On Node Density – Outage Probability Tradeoff in Wireless Networks

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Abstract— A statistical model of interference in wireless networks is considered, which is based on the traditional propagation channel model, a Poisson model of random spatial distribution of the nodes in 1-D, 2-D and 3-D spaces (with both uniform and non-uniform densities), and a threshold-based model of the receiver performance. The power of the dominant interferer is used as a major performance indicator, instead of a traditionally-used aggregate interference power, since the former is an accurate approximation of the latter. This simplifies the problem significantly so that compact closed-form expressions are obtained for the outage probability, including the case when a given number of strongest interferers are suppressed: the outage probability is shown to scale down exponentially in this number. The effect of Rayleigh and log-normal fading can also be included in the analysis. The positive effect of linear filtering (e.g. by directional antennas) is quantified via a new statistical selectivity parameter. The analysis culminates in formulation of an explicit tradeoff relationship between the network density and the outage probability, which is a result of the interplay between random geometry of node locations, the propagation path loss and the distortion effects at the victim receiver.

I. INTRODUCTION

Wireless communication networks have been recently a subject of extensive studies, both from information-theoretic and communication perspectives, including development of practical transmission strategies and fundamental limits (capacity) to assess the optimality of these strategies [1].

Mutual interference among several links (e.g. several users) operating at the same time places a fundamental limit to the network performance. The effect of interference in wireless networks at the physical layer has been studied from several perspectives [2]-[6]. A typical statistical model of interference in a network includes a model of spatial location of the nodes, a propagation path loss law (which includes the average path loss and, possibly, large-scale (shadowing) and small-scale (multipath) fading) and a threshold-based receiver performance model. The most popular choice for the model of the node spatial distribution is Poisson point process on a plane [2]-[6]. Based on this model and ignoring the effect of fading, Sousa [3] has obtained the characteristic function (CF) of the aggregate (total) interference at the receiver, which can be transformed into a closed-from probability density function (PDF) in some special cases, and, based on it, the error rates for direct sequence (DS) and frequency hopping (FH) systems. While using the LePage series representation, Ilow and Hatzinakos [4] have developed a generic technique to obtain the CF of aggregate interference from a Poisson point process on a plane (2-D) and in a volume (3-D), which can be used to incorporate the effects of Rayleigh and log-normal fading in a straightforward way. Relying on a homogeneous Poisson point process on a plane, Weber et al [5] have characterized the transmission capacity of the network subject to the outage probability constraint via lower and upper bounds. In a recent work, Weber et al [6] use the same approach to characterize the network transmission capacity when the receivers are able to suppress some powerful interferers.

A common feature of all these works is the use of aggregate interference (either alone or in the form of signal-to-interference-plus-noise ratio), and a common lesson is that it is very difficult to deal with: while the CF of aggregate interference can be obtained in a closed form, the PDF or CDF are available only in a few special cases. This limits significantly the amount of insight that can be extracted from such a model, especially if no approximations or bounds are used.

To overcome this difficulty, we adopt a different approach: instead of relying on the aggregate interference power as a performance indicator, we use the power of the dominating interfering signal [8]-[10]. While this is clearly an approximation, closed-form performance evaluation becomes feasible and significant insight can be extracted from such a model. Furthermore, since the aggregate interference is dominated by the most powerful interferer in the region of low outage probability (i.e. the practically-important region), both models give roughly the same results. This observation is also consistent with the corresponding results in [5][6], when the "near-field" region contains only one interferer. Thus, in the framework of [5][6], our results represent the (tight) lower bound on the outage probability.

