpha}_{out} &= \arg\min_{\boldsymbol{\alpha}(\gamma_0,\mathbf{H})} \mathbb{P}\left[C(\boldsymbol{\alpha}) < mR\right]
\end{aligned}
$$

Theorem (Instantaneous vs. Average Optimization)

$$
\begin{aligned}
\mathbb{P}\left[C(\overline{\boldsymbol{\alpha}}_C) < mR\right] &\geq \mathbb{P}\left[C(\overline{\boldsymbol{\alpha}}_{out}) < mR\right] \\
&\geq \mathbb{P}\left[C(\boldsymbol{\alpha}_{out}) < mR\right] = P_{out}^{\star} \\
&= \mathbb{P}\left[C(\boldsymbol{\alpha}_C) < mR\right]
\end{aligned}
$$

Unoptimized system

- All $\alpha_i = 1$ (uniform power allocation)
- All per-stream target rates $= R$ (uniform rate allocation)

$$C_u = m \min_i C_i, \quad C_i = \ln(1 + \alpha_i g_i \gamma_0)$$

$$P_{out}^u = \mathbb{P}\left[C_u < mR\right] = 1 - \prod_{i=1}^{m} (1 - \mathbb{P}\left[C_i < R\right])$$

- $g_i = |\mathbf{h}_{i\perp}|^2$ is the $i-$th stream power gain [1].

[1] see e.g. S. Loyka, F. Gagnon, V-BLAST without Optimal Ordering: Analytical Performance Evaluation for Rayleigh Fading Channels, IEEE Trans. Comm., June 2006.

Instantaneous Rate Allocation (IRA)

- All $\alpha_i = 1$
- Per-stream rates are adjusted to match the per-stream capacities C_i

$$C_{IRA} = \sum_{i=1}^{m} C_i, \quad C_i = \ln(1 + g_i \gamma_0)$$

$$P_{out}^{IRA} = \mathbb{P}\left[\sum_i C_i < mR\right]$$

Instantaneous Power Allocation (IPA) [2]

$$C_{IPA} = m \max_{\boldsymbol{\alpha}} \min_i \ln(1 + \alpha_i g_i \gamma_0), \text{ s.t.} \sum_i \alpha_i = m, \ \alpha_i \geq 0$$

Theorem (IPA)

$$C_{IPA} = \begin{cases} m \ln(1 + \overline{g}\gamma_0), & \textit{all } g_i > 0 \\ 0, & \textit{otherwise} \end{cases}$$

where $\overline{g}$ is the harmonic mean per-stream gain,

$$\overline{g} = \left(\frac{1}{m} \sum_i g_i^{-1} \right)^{-1}$$

C_{IPA} *is achieved by the channel inversion:* $\alpha_i = \overline{g}/g_i$

[2] V. Kostina, S. Loyka, Optimum Power and Rate Allocation for Coded V-BLAST: Instantaneous Optimization, IEEE Trans. Comm., accepted, 2011.

Instantaneous Power and Rate Allocation (IPRA)

$$C_{IPRA} = \max_{\boldsymbol{\alpha}} \sum_i \ln(1 + \alpha_i g_i \gamma_0), \text{ s.t.} \sum_i \alpha_i = m, \ \alpha_i \geq 0$$

- Power allocation subject to $\sum_{i=1}^{m} \alpha_i = m$
- Per-stream rates match per-stream capacities $\ln(1 + \alpha_i g_i \gamma_0)$
- Conventional waterfilling (WF): not optimal in V-BLAST!

Fractional waterfilling

- Fractional waterfilling (FWF) = conventional WF on all subsets of streams:
  1. Begin
  2. Select a set of active Tx antennas (streams)
  3. Do WF
  4. Go to 2 until all combinations are done
  5. Select the best active set
  6. End

Performance analysis: Capacities

Proposition (Capacity bounds)

$$\begin{aligned}
\ln\left(1 + m\gamma_0 g_{max}\right) \leq C_{FWF} &\leq m \ln\left(1 + \gamma_0 g_{max}\right) \\
\ln\left(1 + m\gamma_0 g_{max\perp}\right) \leq C_{WF} &\leq m \ln\left(1 + \gamma_0 g_{max\perp}\right) \\
\ln\left(1 + \gamma_0 g_{max\perp}\right) \leq C_{IRA} &\leq m \ln\left(1 + \gamma_0 g_{max\perp}\right) \\
C_u &= m \ln\left(1 + \gamma_0 g_{min\perp}\right)
\end{aligned}$$