Using this model, we study the power distribution of the dominant interferer in various scenarios, which is further used to obtain compact closed-form expressions for the outage probability of a given receiver (or, equivalently, of the link of a given user) in the 1-D, 2-D and 3-D Poisson field of interferers, for both uniform and non-uniform average node densities and for various values of the average path loss exponent. Comparison to the corresponding results in [3] (obtained in terms of the error rates) indicates that the dominant contribution to the error rate is due to the outage events caused by the closest (i.e. dominant) interferer, which increases with the average node density. The proposed method is flexible enough to include the case when a given number of strongest interferers are suppressed. The outage probability is shown to scale down exponentially in this number. Contrary to [6], we do not rely in this case on the simplifying assumption of canceling *all* interferers in the disk with the given average number of interferers; neither we assume that only interferers more powerful than the required signal are cancelled (the last assumption affects significantly the result), i.e. our analysis of interference cancellation is exact. The proposed method can also be used to include the effect of fading. Specifically, using the same technique as in [4] (developed for the aggregate interference), we argue that Rayleigh fading has a negligible effect on the distribution of dominant interferer's power and the effect of log-normal fading (shadowing) is to shift the distribution by a constant non-negligible factor.

Our analysis culminates in the formulation of the outage probability-network density tradeoff: for a given average density of the nodes, the outage probability is lower bounded or, equivalently, for a given outage probability, the average density of the nodes is upper bounded. This tradeoff is a result of the interplay between a random geometry of node locations, the propagation path loss and the distortion effects at the victim receiver.

Using the method developed, we analyze the beneficial effect of linear filtering (e.g. by directional antennas, which attenuate some interferers based on their angles of arrival) on the outage probability and on the tradeoff via a new statistical selectivity parameter (Q-parameter), which is somewhat similar to the traditional antenna gain, but also includes the statistical distribution of interferers over the filtering variables (e.g. angles of arrival).

Our analysis is based on the framework originally developed in [8]-[10].

The paper is organized as follows. In Section II, we introduce the system and network model. In Section III, the distribution of the dominant interference-to-noise ratio is given for this model. Based on this, the node density – outage probability tradeoff is presented in Section IV. The impact of linear filtering is analyzed in Section V.

II. Network and System Model

As an interference model of wireless network at the physical layer, we consider a number of point-like transmitters (Tx) and receivers (Rx) that are randomly located over a certain limited region of space S_m, which can be one ($m=1$), two ($m=2$),or three ($m=3$) -dimensional (1-D, 2-D or 3-D). This can model location of the nodes over a highway or a street canyon (1-D), a residential area (2-D), or a downtown area with a number of high-rise buildings (3-D). In our analysis, we consider a single (randomly-chosen) receiver and a number of transmitters that generate interference to this receiver. We assume that the spatial distribution of the transmitters (nodes) has the following properties: (i) for any two non-overlapping regions of space S_a and S_b, the probability of any number of transmitters falling into S_a is independent of how many transmitters fall into S_b, i.e. non-overlapping regions of space are statistically independent; (ii) for infinitesimally small region of space dS, the probability $\mathcal{P}(k=1,dS)$ of a single transmitter ($k=1$) falling into dS is $\mathcal{P}(k=1,dS)=\rho dS$, where ρ is the average spatial density of transmitters (which can be a function of position). The probability of more than one transmitter falling into dS is negligible, $\mathcal{P}(k>1,dS) << \mathcal{P}(k=1,dS)$ as $dS \to 0$. Under these assumptions, the probability of exactly k transmitters falling into the region S is given by Poisson distribution,

$$\mathcal{P}(k,S)=e^{-\bar{N}}\bar{N}^k/k! \quad (1)$$

where $\bar{N}=\int_S \rho dS$ is the average number of transmitters falling into the region S. If the density is constant, then $\bar{N}=\rho S$. Possible scenarios to which the assumptions above apply, with a certain degree of approximation, are a sensor network with randomly-located non-cooperating sensors; a network(s) of mobile phones from the same or different providers (in the same area); a network of multi-standard wireless devices sharing the same resources (e.g. common or adjacent bands of frequencies) or an ad-hoc network.

Consider now a given transmitter-receiver pair. The power at the Rx antenna output P_r coming from the transmitter is given by the standard link budget equation [7],

$$P_r = P_t G_t G_r g \quad (2)$$

where P_t is the Tx power, G_t, G_r are the Tx and Rx antenna gains, and g is the propagation path gain (=1/path loss), $g=g_a g_l g_s$, where g_a is the average propagation path gain, and g_l, g_s are the contributions of large-scale (shadowing) and small-scale (multipath) fading, which can be modeled as independent log-normal and Rayleigh (Rice) random variables, respectively [7].

The widely-accepted model for g_a is $g_a = a_\nu R^{-\nu}$, where ν is the path loss exponent, and a_ν is constant independent of R [7]. In the traditional link-budget analysis of a point-to-point link, it is a deterministic constant. However, in our network-level model g_a becomes a random variable since the Tx-Rx distance R is random (due to random location of the nodes) and it is this random variable that represents a new type of fading, which we term "network-scale fading", since it exhibits itself on the scale of the whole area occupied by the network. Since g_a does not depend on the local propagation environment around the Tx or Rx ends that affect g_l, g_s but only on the global configuration of the Tx-Rx propagation path (including the distance R, of which g_l, g_s are independent) [7], the network-scale fading in this model is independent of the large-scale and small-scale ones, which is ultimately due to different physical mechanisms generating them. The distribution functions of g_a in various scenarios have been given in [9][10].

III. Interference to Noise Ratio

We consider a fixed-position receiver (e.g. a base station of a given user) and a number of randomly located interfering transmitters (interferers, e.g. mobile units of other users) of the same power P_t [1]. Only the network-scale fading is taken into account in this section, assuming that $g_l = g_s = 1$ (this

[1] following the framework in [8]-[10], this can also be generalized to the case of unequal Tx powers.

assumption is relaxed in section IV). For simplicity, we also assume that the Tx and Rx antennas are isotropic (this assumption is relaxed in section V), and consider the interfering signals at the receiver input.

The statistics of transmitters' location is given by (1). Transmitter i produces the average power $P_{ai} = P_t g_a(R_i)$ at the receiver input, and we consider only the signals exceeding the Rx noise level P_0, $P_{ai} \geq P_0$. We define the interference-to-noise ratio (INR) d_a in the ensemble of the interfering signals via the most powerful (at the Rx input) signal[2],

$$d_a = P_{a1} / P_0 \tag{3}$$

where, without loss of generality, we index the transmitters in the order of decreasing Rx power, $P_{a1} \geq P_{a2} \geq ... \geq P_{aN}$. The most powerful signal is coming from the transmitter located at the minimum distance r_1, $P_{a1} = P_t g_a(r_1)$. The cumulative distribution function (CDF) of the minimum distance can be easily found [8]-[10],

$$F_1(r) = 1 - \exp\left(-\bar{N}\left(V\right)\right) \tag{4}$$

where $\bar{N}(V) = \int_V \rho dV$ is the average number of transmitters in the ball $V(r)$ of radius r. The corresponding PDF can be found by differentiation,

$$f_1(r) = e^{-\bar{N}} \int_{V'(r)} \rho dV \tag{5}$$

where $V'(r)$ is sphere of radius r and the integral in (5) is over this sphere.

The probability that the INR exceeds value D is $\Pr\{d_a > D\} = \Pr\{r_1 < r(D)\} = F_1(r(D))$, where $r(D)$ is such that $P_a(r(D)) = P_0 D$, so that the CDF of d_a is

$$F_d(D) = 1 - \Pr\{d_a > D\} = \exp(-\bar{N}(D)) \tag{6}$$

where $\bar{N}(D) = \int_{V(r(D))} \rho dV$ is the average number of transmitters in the ball $V\left(r(D)\right)$ of the radius $r(D) = (P_t a_\nu / P_0 D)^{1/\nu}$. The corresponding PDF can be obtained by differentiation,

$$f_d(D) = \frac{r(D) e^{-\bar{N}(D)}}{\nu D} \int_{V'(r(D))} \rho dV \tag{7}$$

When the average spatial density of transmitters is constant, $\rho = const$, (6), (7) simplify to [8]-[10],

$$F_d(D) = \exp\left\{-c_m \rho \left(\frac{P_t a_\nu}{P_0 D}\right)^{m/\nu}\right\} = \exp\left\{-\bar{N}_{\max} D^{-m/\nu}\right\},$$