Proposition (Instantaneous capacities)

$$\begin{aligned}
C_u \leq C_{IRA} &\leq C_{WF} \leq C_{FWF} \\
C_u(\gamma_0) \leq C_{IPA}(\gamma_0) &\leq C_u(m\gamma_0) \\
C_{IRA}(\gamma_0) \leq C_{WF}(\gamma_0) &\leq C_{IRA}(m\gamma_0)
\end{aligned}$$

Performance analysis: Outage probabilities

Proposition (Outage probabilities, any fading)

$$P_{out}^{FWF} \leq P_{out}^{WF} \leq P_{out}^{IRA} \leq P_{out}^{u}$$
$$P_{out}^{u}(m\gamma_0) \leq P_{out}^{IPA}(\gamma_0) \leq P_{out}^{u}(\gamma_0)$$
$$P_{out}^{IRA}(m\gamma_0) \leq P_{out}^{WF}(\gamma_0) \leq P_{out}^{IRA}(\gamma_0)$$

Performance analysis: Diversity gains

Diversity gain d:

$$P_{out} \approx \frac{c}{\gamma_0^d} \text{ or } d = -\lim_{\gamma_0 \to \infty} \frac{\ln P_{out}}{\ln \gamma_0}.$$

Proposition (Diversity gains)

In the low outage regime,

$$\begin{aligned} d_u &= n - m + 1 = d_{IPA} \\ &\leq d_{IRA} = \sum_{i=1}^{m} (n - m + i) = d_{WF} \\ &\leq d_{FWF} = nm \end{aligned}$$

The equality is achieved for $m = 1$ only, i.e. only the FWF achieves the full MIMO channel diversity nm for $m > 1$.

Example (high rate)

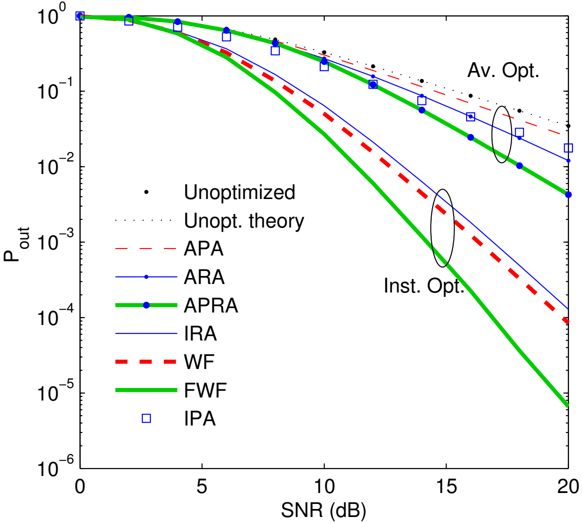


Figure: 2×2 V-BLAST in i.i.d. Rayleigh fading channel at $R = 3$ [nat/s/Hz]

Theorem: Outage probabilities, wideband [3]

$$P_{out}^{u} = 1 - \prod_{i=1}^{m} \left(1 - F_{n-m+i}\left(x\right)\right) \approx \frac{x^{n-m+1}}{(n-m+1)!},$$

$$P_{out}^{IRA} \approx F_{d_{IRA}}\left(mx\right) \approx \frac{1}{d_{IRA}!}\left(mx\right)^{d_{IRA}},$$

$$P_{out}^{WF} \approx \prod_{i=1}^{m} F_{n-m+i}\left(x\right) \approx \frac{x^{d_{IRA}}}{\prod_{i=1}^{m}(n-m+i)!}$$

$$P_{out}^{FWF} \approx F_{n}^{m}\left(x\right) \approx \frac{x^{nm}}{(n!)^{m}}$$

where the second approximation in each case holds at the low outage regime, $x = R/\gamma_0 \ll 1$. $F_k(x) = 1 - e^{-x}\sum_{l=0}^{k-1} x^l/l!$ is the outage probability of k-th order MRC.