$$f_d(D) = \frac{m}{\nu} \bar{N}_{\max} D^{-m/\nu - 1} \exp\left\{-\bar{N}_{\max} D^{-m/\nu}\right\} \tag{8}$$

where $c_1 = 2$, $c_2 = \pi$ and $c_3 = 4\pi/3$, $\bar{N}_{\max} = c_m R_{\max}^m \rho$ is the average number of transmitters in the ball of radius $R_{\max}$, which we term "potential interference zone", and $R_{\max}$ is such that $P_a(R_{\max}) = P_0$, i.e. a transmitter at the boundary of the potential interference zone produces signal at the receiver exactly at the noise level; transmitters located outside of this zone produce weaker signals, which are neglected in the analysis. Note that (8) gives the distribution of the INR as a simple explicit function of the system and geometrical parameters, and ultimately depends on $\bar{N}_{\max}, m, \nu$ only.

When $(k-1)$ most powerful signals, which are coming from $(k-1)$ closest transmitters, do not create any interference (i.e. due to frequency, time or code separation in the multiple access scheme, or due to any other form of separation or filtering), the CDF and PDF of the distance r_k to the most powerful interfering signal of order k can be found in a similar way. The CDF of the INR d_a in this case is given by

$$F_{dk}(D) = e^{-\bar{N}(D)} \sum_{i=0}^{k-1} \bar{N}(D)^i / i! \tag{9}$$

In the case of constant average density $\rho = const$, the CDF and PDF of the INR simplify to [8]-[10],

$$F_{dk}(D) = \exp\left\{-\bar{N}_{\max} D^{-m/\nu}\right\} \sum_{i=0}^{k-1} \frac{1}{i!} \left(\frac{\bar{N}_{\max}}{D^{m/\nu}}\right)^i,$$

$$f_{dk}(D) = \frac{m}{\nu} \frac{\bar{N}_{\max}^k}{(k-1)!} D^{-\frac{km}{\nu} - 1} \exp\left\{-\bar{N}_{\max} D^{-m/\nu}\right\} \tag{10}$$

which are also simple, explicit functions of $\bar{N}_{\max}, m, \nu$.

IV. Outage Probability-Node Density Tradeoff

Powerful interfering signals can result in significant performance degradation due to linear and nonlinear distortion effects in the receiver when they exceed certain limit, which we characterize here via the receiver distortion-free dynamic range (i.e. the maximum acceptable interference-to-nose ratio) $D_{df} = P_{\max} / P_0$, where $P_{\max}$ is the maximum interfering signal power at the receiver that does not cause significant performance degradation. If $d_a > D_{df}$, there is significant performance degradation and the receiver is considered to be in outage, which corresponds to one or more transmitters falling into the active interference zone (i.e. the ball of radius $r(D_{df})$; the signal power coming from transmitters at that zone exceeds $P_{\max}$), whose probability is

$$\mathcal{P}_{out} = \Pr\{d_a > D_{df}\} = 1 - F_d(D_{df}) \tag{11}$$

For given $\mathcal{P}_{out}$, one can find the required distortion-free dynamic range ("outage dynamic range") D_{df}

$$D_{df} = F_d^{-1}(1 - \mathcal{P}_{out}) \tag{12}$$

We note that, in general, D_{df} is a decreasing function of $\mathcal{P}_{out}$, i.e. low outage probability calls for high distortion-free dynamic range. For simplicity of notations, we further drop the subscript and denote the spurious-free dynamic range by D.

<u>*All interfering signals are active (k=1)*</u>: We consider first the case of $k = 1$, i.e. all interfering signals are active. The outage probability can be evaluated using (6) and (11). From practical perspective, we are interested in the range of small outage probabilities $\mathcal{P}_{out} << 1$, i.e. high-reliability communications. When this is the case, $F_d(D) \to 1$ and using MacLaurean series expansion $e^{-\bar{N}} \approx 1 - \bar{N}$, (11) simplifies to

$$\mathcal{P}_{out} \approx \bar{N} = \int_{V(r(D))} \rho dV \tag{13}$$

[2] It can be shown that, in the small outage region, the total interference power (i.e. coming from all transmitters) is dominated by the contribution of the most powerful signal, i.e. the single events dominate the outage probability.

which further simplifies, in the case of $\rho = const$, to

$$\mathcal{P}_{out} \approx \overline{N}_{\max} D^{-m/\nu} \quad (14)$$

Note that, in this case, the outage probability $\mathcal{P}_{out}$ scales linearly with the average number $\overline{N}_{\max}$ of nodes in the potential interference zone, and it effectively behaves as if the number of nodes were fixed (not random) and equal to $\overline{N}_{\max}$. Based on this, we conclude that the single-order events (i.e. when only one signal in the ensemble of interfering signals exceeds the threshold $P_{\max}$) are dominant contributor to the outage. This immediately suggests a way to reduce significantly the outage probability by eliminating (e.g. by filtering) the dominant interferer in the ensemble. Using (14), the required spurious-free dynamic range of the receiver can be found for given outage probability, $D \approx (\overline{N}_{\max} / \mathcal{P}_{out})^{\nu/m}$. Note that higher values of ν and lower values for m call for higher dynamic range. Intuitively, this can be explained by the fact that when the transmitter moves from the boundary of the potential interference zone (i.e. $R = R_{\max}$, $P_a(R) = P_0$) closer to the receiver ($R << R_{\max}$), the power grows much faster when ν is larger, so that closely-located transmitters produce much more interference (compared to those located close to the boundary) when ν is large, which, combined with the uniform spatial density of the transmitters, explains the observed behavior. The effect of m can be explained in a similar way.

To validate the accuracy of approximation in (13), and also the expressions for the dynamic range PDF and CDF in the previous section, extensive Monte-Carlo (MC) simulations have been undertaken. Fig. 1 shows some of the representative results. Note good agreement between the analytical results (including the approximations) and the MC simulations. It can be also observed that the tails of the distributions decay much slower for the $\nu = 4$ case, which indicates higher probability of high-power interference in that case and, consequently, requires higher spurious-free dynamic range of the receiver, in complete agreement with the predictions of the analysis. Note also that the outage probability evaluated via the total interference power coincides with that evaluated via the maximum interferer power, at the small outage region.

Consider now a scenario where the actual outage probability has not to exceed a given value $\mathcal{P}_{out}$ for the receiver with a given distortion-free dynamic range D. Using (8) and (11), the average number of transmitters in the active interference zone (ball of radius $r(D)$) can be upper bounded as $\overline{N} \le -\ln(1-\mathcal{P}_{out})$. Using the expression for $\overline{N}$, one obtains a basic tradeoff relationship between the network density and the outage probability,

$$\overline{N} = \int_{V(r(D))} \rho dV \le -\ln(1-\mathcal{P}_{out}) \quad (15)$$

i.e. for given outage probability, the network density is upper bounded or, equivalently, for given network density, the outage probability is lower bounded.

In the case of uniform density $\rho = const$ and small outage probability, $\mathcal{P}_{out} << 1$, this gives an explicit tradeoff relationship between the maximum distortion-free interference power at the receiver $P_{\max}$, the transmitter power P_t and the average node density for distortion-free receiver operation,

$$\rho \le c_m^{-1} \mathcal{P}_{out} \left(P_{\max} / P_t a_\nu\right)^{m/\nu} \quad (16)$$

or, equivalently, an upper bound on the average density of nodes in the network. As intuitively expected, higher $\mathcal{P}_{out}, P_{\max}, \nu$ and lower P_t, m allow for higher network density. The effect of ν is intuitively explained by the fact that higher ν results in larger path loss or, equivalently, in smaller distance at the same path loss, so that the transmitters can be located more densely without increasing interference level. The effect of the other parameters can be explained in a similar way.