[3] to the best of our knowledge, it is the first time when the WF/FWF outage probability is found in a closed form.

Example (wideband)

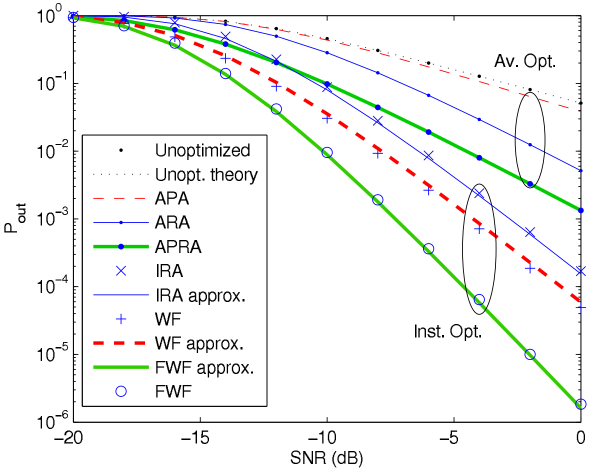


Figure: 2×2 V-BLAST in i.i.d. Rayleigh fading channel at $R = 0.1$ [nat/s/Hz]

Instantaneous capacities, low SNR

In the low SNR regime, $m\gamma_0 \max_i |\mathbf{h}_i|^2 \ll 1$,

$$C_u \approx m\gamma_0 \min_i |\mathbf{h}_{i\perp}|^2$$

$$C_{IRA} \approx \gamma_0 \sum_{i=1}^{m} |\mathbf{h}_{i\perp}|^2$$

$$C_{WF} \approx m\gamma_0 \max_i |\mathbf{h}_{i\perp}|^2$$

$$C_{FWF} \approx m\gamma_0 \max_i |\mathbf{h}_i|^2$$

Corollary

FWF significantly outperforms the WF, $C_{WF} \ll C_{FWF}$, when $\max_i |\mathbf{h}_{i\perp}| \ll \max_i |\mathbf{h}_i|$ and their performance is close otherwise.

Example (fixed channel)

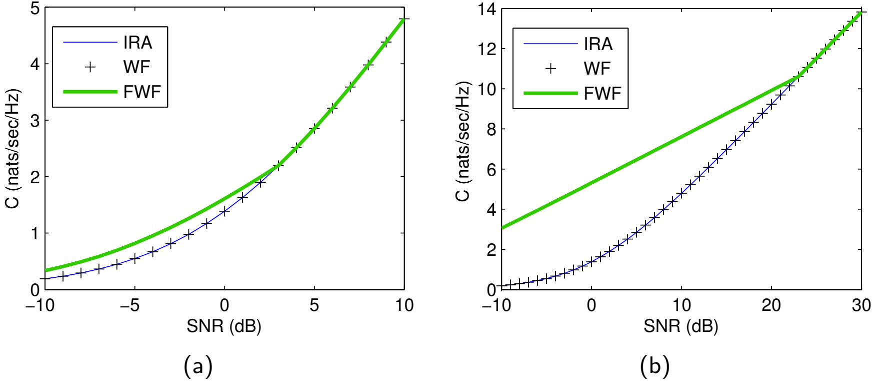


Figure: 2×2 V-BLAST with the FWF and the conventional WF for two channel realizations:
(a) "good": $\mathbf{h}_1 = [1\ 1]^T$, $\mathbf{h}_2 = [0\ 1]^T$
(b) "bad": $\mathbf{h}_1 = [1\ 10]^T$, $\mathbf{h}_2 = [0\ 1]^T$

Conclusion

- Optimum power/rate allocation for coded V-BLAST
- IPA: within a bounded SNR gain ($\leq m$) of U
- IRA: extra diversity gain
- WF: within a bounded SNR gain ($\leq m$) of IRA
- Conventional WF is not optimal in V-BLAST!
- Fractional waterfilling algorithm (FWF)
    - maximizes capacity/minimizes the outage probability
    - significantly outperforms the other strategies (achieves the full channel diversity)
- Closed-form solutions + performance analysis
- Also good for generic multi-stream transmission (e.g. OFDM, MAC, SIC equalizers)