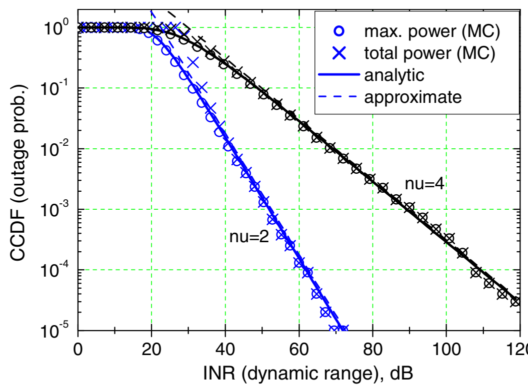

Fig. 1. The CCDF of $d_a = P_{a1} / P_0$ and $d_{tot} = P_{tot} / P_0$ (also the outage probability) evaluated from Monte-Carlo (MC) simulations for $m = 2$, $\nu = 2 \,\&\, 4$, $P_0 = 10^{-10}, P_t = 1, \rho = 10^{-5}$; analytic CCDF of d_a (derived from (8)) and its approximation in (14) are also shown.

$(k-1)$ *strongest interfering signals are inactive*: We now assume that $(k-1)$ strongest interfering signals are eliminated via some means (e.g. by filtering or resource allocation). In this case, (9), (10) apply and (13) generalizes to

$$\mathcal{P}_{out} \approx \tfrac{1}{k!}\overline{N}^k = \tfrac{1}{k!}\left(\overline{N}_{\max} D^{-m/\nu}\right)^k \quad (17)$$

which can be expressed as $\mathcal{P}_{out} = \frac{1}{k!}\mathcal{P}_{out,1}^k \le \mathcal{P}_{out,1}$, where $\mathcal{P}_{out,1}$ is the outage probability for $k=1$ (see (13)). In the small outage region, $\mathcal{P}_{out,1} << 1$ and $\mathcal{P}_{out} << \mathcal{P}_{out,1}$, i.e. there is a significant beneficial effect of removing $(k-1)$ strongest interferers, which scales exponentially with k. Further comparison to the corresponding result in [6] shows that the assumption there of cancelling *all* interferers, which exceed the required signal and are in the disk with the given average number of interferers, affects significantly the result (no exponential scaling). It should also be noted that, contrary to the $k=1$ case, $\mathcal{P}_{out}$ in (17) is super-linear in $\overline{N}_{\max}$: doubling $\overline{N}_{\max}$ increases $\mathcal{P}_{out}$ by the factor $2^k > 2$, i.e. $\mathcal{P}_{out}$ is more sensitive to $\overline{N}_{\max}$ in this case.

In a similar way, the node density-outage probability tradeoff can be formulated. In the for small outage probability region $\mathcal{P}_{out} << 1$, it can be expressed as

$$\overline{N} = \int_{V(r(D))} \rho dV \le \left(k!\mathcal{P}_{out}\right)^{1/k} \quad (18)$$

Comparing (18) to (15), one can clearly see the beneficial effect of "removing" $(k-1)$ most powerful interferers on the outage probability-network density tradeoff, since $\left(k!\mathcal{P}_{out}\right)^{1/k} >> \mathcal{P}_{out}$ in the small outage regime, so that higher node density is allowed at the same outage probability.

In the case of uniform density, (18) reduces to

$$\rho \le c_m^{-1}\left(k!\mathcal{P}_{out}\right)^{1/k}\left(P_{\max}/P_t a_\nu\right)^{m/\nu} \quad (19)$$

which is a generalization of (16) to $k \ge 1$.

Impact of Rayleigh and log-normal fading: Following the same approach as in [4], it can be shown that the impact of Rayleigh and log-normal fading on the distributions above is a shift by a constant factor. In the case of Rayleigh fading, the constant is close to 1 and, thus, can be neglected so that the distributions are roughly not affected. In the case of log-normal fading, the constant is not negligible. The intuition behind this result is that the distributions in (11), (14), (17) are much more heavily-tailed (slowly-decaying) than the Rayleigh distribution so that outage events in the combined distribution are mostly caused by nearby interferers without deep Rayleigh fades and the combined distribution is roughly the same as the original one (without fading). On the other hand, the log-normal distribution is also heavily-tailed, so it cannot be neglected.

V. THE IMPACT OF LINEAR FILTERING

In the previous sections, we considered the interfering signals at the Rx input assuming that the Rx antenna was isotropic, i.e. no measures to eliminate some of the interfering signals e.g by linear filtering at the Rx antenna, its frequency filters etc. were considered. In this section, we explore the effect of linear filtering, which may include filtering by the Rx antenna based on the angle of arrival, polarization and frequency, and by linear frequency filters at the receiver (at RF, IF and possibly basedband). Since, as it follows from the previous section, the average number of interfering signals $\overline{N}$ is a key parameter, which determines the dynamic range of interfering signals (see (6),(9)) and ultimately the network density-outage probability tradeoff (e.g. (15), (18)), we consider the impact of linear filtering on this parameter.

Let $\mathbf{z} = [z_1, z_2 ... z_l]^T$ be the set of filtering variables (i.e. frequency, polarization, angle of arrival etc.) and $f_z(\mathbf{z})$ be the PDF of incoming signals over these variables. The probability of a randomly-chosen input signal (arriving from a randomly-selected node) falling in the interval $d\mathbf{z}$ is $f_z(\mathbf{z})d\mathbf{z}$, and the probability that the filter output power of this signal exceeds the threshold P_0 is

$$\Pr\left\{P_{a,out} > P_0\right\} = \int_{P_0/K(\mathbf{z})}^{\infty} w_a(P)dP = K^{m/\nu}(\mathbf{z}) \quad (20)$$

where $0 \le K(\mathbf{z}) \le 1$ is the normalized filter power gain (e.g. antenna pattern), and $w_a(P) = \frac{m}{\nu} P_0^{m/\nu} P^{-1-m/\nu}$, $P \ge P_0$, is the PDF of the signal's power P. Note that $K^{m/\nu}$ represents the reduction in probability of signal power exceeding the threshold from the input (where it is equal to one) to the output of the filter and thus is a filter gain for given filtering variables. The average number of output signals exceeding the threshold in the interval $d\mathbf{z}$ is $d\overline{N}_{out} = K^{m/\nu}(\mathbf{z}) f_z(\mathbf{z}) d\mathbf{z} d\overline{N}_{in}$, where $d\overline{N}_{in}$ is the average number of input signals exceeding the threshold in the same interval. Finally, the total average number of output signals exceeding the threshold P_0 is

$$\overline{N}_{out} = \overline{N}_{in}/Q, \quad Q = \left(\int_{\Delta\mathbf{z}} K^{m/\nu}(\mathbf{z}) f_z(\mathbf{z}) d\mathbf{z}\right)^{-1} \ge 1 \quad (21)$$

where $\overline{N}_{in}$ is the average number of input signals, Q is the average statistical filter gain, which represents its ability to reduce the average number of visible (i.e. exceeding the threshold) interfering signals, and $\Delta\mathbf{z}$ is the range of filtering variables. This gain further transforms into reduction in the interfering signals' dynamic range (see (6), (9)) or in the outage probability,

$$\mathcal{P}_{out} = 1 - e^{-\overline{N}_{out}} \sum_{i=0}^{k-1} \frac{\overline{N}_{out}^i}{i!} \approx \frac{\overline{N}_{out}^k}{k!} = \frac{1}{k!}\left(\frac{\overline{N}_{in}}{Q}\right)^k \quad (22)$$

and also improves the network density-outage probability tradeoff (i.e. (18), (19)),

$$\overline{N_{in}} = \int_{V(r(D))} \rho dV \le Q\left(k!\mathcal{P}_{out}\right)^{1/k} \quad (23)$$

$$\rho \le Q c_m^{-1}\left(k!\mathcal{P}_{out}\right)^{1/k}\left(P_{\max}/P_t a_\nu\right)^{m/\nu} \quad (24)$$

i.e. the network density ρ can be increased by a factor of Q at the same performance compared to the case of no filtering. Clearly, using directional antennas with highly-directive pattern, for example, results in large Q (similarly to the antenna's gain) and thus the network density can be increased by a large factor Q, as expected intuitively.

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